Ya. I. Perelman

In 1913 in Russian bookshops appeared a book by the outstanding educationalist Yakov Isidorovich Perelman entitled *Physics for Entertainment*. It struck the fancy of the young who found in it the answers to many of the questions that interested them.

Physics for Entertainment not only had an interesting layout, it was also immensely instructive.
In the preface to the 11th Russian edition Perelman wrote: "The main objective of *Physics for Entertainment* is to arouse the activity of scientific imagination, to teach the reader to think in the spirit of the science of physics and to create in his mind a wide variety of associations of physical knowledge with the widely differing facts of life, with all that he normally comes in contact with."

Physics for Entertainment was a best seller.
Ya. I. Perelman was born in 1882 in the town of Byelostok (now in Poland). In 1909 he obtained a diploma of forester from the St. Petersburg Forestry Institute. After the success of *Physics for Entertainment* Perelman set out to produce other books, in which he showed himself to be an imaginative popularizer of science. Especially popular were *Arithmetic for Entertainment*, *Mechanics for Entertainment*, *Geometry for Entertainment*, *Astronomy for Entertainment*, *Lively Mathematics*, *Physics Everywhere*, and *Tricks and Amusements*. Today these books are known to every educated person in the Soviet Union.
He has also written several books on interplanetary travel (*Interplanetary Journeys*, *On a Rocket to Stars*, *World Expanses*, etc.).
The great scientist K. E. Tsiolkovsky thought highly of the talent and creative genius of Perelman. He wrote of him in the preface to *Interplanetary Journeys*: "The author has long been known by his popular, witty and quite scientific works on physics, astronomy and mathematics, which are moreover written in a marvelous language and are very readable."
Perelman has also authored a number of textbooks and articles in Soviet popular science magazines.
In addition to his educational, scientific and literary activities, he has also devoted much time to editing. So he was the editor of the magazines *Nature and People* and *In the Workshop of Nature*.
Perelman died on March 16, 1942, in Leningrad.
Many generations of readers have enjoyed Perelman's fascinating books, and they will undoubtedly be of interest for generations to come.

Ya. I. Perelman

Fun with Maths and Physics

Brain Teasers

Tricks

Illusions

MIR PUBLISHERS
MOSCOW

Я. И. Перельман

ЗАНИМАТЕЛЬНЫЕ ЗАДАЧИ И ОПЫТЫ

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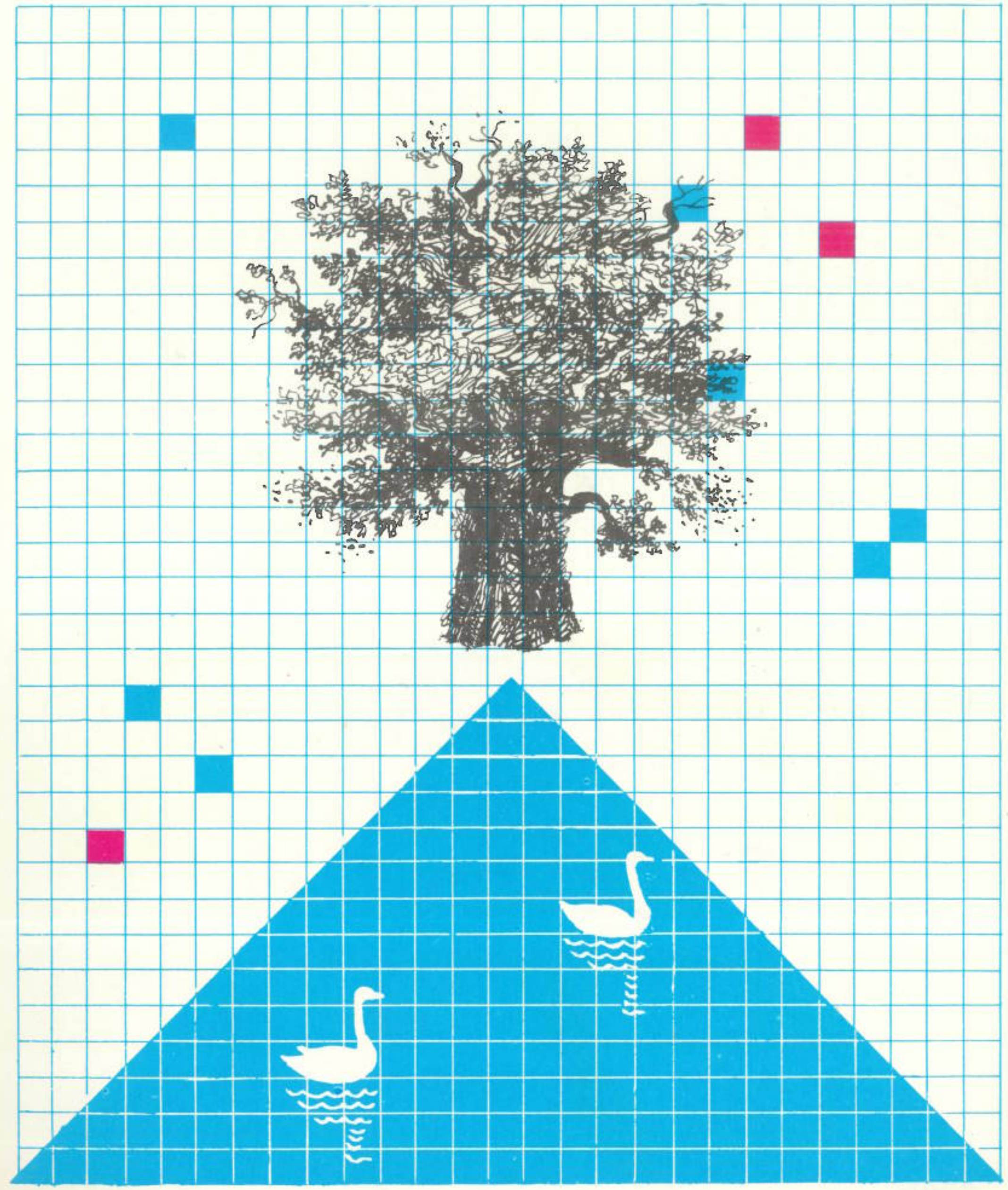

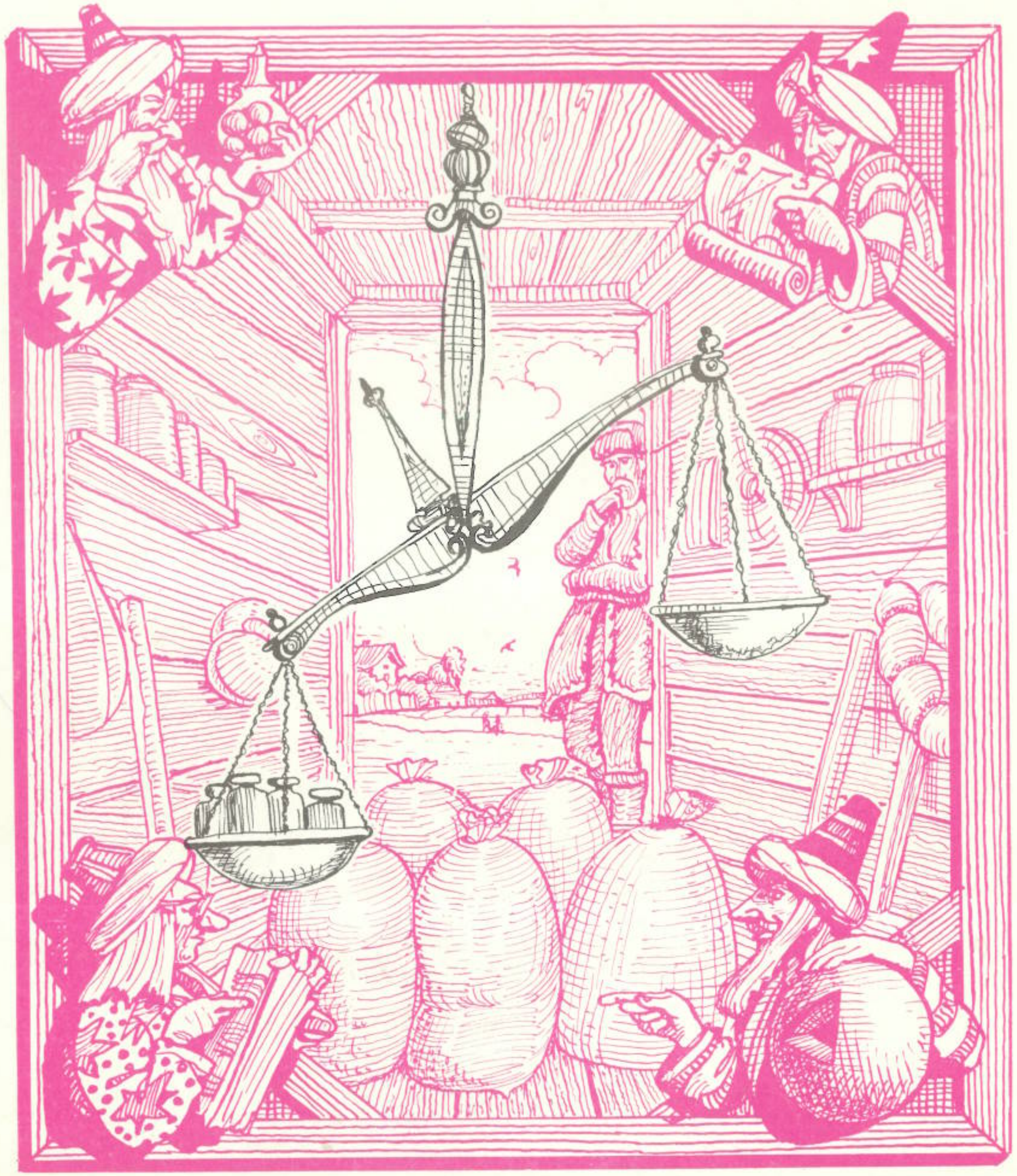

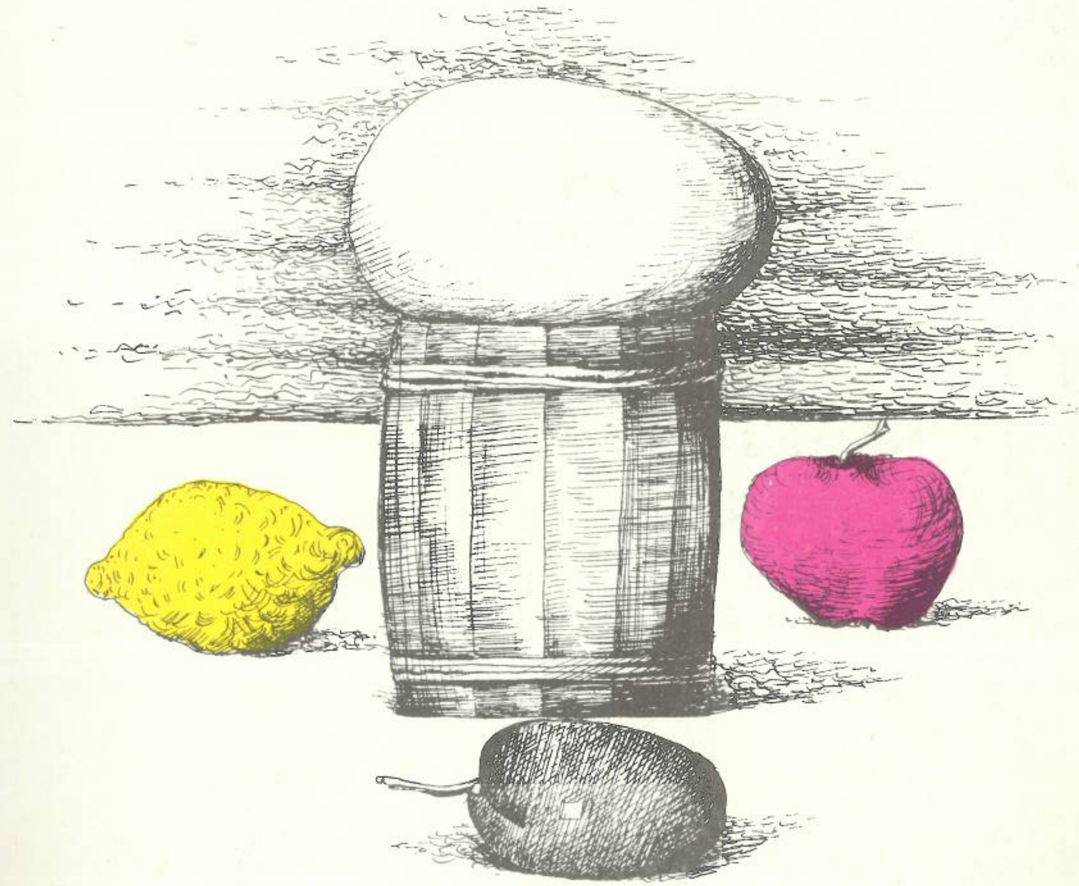

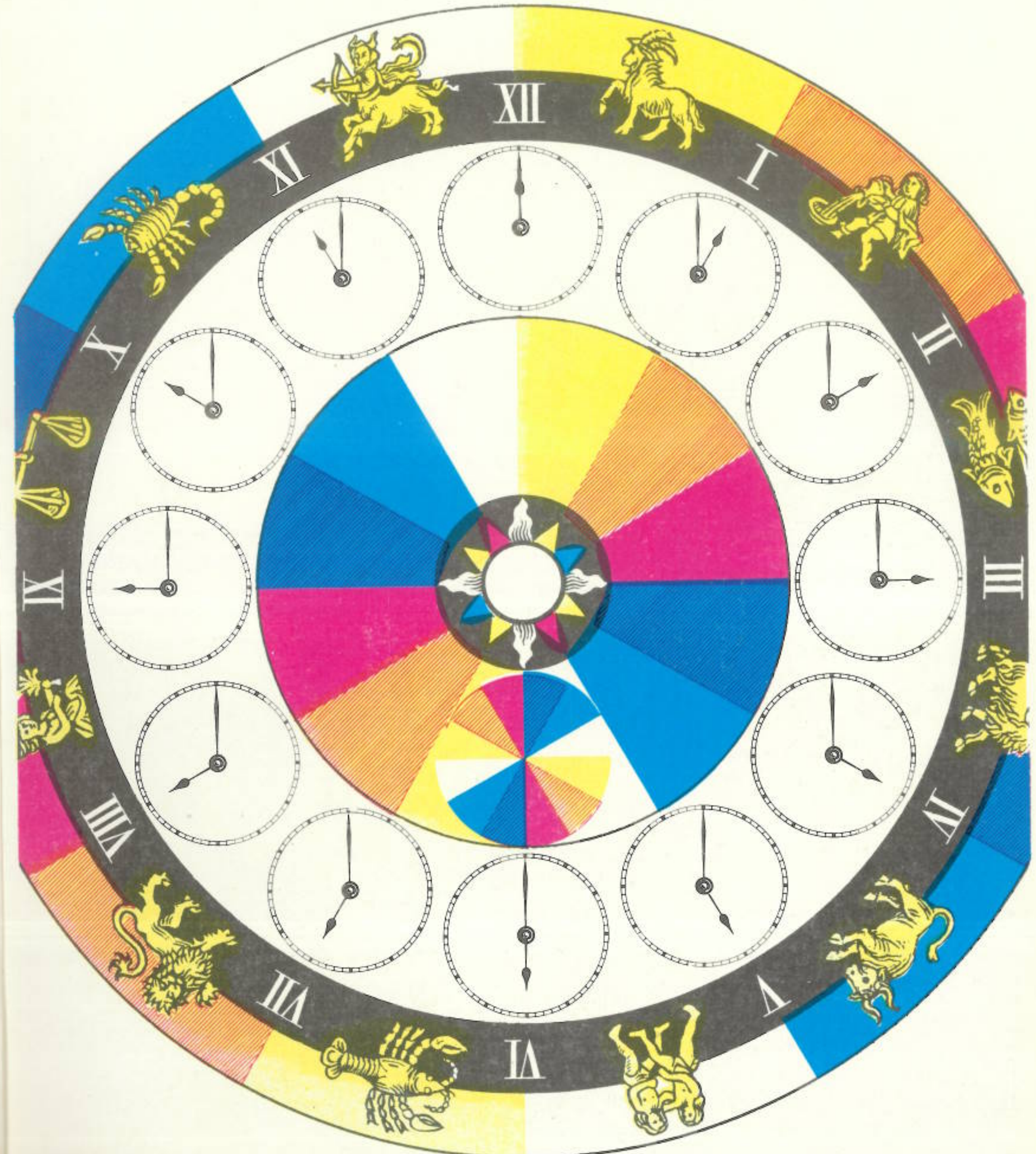

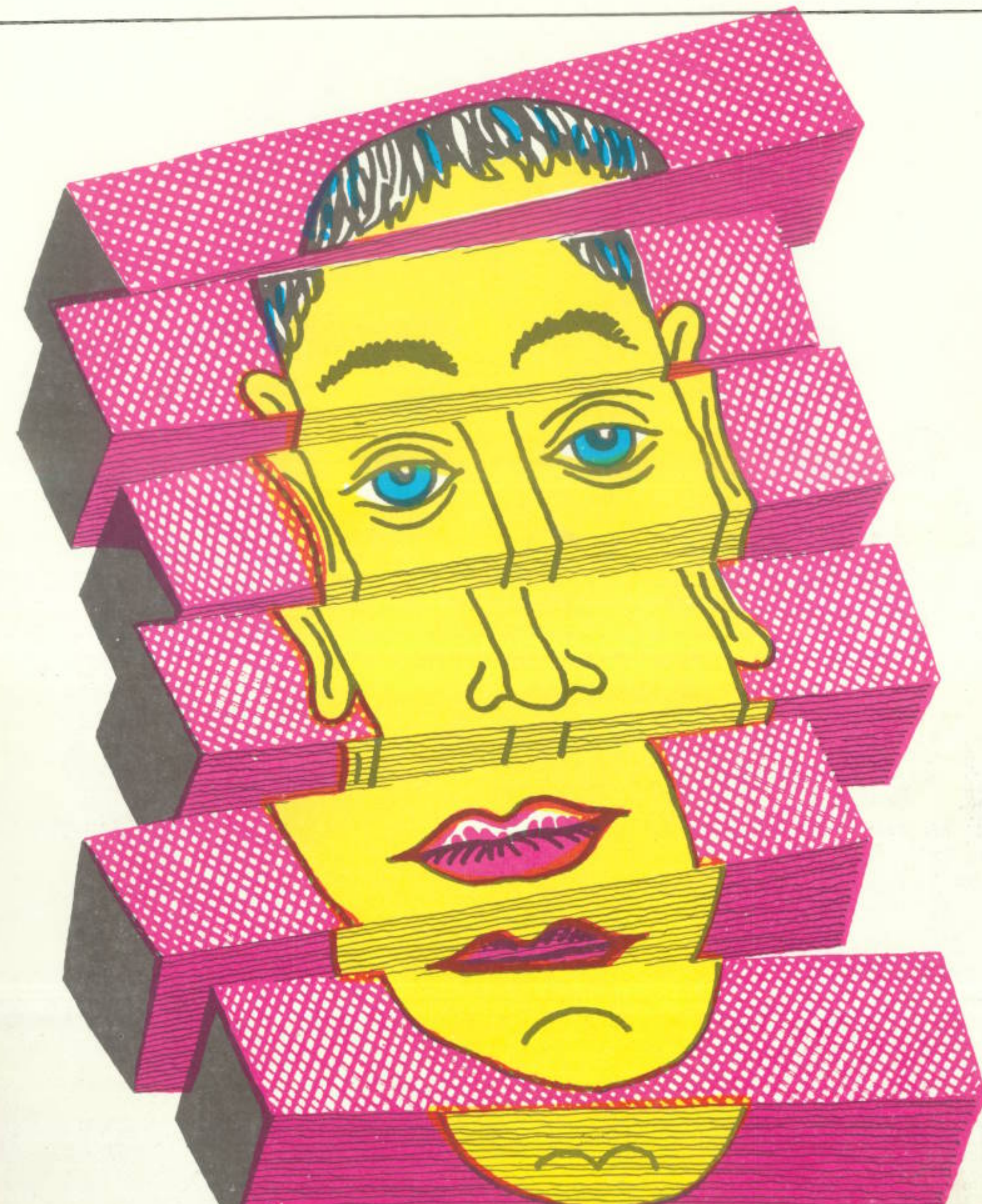

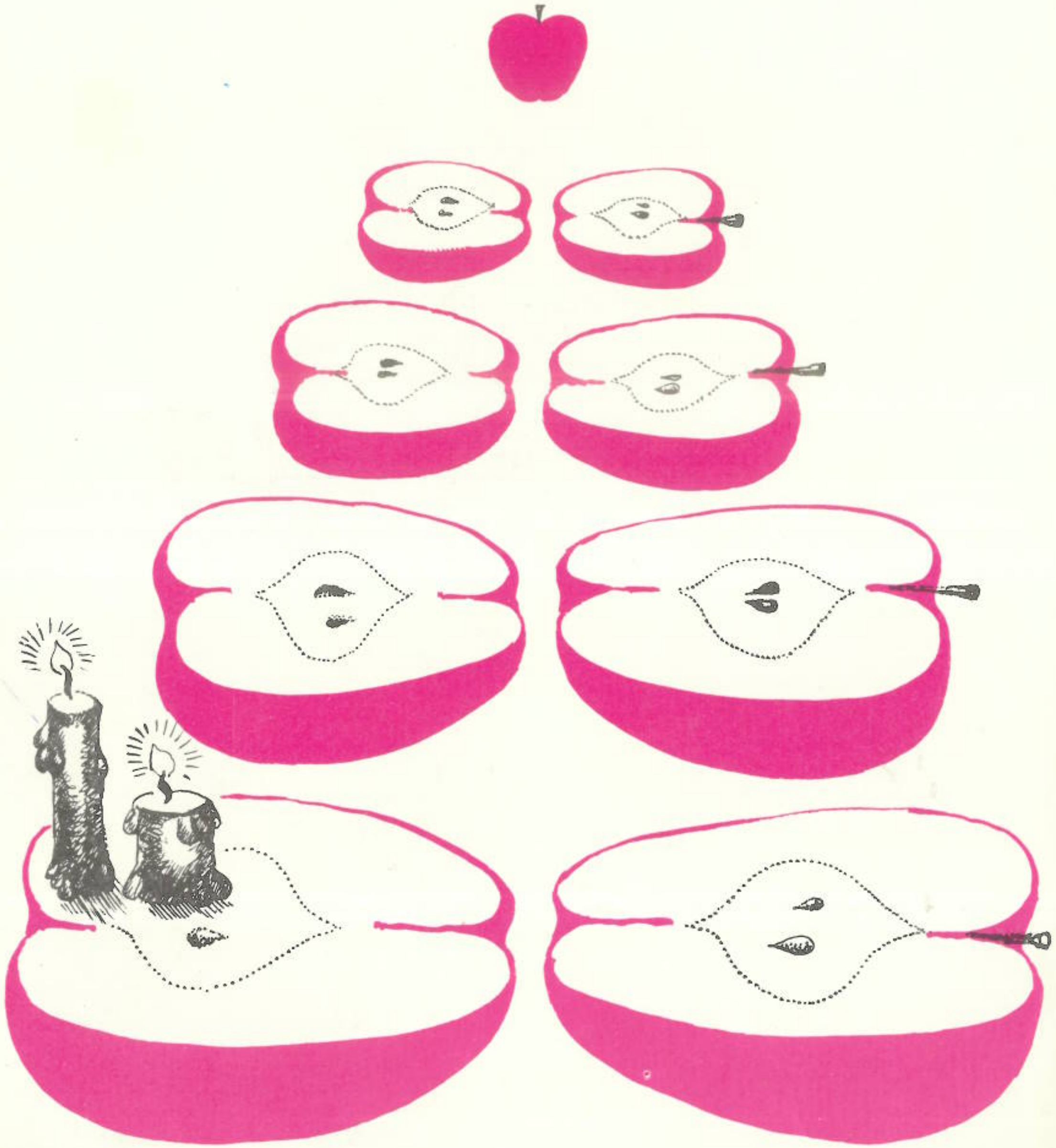

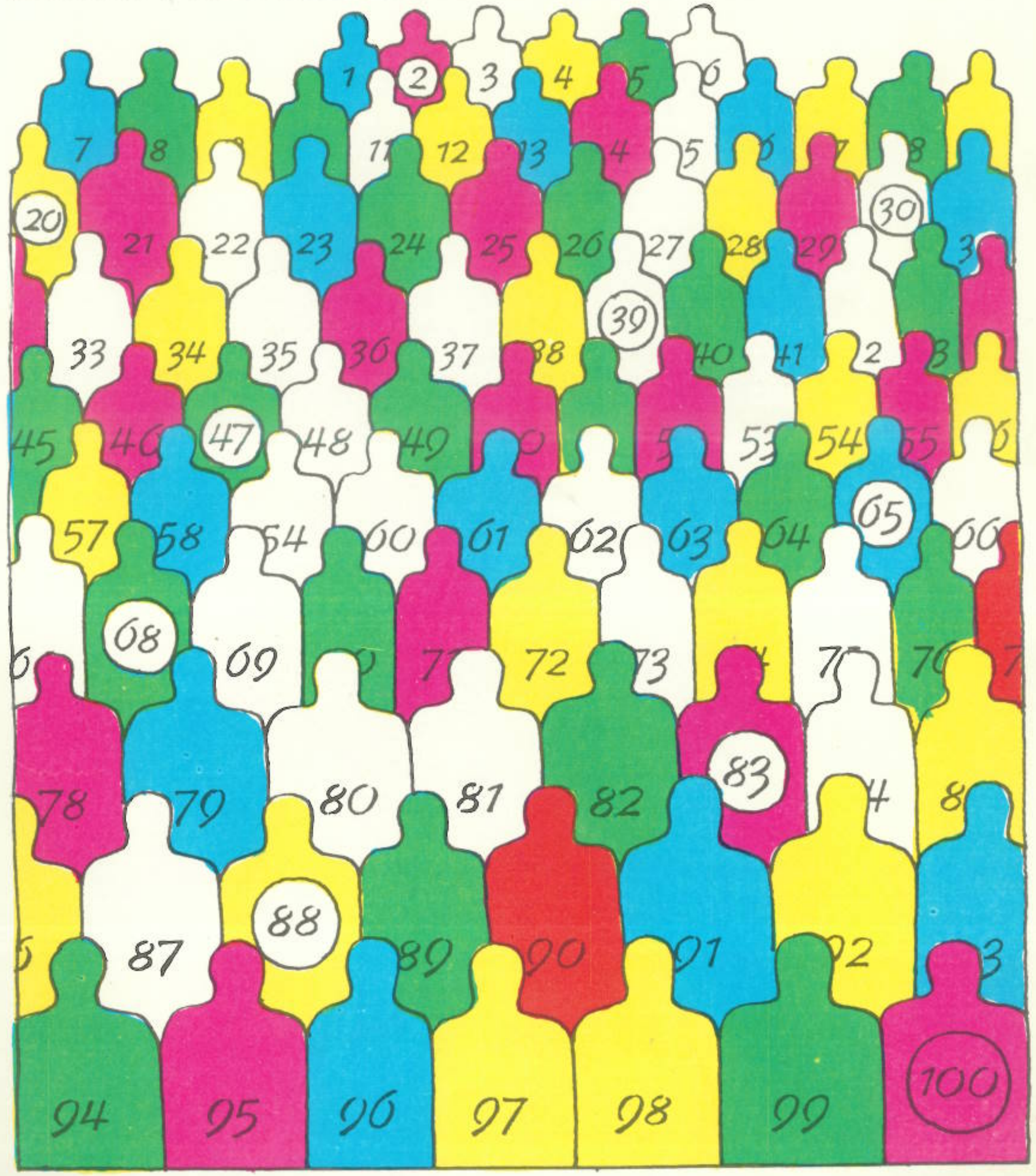

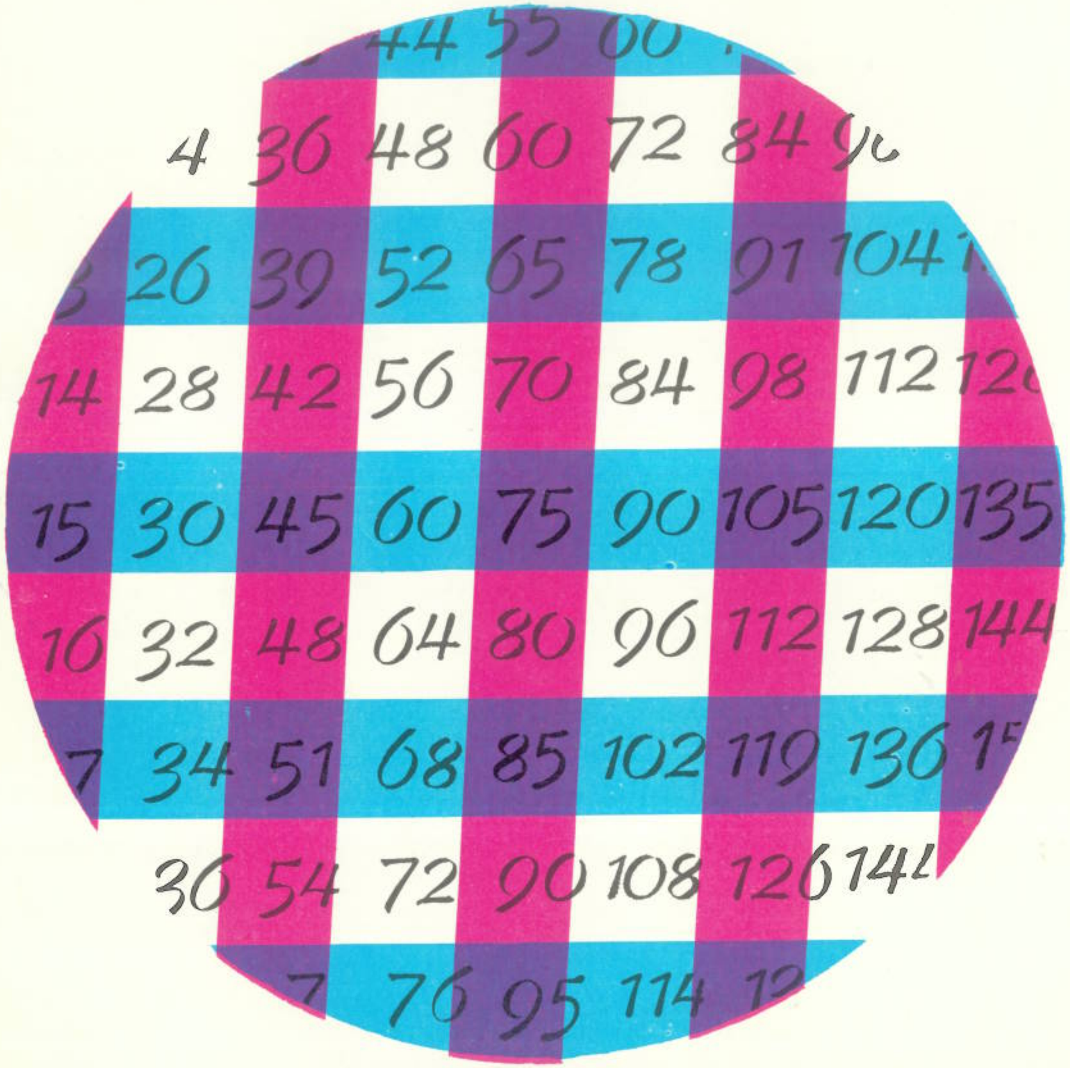
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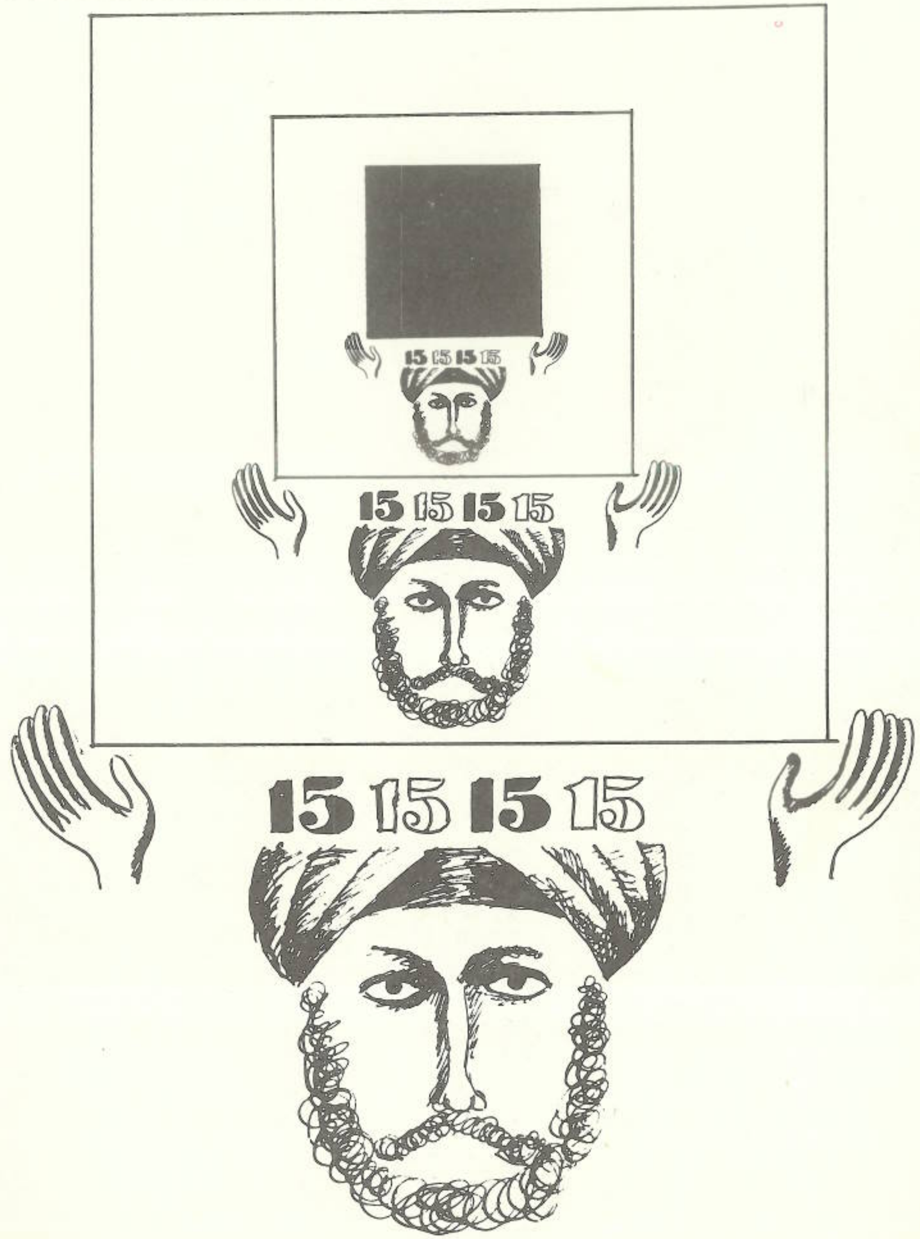
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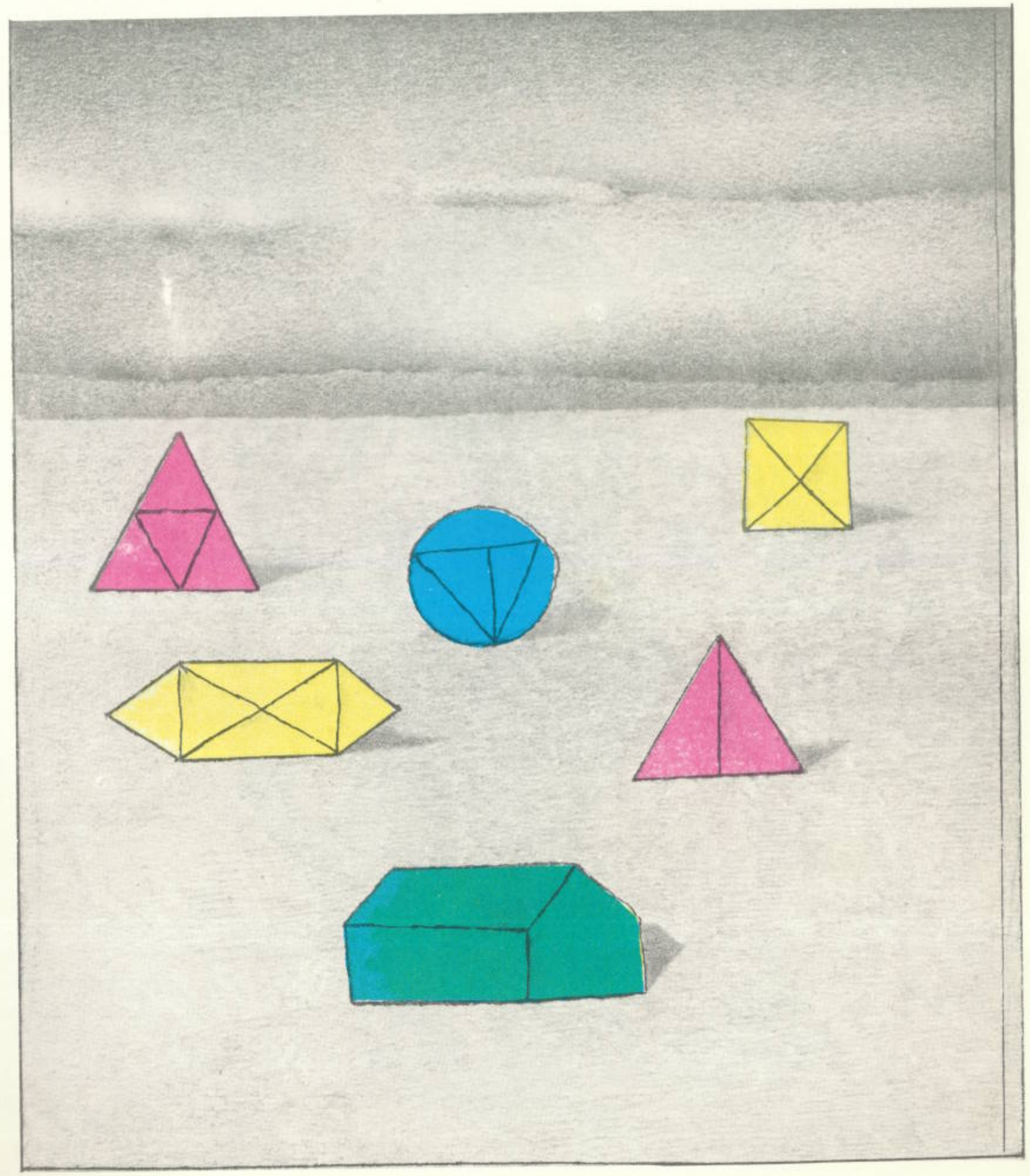


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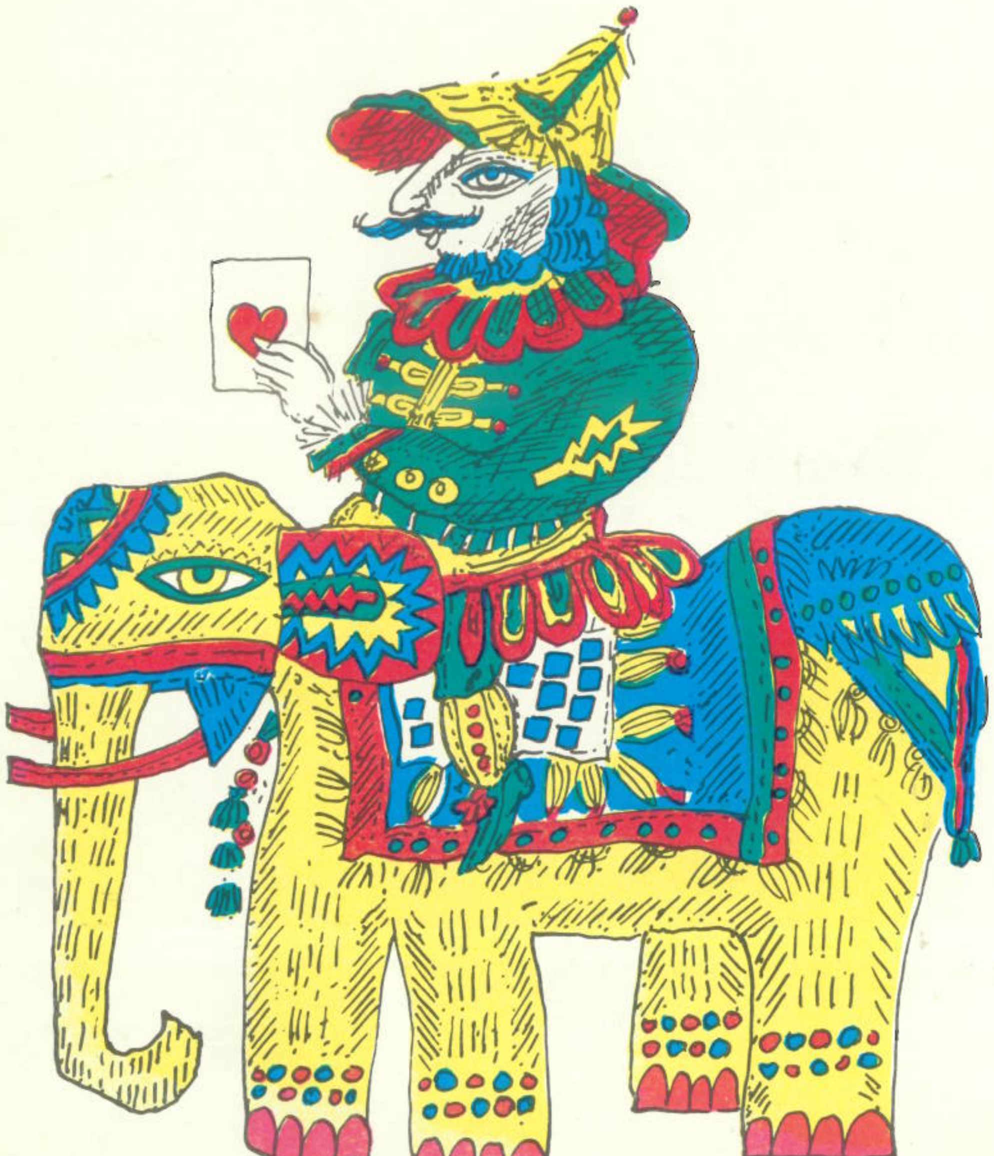


By the Way

Scissors and Paper

1

Three pieces from one cut ● Placing a strip on an edge ● Charmed rings ● Unexpected results of cutting ● Paper chain ● Thread yourself through a sheet of paper.

Perhaps you think, as I once did, that there are some unnecessary things in this world. You're quite mistaken: there is no junk that might not be of help sometime for some purpose. What is useless for one purpose, comes in handy for another, and what is useless for business might be suitable for leisure.

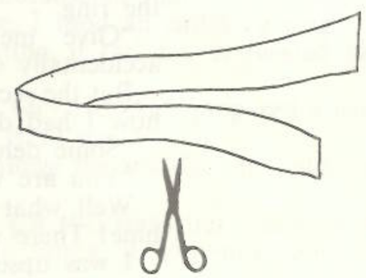
In the corner of a room being repaired I once came across several used postcards and a heap of narrow paper strips that had been trimmed from wall paper. "Rubbish for the fire," I thought. But it turned out that even with this junk one can interestingly amuse oneself. My elder brother Alex showed me some things you could do with them.

He started with the paper strips. Giving me a piece of strip about 30 centimetres long, he said: "Take a pair of scissors and cut the strip in three..."

I was about to cut but Alex stopped me: "Wait a bit, I haven't finished yet. Cut it into three with one cut of the scissors."

This was more difficult. I tried one way and then another, and then began to think my brother had posed a virtually impossible problem. Eventually it occurred to me that it was absolutely intractable.

Figure 1



"You're pulling my leg," I said, "It's impossible."

"Well, think again, maybe you'll work it out."

"I have worked it out that the problem has no solution."

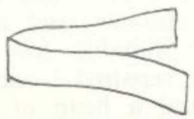
"Too bad. Let me."

Brother took the strip, folded it in two and cut it in the middle to produce three pieces.

"You see?"

"Yes, but you've folded the strip."
"Why didn't you?"
"You didn't say I could."
"But I didn't say you couldn't either. Simply agree that you didn't see the solution."
"All right. Give me another problem. You won't catch me out again."

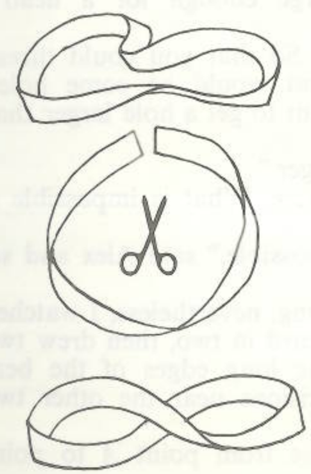
Figure 2



"Here is another strip, put it on its edge."
"So that it stands or falls?" I asked suspecting a trap.
"Standing, of course. If it falls, it will mean that it was laid, not stood on edge."
"So that it stands... on its edge," I muzed, and it suddenly occurred to me that I could bend the strip. I did so and put it on the table.
"There, standing on its edge! You didn't say I couldn't bend it!" I said triumphantly.
"Right."
"More of your problems, please."
"As you like. You see I've glued the ends of several strips and produced paper rings. Take a red-and-blue pencil and draw a blue line all along the outside of this ring and a red along the inside."
"And then?"
"That's all."
A silly job, but somehow it didn't quite come off. When I had joined up the ends of the blue line and wanted to do the red, I found to my surprise I had absent-mindedly drawn the blue line on both sides of the ring.
"Give me another," I was embarrassed, "I've accidentally spoiled the first one."
But the second was a failure, too. I even didn't notice how I had drawn the both sides.
"Some delusion! Again. Give me another."
"You are welcome."
Well, what do you think? Again, both sides appeared blue! There was no room for the red.
I was upset.
"Such a simple thing and you can't do it," smiled brother. "Just look."
He took a paper ring and swiftly drew a red line all round the outside and a blue one on the inside.
Having received a fresh ring, I started as carefully as possible to draw the line along one side, trying very hard not to go over to the other side somehow, and... joined up the line. Dear me! Both sides again. About to weep, I in bewilderment glanced at my brother, and

only then I guessed from his grin that something was wrong.
"Well, you just... Is it a trick?" I asked.
"The rings are magic."
"What magic? Just rings. You've just fixed up something."
"Try to make something else with these rings. For example, can you cut this ring to get two thinner ones?"
"Nothing special."
Having cut the ring, I was about to demonstrate two thin rings I had got when I noticed, much to my surprise, that I had in my hands only one long ring, not two smaller ones.
"Okay, where are your two rings?" Alex asked mockingly.
"Another ring, I'll try again."
"Why? Just cut the one you've got."
I did. This time I had two rings, no kidding. But when I wanted to separate them, it turned out that it was impossible to disentangle them for they were linked together. Brother was right, the ring was enchanted all right!
"The trick is very simple," my brother explained, "You can make such unusual rings for yourself. The key thing is that before you glue the ends of the paper strip twist one of them like this (Figure 3)."
"Is it all because of that?"
"Yes! Sure, I used an ordinary ring... It'll be even more interesting, if the end is twisted twice, not just once."

Figure 3



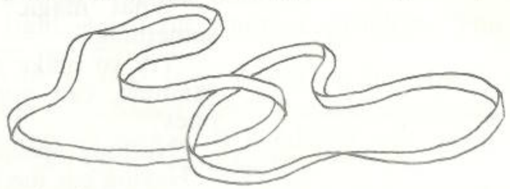
Before my very eyes Alex prepared a ring in this way and handed it to me.
"Cutting along the middle," he said, "and see what happens."
I did and got two rings but one now went through the other. So funny, it was impossible to take them apart.
I prepared three more rings for myself and obtained three more pairs of inseparable rings.
"What would you do," my brother asked again, "if you had to connect all four pairs of rings to form one long open-ended chain?"
"Oh, this is simple: cut one ring in each pair, and glue them together again."
Alex enquired, "So, you would cut three of the rings?"

"Of course."

"But what if you cut less than three?"

"We have four pairs of rings, how can you possibly connect them by only breaking two rings? Impossible!"

Figure 4



I was dead sure.

In answer, my brother took the scissors, cut both rings in one pair and with them connected the remaining three pairs. Lo and behold! a chain of eight rings. Ridiculously simple! No trick in this and I could only be surprised why such a simple idea hadn't occurred to me.

"Enough of these paper rings. You've got some old postcards over there, it seems. Let's have some fun with them, too. For instance, try and cut in a card the largest hole you can."

I punched the card with my scissors and carefully cut a rectangular hole in it, leaving only a narrow edge.

"This is a hole among holes! A larger one is impossible!" I contentedly showed the result of my job to Alex.

Of course, he had another opinion.

"The hole is too small. Just enough for a hand to go through."

"You'd like it to be large enough for a head?" I retorted acidly.

"The head and the body. So that you could thread all yourself through it. That would be some hole."

"Ha-ha! Do you really want to get a hole larger than the paper itself?"

"Exactly. Many times larger."

"No trick will help you here. What is impossible is impossible..."

"And what is possible is possible," said Alex and set out to cut.

Confident that he was joking, nevertheless, I watched him curiously. He bent the card in two, then drew two lines with a pencil near the long edges of the bent postcard and made two incisions near the other two edges.

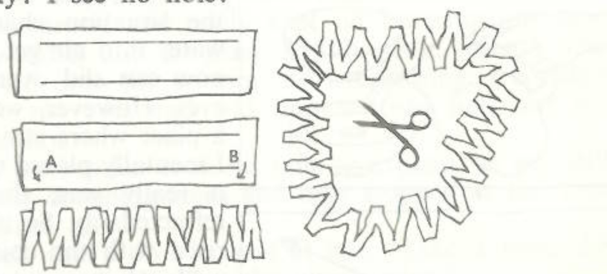
Next he cut the bent edge from point *A* to point

B and began to make a lot of cuts next to each other as shown in Figure 5.

"Finished," proclaimed my brother.

"Why? I see no hole!"

Figure 5



"Take another look."

And Alex expanded the paper. Just imagine: it developed into a long-long chain that he easily threw over my head. It fell to my feet, zigzagging about me.

"How can you get through such a hole? What do you say to that?"

"Big enough for two!" I said with admiration.

At that Alex finished his tricks, promising to treat me next time to a whole heap of new ones, this time only with coins.

Tricks with Coins

A visible and invisible coin ● A bottomless glass ● Where has the coin gone? ● Arranging coins ● Which hand holds the two-pence piece? ● Shifting coins ● An Indian legend ● Problem solutions.

"Yesterday you promised to show me a trick with coins," I reminded my brother at breakfast.

"Tricks? First thing in the morning? Hm, all right. Then empty the washing bowl."

Alex put a silver coin on the bottom of the empty bowl.

"Look into the bowl without moving from your place and without leaning over. See the coin?"

"Yes."

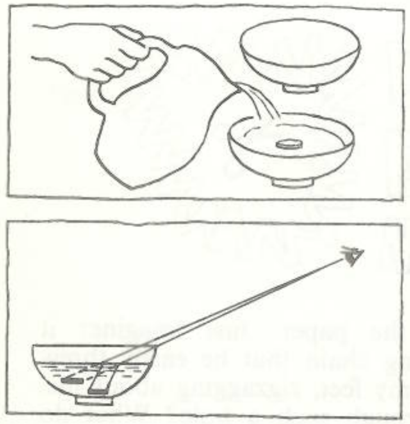
Alex pushed the bowl a bit farther away from me until I couldn't see the coin any more since it was shielded by the side of the bowl.

"Sit still, don't move. I pour water into the bowl. What has happened to the coin?"

"It's visible again, as if it's been lifted up together with the bottom. Why?"

My brother sketched the bowl with the coin in it on a sheet of paper, and then everything became clear to

Figure 6



me. While the coin was at the bottom of the dry bowl, no ray of light could come from the coin because light travels in straight lines and the opaque sides of the bowl were just in the way. When the water was added, the situation changed since light rays coming from water into air get bent (physicists say "refracted") and now can slid over the bowl edge and come into my eyes. However, we are used to seeing things only at a place where straight rays come from and this is why I mentally placed the coin somewhat higher than where it really was, that is along a continuation of the refracted ray. So it seemed to me that the bottom had risen with the coin.

"I advise you to remember this experiment," my brother added. "It will be useful when bathing. In a shallow place where you can see the bottom never forget that you see it higher than really is. And substantially so, for water appears to be shallower by about a quarter of its real depth. Where the actual depth is 1 metre, say, the apparent depth is only 75 centimetres. Bathing children often get into a trouble for this reason: relying on the deceptive appearance, they usually underestimate depth."

"I noticed that when you float slowly in a boat over a place where the bottom is visible, it appears that the greatest depth is just under the boat and it's much shallower everywhere else. But shift to another place and again everywhere is shallow and beneath the boat is deep. It seems as if the deepest place travels with the boat. Why?"

"Now you can understand that easily. The point is that the rays coming straight up out of the water change direction least of all, thus the bottom there appears to be less elevated than the places which send oblique rays to our eyes. Naturally, the deepest place appears to us to lie just beneath the boat even if the bottom were perfectly flat. But now let's do quite another experiment."

Alex filled a glass with water right up to the brim. "What do you think will happen if now I drop a two-pence piece into the glass?"

"The water will overflow, of course."

"Let's try."

Carefully, without jerking, my brother lowered the coin into the brimful glass. Not a drop overflowed.

"Now let's put in another coin," he said.

I warned him, "Now it's sure to overflow."

And I was mistaken: in the full glass there was room for the second coin, too. A third and a fourth coin followed each other into the glass.

"What a bottomless glass!" I exclaimed.

Alex silently and cooly kept on lowering one coin after another into the glass. A fifth, sixth, seventh time coins fell onto the bottom – no overflowing. I couldn't believe my eyes and was impatient to find out the explanation.

But my brother took his time to explain, he was still carefully dropping coins and only stopped at the 15th two-pence piece.

"Well, that'll do," he said at last, "Take a look, the water has bulged up at the glass's edge."

Indeed, the water had bulged above the edge by about the thickness of a match, sloping down at the edges as if it were in a transparent bag.

Alex went on to say, "The answer lies in the bulging. This is where the water is that was expelled by the coins."

"Fifteen coins have displaced so little water?" I was astounded. The stack of 15 two-pence pieces is rather high but here is only a thin layer, just thicker than a penny."

"Take its area into consideration, not only the thickness. The layer may be not thicker than a two-pence piece, but how many times larger is it across?"

I gave some thought to it – the glass was about four times wider than a two-pence.

"Four times wider and the same thickness." I went on to conclude, "The layer is only four times larger than a two-pence. The glass could only receive four coins, but you've already put in 15 and plan, it seems, to add some more. Where's the room?"

"Your calculation is wrong. If a ring is four times larger across than another, its surface area will be 16 times larger, not four."

"Well, I never!"

"You should have known it. How many square centimetres are there in a square metre? One hundred?"

"No, 100 times 100 which is 10,000."

"You see. With rings the same rule holds: if a ring is two times wider than another, it has four times the surface area, three times wider – nine times the area, four times wider – 16 times the area, and so on. So, the

volume of the buldge above the brim is 16 times larger than that of a two-pence piece. You see now where all the room is in the glass. It has even more room because the water can rise up about two two-pence pieces thickness."

"Could you really put 20 coins into the glass?"

"Even more, if only you dip them carefully, without shaking."

"I wouldn't ever have believed that a brimful glass could have enough room for so many coins."

I had to believe it though when I saw the heap of coins inside the glass with my own eyes.

"Now, can you place 11 coins into 10 saucers so that there is only one coin in each saucer?" the brother asked.

"Saucers with water?"

"With or without water, as you please," he laughed, setting 10 saucers in a row.

"Another physics experiment?"

"No, psychological. On with the job."

"Eleven coins in 10 saucers, and one in each... No, I can't," I gave up at once.

"Go ahead, I'll help you. We'll place the first coin in the first saucer and the 11th as well, just for a time."

I did as he said, waiting in bewilderment. What is going to follow?

"Two coins? Well, the third coin goes into the second saucer. The fourth into the third saucer, the fifth into the fourth, and so forth."

When I had placed the 10th coin into the ninth saucer I was surprised to see that the 10th saucer was vacant.

Alex said, "Now that's where we'll place the 11th coin that we put tentatively into the first saucer." He took the extra coin from the first saucer and placed it into the 10th saucer.

Now 11 coins were lying in 10 saucers, one in each... Fantastic!

Brother swiftly collected the coins not caring to explain the trick to me.

"Just think. That'll be both more interesting and more useful than getting ready-made solutions."

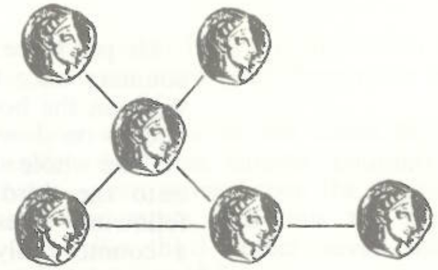
And ignoring my pleads he gave me a fresh problem.

"Here are six coins. Arrange them in three rows so that there are three coins in each."

"That takes nine coins."

"Everyone can do it with nine. No, do it just with six."

Figure 7



"Now again that's something impossible."

"You're too quick to give up. Look, it's simple."

"There are three rows here, with three coins in each," he explained.

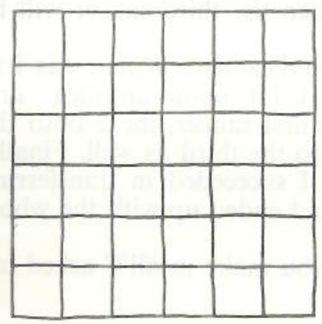
"But the rows criss-cross."

"Perhaps, but did I say that they mustn't?"

"If I'd known that this was allowed, I'd have guessed for myself."

"Well, then, guess how to solve the problem in another way. But not now, sleep on it. Here are three more problems in the same vein. The first one: arrange nine coins in ten rows with three coins in each. The second: arrange ten coins in five rows with four coins in each. The third problem is as follows. I draw a square divided into 36 smaller squares. Now try to arrange 18 coins with one in each small square so that in each row and column there are three coins... Aha, I've just remembered one more trick with coins. Take into one hand a 5 pence, into the other a 10 pence, but don't tell me which coin is in which hand. I'll figure it out. Only do the following mental arithmetic: double what's in the right hand and treble what's in the left, and then add the results. Ready?"

Figure 8



"Yes."

"What's the final result, odd or even?"

"Odd."

"The 10 is in the right and the 5 in the left hand," Alex proclaimed at once and was right on target.

We repeated it once more. The result was even and my brother said without mistake that the 10 was in my left hand.

"About this problem also think at leisure," he said, "and finally, I'll show you a fascinating game with counters. I've just made some counters by cutting out differently sized disks from a sheet of cardboard. The biggest counter is 5 centimetres in diameter, the next biggest 4 centimetres, and so on down to the smallest which is 1 centimetre in diameter."

He put three saucers side by side, and put a stack of counters onto the first saucer: so that the 5 counter went on the bottom on top of that was the 4 counter, and so on down to the 1 counter on top of the stack.

"The whole stack of five counters is to be transferred onto the third saucer but you have to observe the following rules. Rule number 1: each time move 1 counter only. Rule number 2: never put a larger

Figure 9

counter onto a smaller one. Rule number 3: counters may be placed *temporarily* onto the middle saucer but still observing the first two rules and the counters must end up on the third saucer in the initial order. The rules are simple as you can see. Now, go ahead."

I started. First I placed the 1 counter onto the third saucer, the 2 counter onto the middle one... and stopped. Where should the 3 counter go? It was larger than both the 1 and 2 counters.

"Well, then," my brother prompted, "Place the 1 onto the middle saucer, then the third saucer will be vacant for the 3."

I did so. Now a further predicament. Where was I to place the 4? Accidentally, I hit upon an idea: first I transferred the 1 onto the first saucer, the 2 onto the third, and next put the 1 onto the third as well. Finally, after a long series of move I succeeded in transferring the 5 from the first saucer and ended up with the whole stack on the third saucer.

"How many transfer did you make in all?" asked my brother okaying my job.

"Didn't count."

"Well let's count then. After all, it's interesting to find the least number of moves that could lead to the goal. If our stack included only two counters, not five, the 2 and the 1, how many moves would be required?"

"Three: the 1 onto the middle saucer, the 2 onto the third one and then the 1 onto the third."

"Right. Add one more counter, the 3, and count how many moves you need to transfer the stack. We'll proceed as follows: first we transfer the two smaller coins onto the middle saucer one after the other. This takes, as we already know, three moves. We then transfer the 3 onto the vacant third saucer–one more

move. Next we transfer both counters from the middle saucer, too, onto the third one–three more moves. The total is $3+1+3=7$."

"For the four counters, let me count for myself. At first I transfer the three smaller counters onto the middle saucer–seven moves; then the 4 goes onto the third saucer–one move, and now the three smaller coins go onto the third saucer–seven more moves. Thus I get $7+1+7=15$."

"Splendid. And for the five counters?"

"$15+1+15=31$."

"Well, you got it right. But I'll show you a way to simplify the procedure. Note that the numbers involved–3, 7, 15, 31–all represent the product of several twos minus one. Look!" And Alex wrote out the following table.

$$3 = 2 \times 2 - 1$$
$$7 = 2 \times 2 \times 2 - 1$$
$$15 = 2 \times 2 \times 2 \times 2 - 1$$
$$31 = 2 \times 2 \times 2 \times 2 \times 2 - 1$$

"I see, the number of the counters to be transferred equals the number of twos in the product. Now, I could calculate the number of moves for any stack of counters. For instance, for seven counters: it's

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 - 1 = 128 - 1 = 127.$$

"You've thus mastered this ancient game. You need only know one practical rule which is if the stack contains an odd number of counters the first counter is transferred onto the third saucer, if it's even, it goes onto the middle saucer."

"You say it's an ancient game, didn't you invent it yourself?"

"No, I only applied it to counters. But the game as such has a very ancient origin and apparently came from India where there is a marvellous legend associated with it. It says that in the town of Benares is a sanctuary into which the Indian god Brahma, as he was creating the world, installed three diamond sticks and put on one of them 64 golden rings with the largest at the bottom and each of the rest being smaller than the one beneath it. The priests of the sanctuary were obliged ceaselessly to transfer these rings from one stick to another using the third as an auxiliary and observing the rules of our game that is to move one ring at a time

and not to place it onto a smaller one. The legend has it that when all the 64 rings have been transferred the end of the world will come."

"Oh, it means the world should've perished long ago!"

"Perhaps, you think transferring 64 rings won't take much time?"

"Of course. Allowing a second per move, you can make 3600 transferrings in an hour."

"Well."

"And about 100,000 in 24 hours. In 10 days, a million moves. A million would be enough, I think, to transfer even a thousand, not 64 rings."

"You are mistaken. To handle the 64 rings would take as much as 500,000,000,000 years!"

"But, why? After all, the number of moves is only equal to the product of 64 twos, which amounts to..."

"'Only' upwards of 18,000,000,000,000,000,000."

"Wait a bit, I'll now multiply and check."

"Splendid. While you do your multiplying, I'll have time to go to tend to my business," said brother and left.

I first found the product of 16 twos, then multiplied the result by itself. A tedious job, but I was patient and worked it out to the end. I obtained the number 18,446,744,073,709,551,616.

Thus my elder brother was right...

I mustered up courage and set about the problems he had set to me to solve on my own. They didn't turn out to be all that difficult, some were even rather easy. The business of 11 coins in 10 saucers appeared ridiculously simple: we put the first and eleventh coins into the first saucer, next we put the third coin into the second saucer, the fourth coin into the third saucer, and so forth. But what about the second coin? It was ignored and that was the trick. The idea behind guessing which hand had the 10 pence coin was also simple. Doubling 5 gives an even number but trebling it gives an odd one

Figure 10

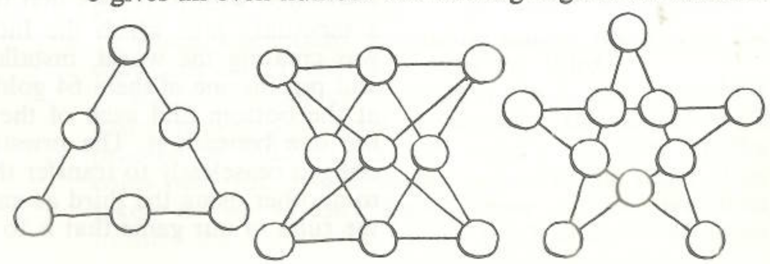
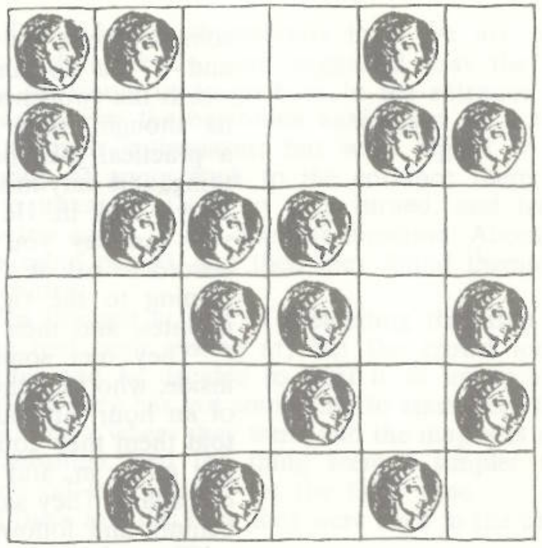


Figure 11



whereas multiplying 10 always gives an even number. Therefore, if the total was even, then the 5 had been doubled, i.e. it must have been in the right hand, and if the total was odd, it is clear that the 5 must have been trebled, i.e. been in the left hand. The solutions to the problems on the coin arrangements are clear from the accompanying drawings (Fig. 10).

Finally, the problem with coins in the small squares works out as shown in Fig. 11. The 18 coins are arranged in the square with 36 small squares and giving three coins in each row.

Wandering in a Maze

Wandering in a maze ● People and rats ● Right- and left-hand rule ● Mazes in ancient times ● Tournefort in a cave ● Solution of the maze problem.

"What are you laughing at in your book? A funny story?" Alex asked me.

"Yes, it's *Three Men in a Boat* by Jerome."

"I remember it had me in stitches! Where are you?"

"Where the crowd of people is wandering about in a garden maze, looking for a way out."

"An interesting story. Read it again for me, please."

So I read the story aloud from the very beginning: "Harris asked me if I'd ever been in the maze at Hampton Court. He said he went in once to show

somebody else the way. He had studied it up in a map, and it was so simple that it seemed foolish–hardly worth the twopence charged for admission. Harris said he thought that map must have been got up as a practical joke because it wasn't a bit like the real thing, and only misleading. It was a country cousin that Harris took in. He said: 'We'll just go in here, so that you can say you've been, but it's very simple. It's absurd to call it a maze. You keep on taking the first turning to the right. We'll just walk round for ten minutes, and then go and get some lunch.'

"They met some people soon after they had got inside, who said they had been there for three-quarters of an hour, and had had about enough of it. Harris told them they could follow him, if they liked; he was just going in, and then should turn round and come out again. They said it was very kind of him, and fell behind, and followed.

"They picked up various other people who wanted to get it over, as they went along, until they had absorbed all the persons in the maze. People who had given up all hopes of ever getting either in or out, or of ever seeing their home and friends again, plucked up courage, at the sight of Harris and his party, and joined the procession, blessing him. Harris said he should judge there must have been twenty people following him, in all; and one woman with a baby, who had been there all the morning, insisted on taking his arm, for fear of losing him.

"Harris kept on turning to the right, but it seemed a long way, and his cousin said he supposed it was a very big maze.

"'Oh, one of the largest in Europe,' said Harris.

"'Yes, it must be, replied the cousin, because we've walked a good two miles already.'

"Harris began to think it rather strange himself, but he held on until, at last, they passed the half of a penny bun on the ground that Harris's cousin swore he had noticed there seven minutes ago. Harris said, 'Oh, impossible!' But the woman with the baby said, 'Not at all,' as she herself had taken it from the child, and thrown it down there, just before she met Harris. She also added that she wished she never had met Harris, and expressed an opinion that he was an impostor. That made Harris mad, and he produced his map, and explained his theory.

"'The map may be all right enough, said one of the

party, if you know whereabouts in it we are now.'

"Harris didn't know, and suggested that the best thing to do would be to go back to the entrance, and begin again. For the beginning again part of it there was not much enthusiasm; but with regard to the advisability of going back to the entrance there was complete unanimity, and so they turned, and trailed after Harris again, in the opposite direction. About ten minutes more passed, and then they found themselves in the centre.

"Harris thought at first of pretending that that was what he had been aiming at; but the crowd looked dangerous, and he decided to treat it as an accident.

"Anyhow, they has got something to start from then. They did know where they were, and the map was once more consulted, and the thing seemed simpler than ever, and off they started for the third time.

"And three minutes later they were back in the centre again.

"After that they simply couldn't get anywhere else. Whatever way they turned brought them back to the middle. It became so regular at length, that some of the people stopped there, and waited for the others to take a walk round, and come back to them. Harris drew out his map again, after a while, but the sight of it only infuriated the mob, and they told him to go and curl his hair with it. Harris said that he couldn't help feeling that, to a certain extent, he had become unpopular.

"They all got crazy at last, and sang out for the keeper, and the man came and climbed up the ladder outside, and shouted out directions to them. But all their heads were by this time, in such a confused whirl that they were incapable of grasping anything, and so the man told them to stop where they were, and he would come to them. They huddled together, and waited; and he climbed down, and came in.

"He was a young keeper, as luck would have it, and new to the business; and when he got in, he couldn't get to them, and then he got lost. They caught sight of him, every now and then, rushing about the other side of the hedge, and he would see them, and rush to get to them, and they would wait there for about five minutes, and then he would reappear again in exactly the same spot, and ask them where they had been.

"They had to wait until one of the old keepers came back from his dinner before they got out."

"They were a bit dense," I said, "To have a plan and

not to find the way out."

"Do you think you'd find at once?"

"With a plan? Certainly!"

"Just wait. It seems to me I've got the plan of that maze," Alex said and began to delve in his bookcase.

"Does this maze really exist?"

"Hampton Court? Of course, it's near London. Been in existence for two centuries... Found at last. Just as

Figure 12

I said *Plan of the Maze at Hampton Court.* It seems rather small, this maze, only 1,000 square metres."

My brother opened the book at a page showing a small plan.

"Imagine you're here in the central area of the maze and want to get out, which way would you go to get to the exit? Sharpen a match and use it to show the way."

I pointed the match at the centre of the maze and bravely drew it along the winding paths of the maze, but the whole affair appeared to be more involved than I had expected. Having wandered a little round about the plan I came... back to the central area, just as Jerome's characters had, the ones I'd just made fun of!

"You see, the plan is no use. But rats solve the task without any plan."

"Rats? What rats?"

"The ones described in this book. Do you think this is a treatise on garden design? No, this book is about the mental abilities of animals. To test the intelligence of animals, scientists make a small plaster model of a maze and put the animals to be tested into it. The book says that rats can find their way about a plaster maze of Hampton Court in only half an hour and that is faster than the people in Jerome's book."

"Judging from the plan, the maze doesn't seem to be very difficult. You would never think that it's so treacherous..."

"There's a simple rule. If you know it, you can safely enter any maze without any fear of getting lost."

"Which rule?"

"You should follow the paths touching its wall with your right hand, or left for that matter–it makes no difference. But with one hand, all the time."

"Just this?"

"Yes. Now try and use the rule in reality, mentally wandering about the plan."

I ran my match along the paths, being guided by the rule. Truly, I soon came from the entrance to the centre and back again, to the exit.

"A beautiful rule."

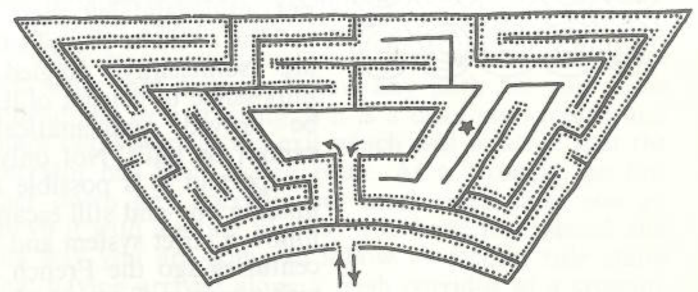
"Not really," Alex objected, "The rule is good so long as you simply don't want to be lost in a maze, but it's no good if you want to walk along all of its paths without exception."

"But I've just been in all the alleys on the plan. I didn't miss one."

"You are mistaken. Had you marked with a dash line the way you went, you'd have found that one alley wasn't covered."

"Which one?"

Figure 13



"I've marked it with a star on this plan (Fig. 13). You haven't been down this alley. In other mazes the rule would guide you past large sections of it so that even though you'd find your way out safely, you wouldn't see much of it."

"Are there many different kinds of maze?"

"A lot. Nowadays they are only in garden and parks and you wander around in the open air between high green walls of hedge, but in ancient times they used to put mazes inside large houses or dungeons. That was

done with the cruel aim of dooming the unhappy people thrown into them to wander hopelessly about the intricate tangle of corridors, passages and halls, eventually leading them to starve to death. One such, for instance, was the proverbial maze on the island of Crete and the legend has it that an ancient king called Minos had it built. Its paths were so tangled that its own creator, a man called Daedalus, allegedly couldn't find his way out of it," brother continued, "The aim of other ancient mazes was to guard the tombs of kings, to protect them from robbers. A tomb was located at the centre of a maze so that if a greedy seeker after buried treasure even succeeded in reaching it, he wouldn't be able to find his way out – the grave of the king would become his grave, too."

Figure 14

"Why didn't they use the rule for walking round mazes you've just told me about?"

"For one thing, apparently, in ancient times nobody knew about the rule. For another, I've already told you that it doesn't always let you visit every part of the maze. A maze can be contrived in such a way that the user of the rule will miss the place where the treasure is kept."

"But is it possible to make a maze from which there is no escaping? Of course, someone who enters it using your rule will get out eventually, but suppose a man is put inside and left there to wander?"

"The ancients thought that when the paths of a maze are sufficiently tangled, it would be absolutely impossible to get out of it. This isn't true because it can be proved mathematically that inescapable mazes cannot be built. Not only that but every maze has an escape and it is possible to visit every corner without missing one and still escape to safety. You only need to follow a strict system and take certain precautions. Two centuries ago the French botanist Tournefort dared to visit a cave in Crete which was said to be an inescapable maze because of its innumerable paths. There are several such caves in Crete and it may be possible that they gave rise to the ancient legend about the maze of King Minos. What did the French botanist do in order not to be lost? This is what his fellow-countryman, the mathematician Lucas, said about it."

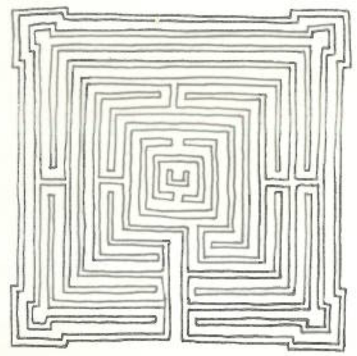
My brother took down from the bookcase an old book entitled *Mathematical Amusements* and read aloud the following passage (I copied it later):

"Having wandered for a time with our companions

about a network of underground corridors, we came to a long wide gallery that led us into a spacious hall deep in the maze. We had counted 1460 steps in half an hour along this gallery, deviating neither right nor left... On either side there were so many corridors that one would be bound to get lost there unless some necessary precautions were taken. But as we had a strong desire to be out of the maze, we saw to it to provide for our return.

"First, we left one of our guides at the entrance to the cave, having instructed him to call for the people from a neighbouring village to rescue us should we not return by night fall. Second, each of us had a torch. Third, at every turn which, it seemed, might be difficult to find later we attached numbered papers to the wall. Finally, one of our guides put on the left side bunches of blackthorn he had prepared beforehand, and the other side of the path he sprinkled with chopped straw, which he carried in a bag."

Figure 15



Alex finished reading and said, "All these laborious precautions might not seem all that necessary to you. In the times of Tournefort, however, there was no other way since the problem of mazes had not yet been solved. These days the rules for walking around mazes have been worked out that are less burdensome but no less reliable than his precautions."

"Do you know these rules?"

Figure 16

"They aren't complicated. A first rule is that when you walk into a maze, follow any path till you reach a dead end or a crossing. If it is a dead end, return and place two stones at the exit which will indicate that the corridor has been passed twice. At a crossing, go further down any corridor but mark each time you go down it with a stone the way you have just passed and the way you are going to follow. A second rule states that having arrived along a fresh corridor at a crossing that has earlier been visited (as seen by the stones), go back at once and place two stones at the end of the corridor. Finally, a third rule requires that having come to a visited crossing along a corridor that has already been walked, mark the way with a second stone and go along one of new corridors. If there doesn't happen to be such a corridor, take one whose entrance has only one stone (that is a corridor that has only been passed once). Abiding by these rules you can pass twice, that is there and back, every corridor of the maze without missing any corner and return back to safety. Here I've

7 got several plans of mazes I've cut out at different times from illustrated magazines (Figs. 14, 15, and 16). If you wish, you can try and travel about them. I hope that now that you know so much you shouldn't be in any danger of getting lost in them. If you've enough patience, you could actually make a maze like, say, the Hampton Court one that Jerome mentioned–you could construct it with your friends out of snow in the yard."

For Young Physicists

2

More Skilled Than Columbus

A schoolboy once wrote in a composition: "Christopher Columbus was a great man because he discovered America and stood an egg upright." This young scholar had thought both deeds equally amazing. On the contrary, the American humorist Mark Twain saw nothing special about Columbus discovering America: "It would have been strange if he hadn't found it there."

The other feat the great navigator had performed is not really all that marvellous. Do you know how Columbus stood an egg upright? He simply pressed it down onto a table crushing the bottom of the shell. He had, of course, changed the shape of the egg. But how can one possibly stand an egg on end without changing its shape, the navigator didn't know.

Meanwhile it is easier by far than discovering America or even one tiny island. I'll show you three methods: one for boiled eggs, one for raw eggs, and one for both.

A boiled egg can be stood upright simply by spinning it with your fingers or between your palms like a top. The egg will remain upright as long as it spins. After two or three trials the experiment should come out well.

This won't work if you try to stand a raw egg upright, you may have noticed that raw eggs spin poorly. This, by the way, is used to distinguish a hard-boiled egg from a raw one without breaking the shell. The liquid contents of a raw egg is not carried along by the spinning as fast as the shell and, therefore, sort of damps the speed down. We have to look for another way of standing eggs and one does exist. You have to shake an egg intensely several times. This breaks down the soft envelope containing the yolk with the result that the yolk spreads out inside the egg. If you then stand the egg on its blunt end and keep it this way for a while, then the yolk, which is heavier than the white, will pour down to the bottom of the egg and concentrate there. This will bring the centre of mass of the egg down making it more stable than before.

Finally, there is a third way of putting an egg upright. If an egg is placed, say, on the top of a corked bottle and another cork with two forks stuck into it is placed on the top as shown in Fig. 17, the whole

Figure 17

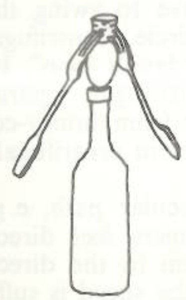


Figure 18

system (as a physicist would put it) is fairly stable and remains in equilibrium even if the bottle is slightly inclined. But why don't the egg and cork fall down? For the same reason that a pencil placed upright on a finger doesn't fall off when a bent penknife is stuck into it as shown. A scientist would explain: "The centre of mass of the system lies below the support." This means that the point at which the weight of the system is applied lies below the place at which it is supported.

Centrifugal Force

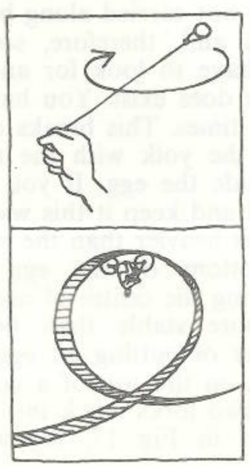
Open an umbrella, put its end on the floor, spin it and drop a ball into it. The ball could be a balled piece of paper or handkerchief, or any other light and unbreakable thing. Something will happen you probably wouldn't expect. The umbrella does not, as it were, desire to accept the present and the thing itself crawls up the edge and then flies off in a straight line.

Figure 19

The force that threw the ball out in this experiment is generally called the "centrifugal force", although it would be more appropriate to dub it "inertia". Centrifugal force manifests itself when a body travels in a circle but this is nothing but an example of inertia which is the desire of a moving body to maintain its speed and direction.

We come across centrifugal force more often than you might suspect. If you whirl a stone tied to a piece of string, you can feel the string become taut and seem to be about to break under the action of the centrifugal force. The ancient weapon for hurling stones, the sling, owes its existence to the force. Centrifugal force bursts a millstone, if it is spun too fast and is not sufficiently strong. If you are adroit enough, this force will help you to perform a trick with a glass from which the water doesn't escape, even though it is upside down. In order to do this you'll only have to swing the glass quickly above your head in a circle. Centrifugal force helps a circus bicyclist to do a "devil's loop". It is put to work. In the so-called centrifugal separators it churns cream; it extracts honey from honey-comb; it dries washing by extracting water in centrifugal driers, etc., etc.

Figure 20



When a tram travels in a circular path, e. g. as it turns at a crossing, the passengers feel directly the centrifugal force that pushes them in the direction of the outer wall of the carriage. If the speed is sufficiently large, the carriage could be overturned by the force if the outer rail wasn't laid a bit higher than the inner one: which is why a tram is slightly inclined inwards when it turns. It sounds rather unusual but an inclined tram is more stable than an upright one!

But this is quite the case, though. A small experiment will help explain this to you. Bend a cardboard sheet to form a wide funnel, or better still take a conical bowl if available. The conical shield (glass or metallic) of an electrical lamp would be suitable for our purposes. Roll a coin (small metal disk, or ring) around the edge of any of these objects. It will travel in a circle bending in noticeably on its way. As the coin slows down, it will travel in ever decreasing circles approaching the centre of the funnel. But by slightly shaking the funnel the coin can easily be make roll faster and then it will move away from the centre describing increasingly larger circles. If you overdo it a bit, the coin will roll out.

For cycling races in a velodrome special circular tracks are made and you can see that these tracks, especially where they turn abruptly have a noticeable slope into the centre. A cyclist rides along them in an inclined position (like the coin in the funnel) and not only does he not turn over but he acquires special stability. Circus cyclists used to amaze the public by racing along a steep deck. Now you can understand that there is nothing special about it. On the contrary, it would be a hard job for a cyclist to travel along a horizontal track. For the same reason a rider and his horse lean inwards on a sharp turn.

Let's pass on from small to large-scale phenomena. The Earth, on which we live, rotates and so centrifugal force should manifest itself. But where and how? By making all the things on its surface lighter. The closer something is to the Equator, the larger the circle in which it moves and hence it rotates faster, thereby losing more of its weight. If a 1-kg mass were to be brought from one of the poles to the Equator and reweighed using a spring balance, the loss in weight would amount to 5 grammes. That, of course, is not very much of a difference, but the heavier a thing, the larger the difference. A locomotive that has come from Stockholm to Rome loses 60 kg, the weight of an adult. A battle ship of 20,000-tonne displacement that has come from the White Sea to the Black Sea will have lost as much as 80 tonnes, the weight of a locomotive!

Why does it happen? Because as the globe rotates, it

tries to throw everything off its surface just like the umbrella in our earlier experiment. It would succeed were it not for the terrestrial attraction that pulls everything back to the Earth's surface. We call this attraction "gravity". The rotation cannot throw things off the Earth's surface, but it can make them lighter.

The faster the rotation, the more noticeable the reduction in weight. Scientists have calculated that if the Earth rotated 17 times faster, things at the Equator would lose their weight completely to become weightless. And if it rotated yet quicker, making, say, one turn every hour, then the weight lessness would extend to the lands and seas farther away from the Equator.

Just imagine things losing their weight. It would mean there would be nothing you could not lift, you would be able to lift locomotives, boulders, cannons and warships as easily as you could a feather. And should you drop them – no danger, they could hurt nobody since they wouldn't fall down at all, but would float about in mid-air just where you'd let go of them. If, sitting in the cabin of an airship, you wanted to throw something overboard, it wouldn't drop, but would stay in the air. What a wonder world it would be. So you could jump as high as you've never dreamed, higher than sky-scrapers or the mountains. But remember, it would be easy to jump up but difficult to return back to ground. Weightless, you'd never come back on your own.

There would also be other inconveniences in such a world. You've probably realized yourself that everything, whatever its size, would, if not fixed, rise up due to the slightest motion of air and float about. People, animals, cars, carts, ships – everything would move about in the air disorderly, breaking, maiming and destroying.

That is what would occur if the Earth rotated significantly faster.

Ten Tops

The accompanying figures show 10 types of tops. These will enable you to do a number of exciting and instructive experiments. You don't need any special skill to construct them so you can make them yourself without any help or expense.

This is how the tops are made:

56–57

Figure 21

Figure 22

Figure 23

Figure 24

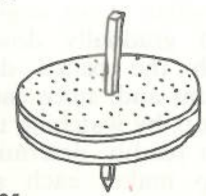


Figure 25

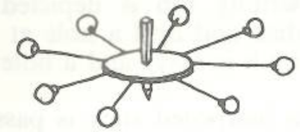
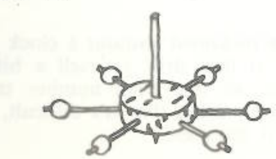


Figure 26



1. If a button with five holes comes your way, like the one shown in the figure, then you can easily make it into a top. Push a match with a sharpened end through the central hole, which is the only one needed, wedge it in and then... the top is ready. It will rotate on both the blunt and pointed end, you only need to spin it as usual by twisting the axle between your fingers and dropping the top swiftly on its blunt end. It will spin rocking eccentrically.

2. You could do without a button, a cork is nearly always at hand. Cut a disk out of it, pierce the disk with a match, and you have top No. 2 (Fig. 22).

3. Figure 23 depicts a rather unusual top – a walnut that spins on the pointed end. To turn a suitable nut into a top just drive a match into the other end, the match being used for spinning the top.

4. A better idea is to use a flat wide plug (or the plastic cover of a small can). Heat an iron wire or knitting-needle and burn through the plug along the axis to form a channel for the match. A top like this will spin long and steadily.

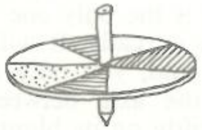
5. Figure 24 shows another top: a flat round box pierced by a sharpened match. For the box to fit tightly without sliding along the match, seal the hole with wax.

6. A fancy top you see in Figure 25. Globular buttons with an eye are tied to the edge of a cardboard disk with pieces of string. As the top rotates the buttons are thrown off radially, stretching the strings out taut and graphically demonstrating the action of our old friend, the centrifugal force.

7. The same principle is demonstrated in another way by the top in Figure 26. Some pins are driven into the cork ring of the top with coloured beads threaded onto them so that beads can slide along the pin. As the top spins the beads are pushed away to the pin heads. If the spinning top is illuminated, the pins merge into a solid silvery belt with a coloured fringe of the merged beads. In order to enjoy the illusion spin the top on a smooth plate.

8. A coloured top (Fig. 27). It is fairly laborious to make but the top will reward your efforts by demonstrating an astounding behaviour. Cut a piece of cardboard into a disk, make a hole at the centre to receive a pointed match. Clamp the match on either side of the disk with two cork disks. Now divide the cardboard disk into equal sectors by straight radial lines in the same way a round cake is shared out.

Figure 27



Colour the sectors alternately in yellow and blue. What will you see as the top rotates? The disk will appear neither blue nor yellow, but green. The blue and yellow colours merge in your eye to give a new colour, green.

You can continue your experiments on colour blending. Prepare a disk with sectors alternately coloured in blue and orange. Now, when the disk is spun it will be white, not yellow (actually it will be light grey, the lighter the purer the paints). In physics two colours that, when blended, give white are called "complementary". Consequently, our top has shown that blue and orange are complementary.

If you have a good set of paints you can try an experiment that was first done 200 years ago by the great English scientist Isaac Newton. Paint the sectors of a disk with the seven colours of the rainbow which are: violet, indigo, blue, green, yellow, orange, and red. When all the seven colours are rotated together they will produce a greyish-white. The experiment will help you to understand that the sunlight is composed of many colours.

Figure 28

These experiments can be modified as follows: as the top spins throw a paper ring onto it and the disk will change its colour at once (Fig. 28).

Figure 29

9. The writing top (Fig. 29). Make the top as just described, the only difference being that its axle will now be a soft pencil, not match. Make the top spin on a cardboard sheet placed somewhat at an angle. The top will, as it spins, descend gradually down the inclined cardboard sheet, with the pencil drawing flourishes. These are easy to count and, since each one corresponds to one turn of the top, by watching the top with a clock in hand* you can readily determine the number of revolutions the top makes each second. Clearly, this would be impossible in any other way.

Figure 30

A further form of the writing top is depicted in Fig. 30. Find a small lead disk and drill a hole at the centre (lead is soft and drilling it is easy), and a hole on either side of it.

Through the centre hole a sharpened stick is passed, and through one of the side holes a piece of fishing-line

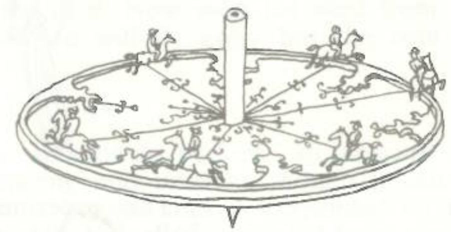
* By the way, seconds can also be reckoned without a clock just by counting. To do so, you should at first drill yourself a bit to pronounce "one", "two", "three", etc., so that each number takes exactly one second to pronounce. Don't think that it's difficult, the practice shouldn't take more than 10 minutes.

(or bristle) is threaded so that it protrudes a bit lower than the end of the top axle. The fishing-line is fixed in with a piece of match. The third hole is left as it is, its only purpose is to balance the disk since otherwise the top won't spin smoothly.

Our writing top is ready, but to experiment with it we need a sooty plate. Hold a plate over a smoky flame until it is covered with a uniform layer of dense soot. Then send the top spinning over the sooty surface. It will slide over the surface and the end of the fishing-line will draw, white on black, an intricate and rather attractive ornament.

10. Our crowning effort is the last rig, the merry-go-round top. However, it is much easier to make that it might seem. The disk and stick here are just as in the

Figure 31



earlier coloured top. Into the disk, pins with small flags are stuck symmetrically about the axis, and tiny paper riders are glued in-between the pins. Thus, you have a toy merry-go-round to amuze your younger brothers and sisters.

Impact

When two boats, trams or croquet balls collide (an incident or move in a game) a physicist would call such an event just "impact". The impact lasts a split second, but if the objects involved are elastic, which is normally the case, then a lot happens in this instant. In each elastic impact physicists distinguish three phases. In the first phase both colliding objects compress each other at the place of contact. Then comes the second phase when the mutual compression reaches a maximum, the internal counteraction begins in response to the compression and prevents the bodies from compressing further, so balancing the thrusting force. In the third phase the counteraction, seeking to restore the body's shape deformed during the first phase, pushes the objects apart in opposite directions. The receding

object, as it were, receives its impact back. In fact, when we observe, say, one croquet ball striking another, stationary, ball of the same mass, then the recoil makes the oncoming ball stop and the other ball roll forward with the velocity of the first.

It is very interesting to observe a ball striking a number of other balls arranged in a file touching each other. The impact received by the first ball is, as it were, transferred through the file, but all the balls remain at rest and only the outermost one jumps away as it has no adjacent ball to impart the impact to and receive it back.

Figure 32

This experiment can be carried out with croquet balls, but it is also a success with draughts or coins. Arrange the draughts in a straight line, it can be a very long one, but the essential condition is that they touch one another. Holding the first draught with a finger strike it on its edge with a wooden ruler, as shown. You will see the last draught jump away, with the rest of the draughts remaining in their places.

An Egg in a Glass

Circus conjurers sometimes surprise the public by jerking the cloth from a laid table so that everything–plates, glasses, and bottles–remain safely in place. This is no wonder or deceit, it is simply a matter of dexterity acquired by prolonged practice.

Such a sleight-of-hand is too difficult for you to attain but on a smaller scale a similar trick is no problem. Place a glass half-filled with water on a table and cover it with a postcard (or half of it). Further, borrow a man's wide ring and a hard-boiled egg. Put the ring on the top of the card, and stand the egg on the ring. It possible to jerk the card away so that the egg doesn't roll down onto the table?

Figure 33

At first sight, it may seem as difficult as jerking the

table-cloth from under the table things. But a good snap with a finger on the edge of the card should do the trick. The card flies away and the egg... plunges with the ring safely into the water. The water cushions the blow and the shell remains intact.

With some experience, you could try the trick with a raw egg.

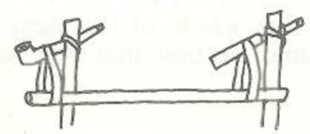
This small wonder is explained by the fact that during the fleeting moment of the impact the egg doesn't receive any observable speed but the postcard that was struck has time to slip out. Having lost its support, the egg drops into the glass.

If the experiment is not at first a success, first practice an easier experiment in the same vein. Place half a postcard on the palm of your hand and a heavy coin on top of it. Now snap the card from under the coin. The card will fly away but the coin will stay.

Unusual Breakage

Conjurers sometimes perform an elegant trick that seems amazing and unusual, though it can be easily explained. A longish stick is suspended on two paper rings. One of the rings is suspended from a razor blade, the other, from a clay pipe. The conjurer takes another stick and strikes the first one with all his strength. What happens? The suspended stick breaks but the paper rings and the pipe remain absolutely intact!

Figure 34

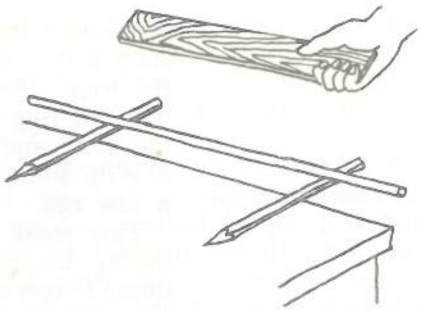


The trick can be accounted for in much the same way as the previous one. The impact was so fast that it allowed no time for the suspended stick's ends and the paper rings to move. Only the part of the stick that is directly subjected to the impact moves with the result that the stick breaks. The secret is thus that the impact was very *fast* and *sharp*. A slow, sluggish impact will not break the stick but will break the rings instead.

The most adroit conjurers even contrive to break a stick supported by the edges of two thin glasses leaving the glasses intact.

I do not tell you this, of course, to encourage you to do such tricks. You'll have to content yourself with a more modest form of them. Put two pencils on the edge of a low table or bench so that part of them overhang and place a thin, long stick on the overhanging ends. A strong, sharp stroke with the edge of a ruler at the middle of the stick would break it in two, but the pencils would remain in their places.

Figure 35



Now it should be clear to you why it is difficult to crack a nut by the strong pressure of a palm, but the stroke of a fist does the job easily. When you hit it, the impact has no time to propagate along the flesh of your fist so that your soft muscles do not yield under the upthrust of the nut and act as a solid.

For the same reason a bullet makes a small round hole in the window-pane, but a small stone traveling at a far slower speed breaks the pane. A slower push makes the window frame turn on its hinges, something neither the bullet nor the stone can make it do.

Finally, one more example of the phenomenon is being able to cut a stem of grass by a stroke of a cane. By slowly moving the cane you can't cut a stem, you only bend it. By striking it with all your strength you will cut it, if, of course, the stem is not too thick. Here, as in our earlier cases, the cane moves too fast for the impact to be transferred to the whole of the stem. It will only concentrate in a small section that will bear all the consequences.

Just Like a Submarine

A fresh egg will sink in water, a fact known to every experienced housewife. If she wants to find out whether an egg is fresh, she tests it in exactly this way. If an egg sinks, it is fresh; and if it floats, it is not suitable for eating. A physicist infers from this observation that a fresh egg is heavier than the same volume of fresh water. I say "fresh water" because impure (e.g. salt) water weighs more.

It is possible to prepare such a strong solution of salt that an egg will be lighter than the amount of brine displaced by it. Then, following the principle of floating discovered in olden days by Archimedes, even the freshest of eggs will float in the solution.

Figure 36

Use your knowledge in the following instructive experiment. Try to make an egg neither sink nor float, but hang in the bulk of a liquid. A physicist would say that the egg is "suspended". You'll need a water solution of salt that is so strong that an egg submerged in it displaces exactly its own weight in the brine. The brine is prepared by the trial-and-error method: by pouring in some water if the egg surfaces and adding some stronger brine if it sinks. If you've got patience, you'll eventually end up with a brine in which the submerged egg neither surfaces nor sinks, but is at rest within the liquid.

This state is characteristic of a submarine. It can stay under water without touching the ground only when it weighs exactly as much as the water it displaces. For this weight to be reached, submarines let water from the outside into a special container; when the submarine surfaces the water is pushed out.

A dirigible – not an aeroplane but just a dirigible – floats in the air for the very same reason: just like the egg in the brine it diplaces precisely as many tonnes of air as it weighs.

Floating Needle

Is it possible to make a needle float on the surface of water like a straw? It would seem impossible: a solid piece of steel, although it's small, would be bound to sink.

Many people think this way and if you are among the many, the following experiment will make you change your mind.

Get a conventional (but not too thick) sewing needle, smear it slightly with oil or fat and place it carefully on the surface of the water in a bowl, pail, or glass. To your surprise, the needle will not go down, but will stay on the surface.

Why doesn't it sink, however? After all, steel is heavier than water? Certainly, it is seven to eight times as heavy as water, if it were under the water it wouldn't be able to surface like a match. But our needle doesn't submerge. To find a clue, look closely at the surface of the water near the floating needle. You'll see that near the needle the surface forms sort of a valley at the bottom of which lies our needle.

The surface curvature is caused by the oil-smeared needle being not wetted by the water. You may have

noticed that when your hands are oily, water doesn't wet the skin. The feathers of water birds are always covered with oil exuded by a gland, which is why water doesn't wet feathers ("like water off a duck's back"). And again this is the reason why without soap, which dissolves the oil film and removes it from the skin, you cannot wash your oily hands even by hot water. The needle with oil on it is not wetted by water either and lies at the bottom of a concavity supported by the water "film" created by surface tension. The film seeks to straighten and so pushes the needle out of the water, preventing it from sinking.

As our hands are always somewhat oily, if you handle a needle it will be covered by a thin layer of oil. Therefore, it is possible to make the needle float without specially covering it with oil–you'll only have to place it extremely carefully on some water. This can be made as follows: place the needle on a piece of tissue-paper, then gradually, by bending down the edges of the paper with another needle, submerge the paper. The paper will descend to the bottom and the needle will stay on the surface.

Figure 37

Now if you came across a pondskater scuttling about the water surface, you won't be puzzled by it. You'll guess that the insect's legs are covered with oil and are not wetted by the water and that surface tension supports the insect on the surface.

Diving bell

This simple experiment will require a basin, but a deep, wide can would be more convenient. Besides, we'll need a tall glass (or a big goblet). This'll be our diving bell, and the basin with water will be our "sea" or "lake".

There is hardly a simpler experiment. You just hold the glass upside down, push it down to the bottom of the basin holding it in your hand (for the water not to push it out). As you do so you'll see that the water doesn't find its way into the glass–the air doesn't let it in. To make the performance more dramatic, put something easily soaked, e.g. a lump of sugar, under your "bell". For this purpose, place a cork disk with a lump of sugar on it on the water and cover it by the glass. Now push the glass into the water. The sugar will appear to be below the water surface, but will remain dry, as the water doesn't get under the glass.

Figure 38

You can perform the experiment with a glass funnel,

if you push it into the water, its wider end down and its narrow end covered tightly with a finger. The water again doesn't get inside the funnel, but once you remove your finger from the hole, thereby letting the air out of the funnel, the water will promptly rise into the funnel to reach the level of the surrounding water.

You see that air is not "nothing", as some think, it occupies space and doesn't let in other things if it has nowhere to go.

Besides, these experiments should graphically illustrate the way in which people can stay and work under water in a diving bell or inside wide tubes that are known as "caissons". Water won't get into the bell, or caisson, for the same reason as it can't get into the glass in our experiment.

Why Doesn't It Pour Out?

The following experiment is one of the easiest to carry out, it was one of the first experiments I performed when I was a boy. Fill a glass with water, cover it with a postcard or a sheet of paper and, holding the card slightly with your fingers, turn the glass upside down. You can now take away your hand, the card won't drop and the water won't pour out if only the card is strictly horizontal.

You can safely carry the glass about in this position, perhaps even more comfortably than usually since as the water won't spill over. As the occasion serves, you can astound your friends (if asked to bring some water to drink) by bringing water in a glass upside down.

Figure 39

What then keeps the card from falling, i.e. what overcomes the weight of the water column? The pressure of air! It exerts a force on the outside of the card that can be calculated to be much greater than the weight of the water, i.e. 200 grammes.

The person who showed me the trick for the first time also drew my attention to the fact that the water must fill the glass completely for the trick to be a success. If it only occupies a part of the glass, the rest of the glass being filled by air, the trial may fail because the air inside the glass would press on the card balancing off the pressure of the outside air with the result that it might fall down.

When I was told this, I set out at once to try it with a glass that wasn't fully filled in order to see for myself of the card would drop. Just imagine my astonishment

when I saw that in that case too, it didn't fall! Having repeated the experiment several times, I made sure that the card held in place as securely as with the full glass.

This has taught me a good lesson about how the facts of nature should be perceived. The highest authority in natural science must be *experiment*. Every theory, however plausible it might seem, must be tested by experiment. "Test and retest" was the motto of the early naturalists (Florentine academicians) in the 17th century, it is still true for 20th century physicists. And should a test of a theory indicate that experiment doesn't bear it out, one should dig for the clues to the failure of the theory.

In our case we can easily find a weak point in the reasoning that once had seemed convincing. If we carefully turn back a corner of the card covering the overturned, partially filled glass, we'll see an air bubble come up through the water. What is it indicative of? Obviously the air in the glass was slightly rarefied, otherwise the outside air wouldn't rush into the space above the water. This explains the trick: although some air remained in the glass, it was slightly rarefied, and hence exerted less pressure. Clearly, when we turn the glass over, the water, as it goes down, forces some of the air out of the glass. The remaining air, which now fills up the same space, becomes rarefied and its pressure becomes weaker.

You see that even simplest physical experiments, when treated attentively, can suggest fundamental ideas. These are those small things that teach us great ideas.

Dry Out of Water

You'll now see that the air surrounding us on all sides exerts a significant pressure on all the things exposed to it. The experiment I'm going to describe will show you more vividly the existence of what physicists call "atmospheric pressure".

Figure 40

Place a coin (or metal button) on a flat plate and pour some water over it. The coin will be under water. It's impossible, you are sure to think, to get it out from under the water with your bare hands without getting your fingers wet or removing the water from the plate. You're mistaken, it is possible.

Proceed as follows. Set fire to piece of paper inside a glass and when the air has heated, upend the glass and put it on the plate near the coin. Now watch, you

won't have to wait long. Of course, the paper under the glass will burn out soon and the air inside the glass will begin to cool down. As it does so, the water will, as it were, be sucked in by the glass and before long it will be all there, exposing the plate's bottom.

Wait a minute for the coin to dry and take it without wetting your fingers.

The reason behind these phenomena is not difficult to understand. On heating, the air in the glass expanded, just as all bodies would do, and the extra amount of air came out of the glass. But when the remaining air began to cool down, its amount was no longer enough to exert its previous pressure, i.e. to balance out the external pressure of the atmosphere. Therefore, each square centimetre of the water under the glass was now subject to less pressure than the water in the exposed part of the plate and so no wonder it was forced under the glass by the extra pressure. In consequence, the water was not really "sucked in" by the glass, as it might seem, but pushed under the glass from the outside.

Now that you know the explanation of the phenomenon in question, you will also understand that it is by no means necessary to use in the experiment a burning piece of paper or cotton wool soaked in alcohol (as is sometimes advised), or any flame in general. It suffices to rinse the glass with boiling water and the experiment will be as much of a success. The key thing here is to heat the air in the glass, no matter how that is done.

The experiment can be performed simply in the following form. When you have finished your tea, pour a little tea into your saucer, turn your glass upside down while it is still hot, and stand it in the saucer and tea. In a minute or so the tea from the saucer will have gathered under the glass.

Parachute

Make a circle about a metre across out of a sheet of tissue-paper and then cut a circle a few centimetres wide in the middle. Tie strings to the edges of the large circle, passing them through small holes; tie the ends of the strings, which should be equally long, to a light weight. This completes the manufacture of a parachute, a scaled-down model of the huge umbrella that saves lives of airmen who, for some reason or other, are

Figure 41

compelled to escape from their aircraft.

To test your miniature parachute in action drop it from a window in a high building, the weight down. The weight will pull on the strings, the paper circle will blossom out, and the parachute will fly down smoothly and land softly. This will occur in windless weather but on a windy day your parachute will be carried away however weak the wind and it will descend to the ground somewhere far from the starting point.

The larger the "umbrella" of the parachute, the heavier the weight the parachute will carry (the weight is necessary for the parachute not to be overturned), the slower it will descend without a wind and the farther it will travel with a wind.

But why should the parachute keep up in the air so long? Surely, you've guessed that the air stops the parachute from falling at once. If it were not for the paper sheet, the weight would hit the ground quickly. The paper sheet increases the surface of the falling object, yet adding almost nothing to its total weight. The larger the surface of an object, the more drag there is on it.

If you've got it right, you'll understand why particles of dust are carried about by the air. It is widely believed that dust floats in air because it is lighter. Nonsense!

What are particles of dust? Tiny pieces of stone, clay, metal, wood, coal, etc., etc. But all of these materials are hundreds and thousands of times heavier than air; stone, is 1,500 times heavier; iron, 6,000 times; wood, 300 times, and so on.

A speck of solid or liquid should infailingly fall down through the air, it "sinks" in it. It does fall, only falling it behaves like a parachute does. The point is that for small specks the surface-to-weight ratio is larger than for large bodies. Stated another way, the particles' surfaces are relatively large for their weight. If you were to compare a round piece of lead shot with a round bullet that is 1,000 times as heavy as the shot, the shot's surface is only 100 times smaller than the bullet's. This implies that the shot's surface per unit weight is 10 times larger than the bullet's. Imagine that the shot shrinks until it becomes one million times lighter than the bullet, that is, turns into a speck of lead. Its "specific" surface would be 10,000 times larger than the bullet's. Accordingly, the air would hinder its motion 10,000 times more strongly than it does the bullet's.

That's why it would *hover* in the air hardly falling and being carried by the slightest wind away and even upwards.

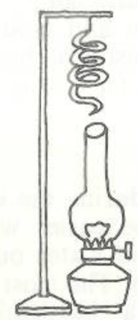
A Snake and a Butterfly

Figure 42

Cut a circle about the size of a glass hole from a postcard or a sheet of strong paper. Cut a spiral in it in the form of a coiled-up snake, as shown in Fig. 42. Make a small recess in the tail to receive a knitting-pin fixed upright. The coils of the snake will hang down forming sort of a spiral stairs.

Now that the snake is ready, we can set out to experiment with it. Place it near a hot kitchen stove: the snake will spin and the faster, the hotter the stove. Near any hot object (a lamp or tea kettle, etc.) the snake will rotate while the object remains hot. So, the snake will spin very fast if suspended above a kerosene lamp from a piece of string.

Figure 43



What makes the snake rotate? The same thing that makes the arms of a windmill rotate – the flow of air. Near every heated object, there is an air flow moving upwards. This flow occurs because air, just like any other material, expands on heating and becomes thinner, i. e. lighter. The surrounding air, which is colder and thus denser and heavier, displaces the hotter air, making it rise, and occupies its place. But the fresh portion of air heats at once and, just like the first one is ousted by a yet fresher amount of colder air. In this way, each heated object gives rise to an ascending flow of air around it, which is maintained all the time the object is warmer than the surrounding air. In other words, a barely noticeable warm wind blows upwards from every heated object. It strikes the coils of our paper snake making it rotate, just as wind makes the arms of a windmill rotate.

Instead of a shake you can use a piece of paper in another shape, for example a butterfly. Cut it out of tissue-paper, bind in the middle and suspend from a piece of a very thin string or hair. Hang the butterfly above a lamp and it will rotate like a live one. Also, the butterfly will cast its shadow on the ceiling and the shadow will repeat the motions of the rotating paper butterfly magnified up. It'll seem to an uninitiated person that a large black butterfly has flown into the room and is hectically hovers under the ceiling.

You can also make as follows: strick a needle into

a cork and place the paper butterfly on the needle's tip at the point of equilibrium which can be found by trial and error. The butterfly will rotate quickly if placed above a warm thing, in fact putting your palm under it will be enough for the butterfly to rotate.

We come across the expansion of air as it heats and ascending warm currents everywhere.

It is well known that the air in a heated room is the warmest near the ceiling and the coldest near the floor. That's way it seems sometimes that there is a draught near our feet when the room hasn't properly heated up. If you leave the door from a warm room to a colder one ajar, cold air will flow into the warm one along the floor and warm air will flow out along the ceiling. The flame of candle placed near the door will indicate the direction of these flows. If you want to keep the warmth in a heated room you should see to it that no cold air comes in from under the door. You need only to cover the gap by a rug or just a newspaper. Then the warm air won't be ousted from below by the colder one and won't leave the room through holes higher up in the room.

And what is the draught in a furnace or a chimney stack but an ascending flow of warm air?

We could also discuss the warm and cold flows in the atmosphere, trade winds, monsoons, breezes and the like but it would lead us too far astray.

Ice in a Bottle

Is it easy to get a bottle full of ice during the winter? It would seem that nothing could be easier when it is frosty outdoors. Just put a bottle of water outside the window and let the frost do the job. The frost will cool the water and you will have a bottleful of ice.

But if you actually try to do the experiment, you'll see that it is not that easy. You will obtain ice but the bottle will be destroyed in the process, it'll burst under the pressure of the freezing water. This occurs because water, on freezing, expands markedly, by about a tenth of its volume. The expansion is so powerful that it bursts both a corked bottle and the bottleneck of an open bottle, the water frozen in the neck becomes, as it were, an ice cork.

The expansion of freezing water can even break metal walls if they are not too thick. So, water can break the 5-cm walls of a steel bomb. No wonder that

Figure 44

water pipes burst so often in winter.

The expansion of water on freezing also accounts for the fact that ice floats on water and doesn't sink. If water contracted on cooling, just like all other liquids do, then ice wouldn't float on the water's surface but would go down to the bottom. And those of us in northern countries wouldn't enjoy skating and travelling on the ice of our rivers and lakes.

To Cut Ice and... Leave It One Piece

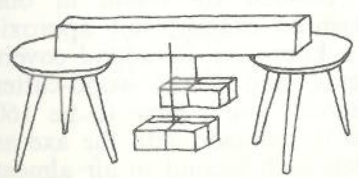
You may have heard that pieces of ice "freeze up" under pressure. This doesn't mean that pieces of ice freeze up more strongly when exposed to pressure. On the contrary, under strong pressure ice *melts*, but once the cold water produced in the process is free of the pressure, it refreezes (as its temperature is below 0 °C). When we compress pieces of ice, the following occurs. The ends of the parts that contact each other and are subject to high pressure melt, yielding water at a temperature below zero. This water fills in tiny interstices between the parts that are sticking out and when it is not subjected to the high pressure any more it freezes at once, thus soldering the pieces of ice into a solid block.

You can test this by an elegant experiment. Get a beam of ice, and support its ends by the edges of two stools, chairs or the like. Make a loop of a thin steel wire 80 centimetres long and put it round the beam, the wire should be 0.5 millimetre or a little less thick. Finally, suspend something heavy (about 10 kilogrammes) from the ends of the wire. Under the pressure of the heavy object the wire will bite into the ice and cut slowly through the whole of the beam but... the beam will still remain one piece. You may safely take it in your hands as it will be intact as if it had not been cut!

After you've learned about the freezing up of ice, you'll see why it works. Under the thrust of the wire the ice melted but the water flowed over the wire and free of the pressure refroze at once. In plain English, while the wire cut the lower layers, the upper layers were freezing again.

Ice is the only material in nature with which you can do this experiment. It's for this reason that we can skate and toboggan over ice. When a skater presses all his weight on his skates, the ice melts under the

Figure 45



pressure (if the frost is not too severe) and the skate slides along where it again melts some ice and the process occurs continuously. Wherever the skate goes a thin layer of ice turns into water that when free of the pressure refreezes. Therefore, although the temperature might be below freezing point, the ice is always "lubricated" with water under skates. That's why it's so slippery.

Sound Transmission

You may have observed from a distance a man using an axe or a carpenter driving in nails. You may then have noticed an unusual thing, you do not hear the stroke when the axe touches the tree or when the hammer hits a nail, but you hear it later when the axe (or hammer) is ready for the next stroke.

Next time you happen to observe something similar, move a little forward or backward. After trying several times you'll find a place where the sound of a stroke comes just at the moment of a visible stroke. Then return to where you started and you'll again notice the lack of coincidence between the sound and the visible stroke.

Now it should be easy for you to guess the reason behind this enigmatic phenomenon. Sound takes some time to cover the distance from the place where it originated to your ear; on the other hand, light does it nearly instantaneously. And it may happen that while the sound is travelling through the air to your ear, the axe (or hammer) will have been raised for a new stroke. Then your eyes will see what your ears hear and it'll seem to you that the sound comes when the tool is up and not when the tool is down. But if you move in either direction just a distance, covered by the sound during a swing of the axe, then by the time the sound reaches your ear the axe will strike again. Now, of course, you'll see and hear a stroke simultaneously, only it'll be *different* strokes since you'll hear an *earlier* stroke, perhaps the last but one or even earlier.

What is the distance covered by sound in one second? It has been measured exactly, but approximately it is about 1/3 of a kilometre. So sound covers one kilometre in 3 seconds, and if the wood-cutter swings his axe twice a second, you'll have to be 160 metres away for the sound to coincide with the axe as he raises it. But light travels each second in air almost

a million times as far as sound. So you can understand that for any distances on earth we can safely take the speed of light to be infinite.

Sound is transmitted not only through the air but also through other gases, liquids and solids. So in water, sound travels four times faster than in air, and under water sound can be heard distinctly. People working in underwater caissons can hear sounds from the shore perfectly and anglers will tell you how fish scatter at the slightest suspicious noise from the shore.

Elastic solids are better still as sound transmitters, e.g. cast iron, wood, and bone. Put your ear to the end face of a long wood beam or a block and ask somebody to tap it slightly at the other end. You'll hear the dull sound of the stroke transmitted through the entire length of the beam. If it's rather quiet and spurious noises don't interfere, you can even hear a clock ticking at the opposite end of your beam. Sound is transmitted equally well along iron rails or beams, cast iron tubes, and even soil. If you put your ear to the ground you can hear the clatter of horses' hoofs long before the sound comes through the air and in this way you can hear thunder that is so far away that no sound comes to you by air at all.

Only *elastic* solids transmit sound so well, soft tissues and loose, inelastic materials are very poor sound transmitters since these "absorb" it. That's why they hang thick curtains near doors if they don't want any sound to reach an adjacent room. Carpets, soft furniture and clothes have the same effect on sound.

A Bell

Among the materials distinguished for their perfect sound transmission I've mentioned *bones*. Do you want to make sure that the bones of your skull have this property? Hold the ring of a pocket watch with your teeth and close your ears with your hands. You'll still quite distinctly "hear" the measured strokes of the balance, and they'll be louder than the ticking perceived through the ear. This sound comes to your ears through the bones of your skull.

A further fascinating experiment testifying to the good transmission of sound through your skull. Tie a soup spoon in the middle of a piece of string so that the string has two loose ends. Press these ends with your fingers to your closed ears and, leaning forward

Figure 46

for the spoon to swing freely, make it strike something solid. You'll hear a low-pitched drone as if a bell is ringing near your ears.

The experiment comes out better if you use something heavier instead of the spoon.

A Frightening Shadow

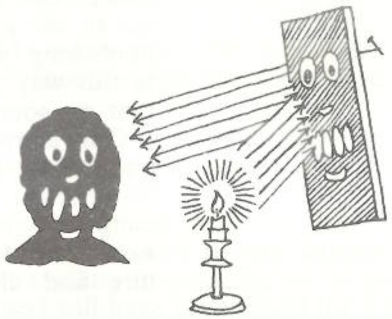
One evening my brother Alex asked, "Want to see something unusual? Come into this room."

The room was dark. Alex took a candle and we walked in. I led the way and so was the first to enter the room. But suddenly I was stunned: an incongruous monster eyed me from the wall. Flat as a shadow, it stared at me.

To tell the truth, I got a little frightened. I might have taken to my heels had it not been for my brother's laughter behind me.

I turned round and saw the reason. There was

Figure 47



a mirror on the wall covered with a sheet of paper that had eyes, a nose, and a mouth cut in it. Alex had so directed the candle's light that these parts of the mirror reflected directly onto my shadow.

Thus, I was scared by my own shadow.

But later, when I attempted to play this joke on my friends, it turned out that arranging the mirror properly is not that easy. It took a lot of practice to master the art. Light rays are reflected in a mirror according to the following rule: the angle at which light rays strike the mirror equals the angle at which they are reflected. After I'd learned the rule it was no problem to work out where to locate the candle with respect to the mirror for the light spots to be cast at the required place on the shadow.

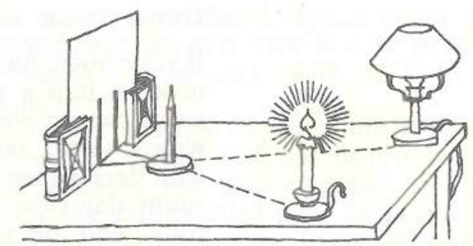
To Measure Light Brightness

At twice the distance, clearly, a candle illuminates much weaker. But how many times weaker? Two? No, if you place two candles at double the distance, you won't obtain the previous illumination. In order to obtain the earlier illumination at double the distance you'd have to put two times two, i.e. four candles, not two. At triple the distance you'd need three times three, i.e. nine candles, not three, and so forth. It follows that at twice the distance illumination is four times weaker; at three times the distance, nine times weaker; at four times the distance, 16 times; and at five times the distance, 5×5 or 25 times weaker, and so on. This is the law of weakening illumination with distance. Note in passing that this is also the law of sound attenuation with distance. For example, sound attenuates on six times the original distance by 36 times, not by 6 times*.

Knowing this law we can make use of it to compare the brightness of two lamps, or any two light sources in general. For instance, you wish to compare the brightness of your lamp with that of a conventional candle, in other words, you want to find out how many candles you need to replace the lamp to obtain the same illumination.

For this purpose place the lamp and a burning candle at one end of a table and at the other you stand a sheet of white cardboard clamped between books as

Figure 48



* This explains why a whisper from your neighbour drowns the loud voice of an actor on the stage in a theatre. If the actor is only 10 times farther away from you than your neighbour, then the actor's voice is attenuated 100 times more than what you'd hear if the same sound came to you from the lips of your neighbour. It's not surprising then that for you the actor's voice is weaker than the whisper. For exactly the same reason it's important for students to keep quiet when the teacher speaks. The teacher's words reaching students (especially those far away) are so attenuated that even a soft whisper from a neighbour will muffle them completely.

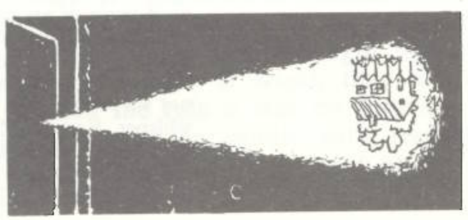
shown. Just in front of the sheet fix up a stick, e.g. a pencil, also upright. It will cast two shadows onto the cardboard, one from the lamp and the other from the candle. The density of the two shadows is, generally speaking, different because both are lit, one by the bright lamp, the other by the dimmer candle. By bringing the candle nearer you can achieve a situation in which both shadows will have the same "blackness". This will mean that the illumination due to the lamp just equals that due to the candle. But the lamp is farther away from the cardboard than the candle. Measuring how many times farther away will tell you how many times the lamp is brighter than the candle. If, say, the lamp is three times farther away from the cardboard than the candle, then its brightness is 3×3, i.e. nine times the brightness of the candle. Remember the law?

Another way of comparing the luminous intensity of two sources relies on the use of an oil spot on a sheet of paper. The spot will seem light if illuminated from behind, and dark if lit from the front. So the sources to be compared can be placed at such distances that the spot will seem equally illuminated on either side. Then it only remains to measure the respective distances and repeat the previous process. And in order that both sides of the spot might be compared it is a good idea to place the paper near a mirror, you should know how.

Upside Down

If your home has a room facing south, you could easily make it into a physical device that has an old Latin name *camera obscura.* You'll have to close the window with a shield, made of a plywood or cardboard glued with dark paper with a small hole made in it. On a fine sunny day close the doors and windows to darken the room and place a large sheet of paper or a sheet opposite the hole. This will be your "screen". You'll immediately see on it a reduced image of what can be

Figure 49



seen from the room through the hole. Houses, trees, animals, and people, everything will appear on the screen in its natural colours, but... upside down.

What does this experiment prove? That light propagates in straight lines. The rays from the upper and lower parts of an object cross in the hole and travel on so that now the top rays appear below and the bottom rays above. If the rays were not straight but curved or broken, you'd have got something different.

Significantly, the shape of the hole has no effect whatsoever on the image. You might drill a round hole, or make a square, triangular, hexagonal, or other hole – the image on the screen would be the same. Did you happen to observe oval light circles under a dense tree? These are nothing but images of the Sun painted by the rays that pass through various gaps between the leaves. The images are roundish because the sun is round, and elongated because the rays are obliquely incident on the ground. Put a sheet of paper at right angles to the solar rays and you'll obtain round spots on it. During solar eclipses when the dark sphere of the moon blots out the sun leaving only a bright crescent, the small spots under trees turn into small crescents as well.

The old photographer's camera, too, is nothing but a *camera obscura,* the only difference being that at the hole an objective lens is fitted for the image to be brighter and clearer. The back wall is a frosted glass on which the image is produced, upside down of course. The photographer can only view it if he covers himself and the camera by a dark cloth to keep out any spurious light.

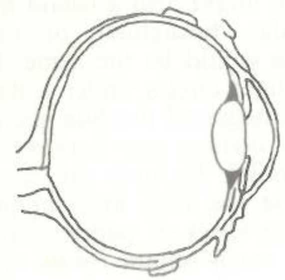
You can make a simple model of this sort of camera. Find a closed elongated box and drill a hole through one wall. Remove the wall opposite the hole and stretch over the gap an oiled piece of paper instead – a substitute for the frosted glass. Bring the box into our dark room and place it so that its hole is just opposite the hole in the darkened window. On the back side you'll see a distinct image of the outside, again upside down of course.

Your camera is convenient in that you no longer need a dark room and you can bring it out into the open and put it where it suits you. You'll only need to cover your head and the camera with a dark cloth for the spurious light not to interfere.

Overturned Pin

We have just discussed the *camera obscura* and a way of manufacturing it but we omitted one interesting thing: every human being always has a pair of small cameras like that about him or her. These are our eyes. Just fancy, your eye is just like the box that you were shown how to make above. What we call the *pupil* of the eye is not a black circle on the eye but a hole leading into the inside of your organ of sight. The hole is covered with a transparent envelope on the outside and with a jelly-like and transparent substance underneath. Next to the pupil behind it is the *crystalline lens* having the form of a convexo-convex glass, and the inner cavity of the eye between the crystalline lens and the back wall, on which the image is produced, is filled with a transparent substance. A cross section through the eye is given in Fig. 50. Despite all these distinctions the eye is still a *camera obscura*, only an improved one, as the eye produces high-quality, distinct images. The images at the back of the eye are minute. So, an 8-m high lamp-post seen 20 metres away from the eye is only a tiny line, about 5 millimetres long, at the back of the eye.

Figure 50

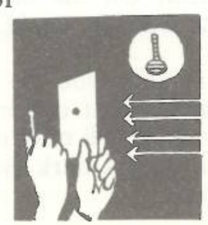


But the most interesting thing here is that although all the images are upside down, we perceive them as they are. This turning over is due to long habit. We are used to seeing with our eyes so that each visual image obtained is converted into its natural position.

That this is really true, you could test by an experiment. We'll attempt to contrive it so that we get at the back of the eye not an inverted, but direct image of an object. What will we see then? Since we are used to inverting every visual image, we'll invert this one as well. Accordingly, we'll in this case too see an inverted image, not a direct one. In actual fact that is exactly what happens and the following experiment will demonstrate it in a fairly graphic manner.

Make a pinhole in a postcard and hold it against a window or a lamp about 10 centimetres away from your right eye. Hold the pin between you and the postcard so that its head is opposite the hole. With this arrangement you'll see the pin as if it were *behind* the hole, and what is of the more importance here, *upside down*. This unusual situation is presented in Fig. 51. Move the pin to the right and your eyes will tell you it's moved to the left.

Figure 51



The explanation is that the pin at the back of your eye is here depicted not upside down but directly. The hole in the card plays the role of a light source producing the shadow of the pin. The shadow falls on the pupil and its image is not inverted as it's too close to the pupil. On the back wall of the eye a light spot is produced – the image of the hole in the card. On it the dark silhouette of the pin is seen which is its shadow, the right way up. But it seems to us that through the hole in the card we see the pin behind the card (as only the part of the pin that gets in the hole is seen) and inverted at that because our eyes are in the habit of turning images upside down.

Igniting with Ice

When a boy, I liked watching my brother lighting a cigarette with a magnifying glass. He would put the glass in the sunlight and train the spot of light on the cigarette end. After a while it would begin to give off a bluish smoke and smolder.

One winter day Alex said, "You know, it's possible to light a cigarette with ice, too."

"With ice?"

"Ice doesn't ignite it, of course, the sun does. Ice only collects solar rays, just like this glass."

"So you want to make a magnifying glass out of ice?"

"I can't make glass of ice, nobody can, but we could make a burning lens from ice."

"What's a lens?"

"We'll shape a piece of ice like this glass and it will be a lens: round and convex which means thick in the middle and thin at the edges."

"And will it ignite things?"

"Yes."

"But it's cold!"

"What of it? Let's try."

To begin with, my brother told me to fetch a washing basin. When I did he rejected it: "Nothing doing. You see, the bottom is flat. We need a curved bottom."

When I brought a suitable basin, Alex poured some clean water into it and put it outside, the temperature outdoors being below freezing point.

"Let it freeze down to the bottom. We'll then have an ice lens with one side flat and the other convex."

"So big?"

"The bigger, the better, it'll catch more sunlight."

First thing in the morning I ran to inspect our basin. The water had frozen right through to the bottom.

"What a good lens we'll have," Alex said tapping the ice with finger. "Let's take it out of the basin."

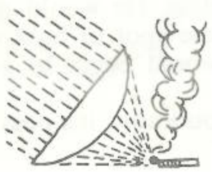
This turned out to be no problem. Alex put the icy basin into another one containing hot water and the ice at the walls melted quickly. We got the ice basin out into the yard and placed the lens on a board.

Figure 52

"Good weather, ins't it!" Alex screwed up his eyes in the sunlight, "Ideal for igniting. Just hold the cigarette."

I did so and my brother, taking hold of the lens with both hands turned it towards the sun but so that he wasn't in the way of the rays himself. He took aim painstakingly but eventually succeeded in training the lightspot directly on the end of the cigarette. When the spot rested on my hand, I felt it was hot and already I had no doubt that the ice would light the cigarette.

Figure 53



Indeed, when the spot got onto the end of the cigarette and had stayed there for about a minute, the tobacco smoldered and discharged some bluish smoke.

My brother took a puff at the cigarette, "Here you are, we've lit it with ice. In this way you could make a fire without matches even at the pole, if only you had firewood."

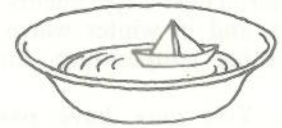
Magnetic Needle

You can already make a needle float on the surface of water. Here you'll have to use your skill in a new and more impressive experiment. Find a magnet, if only a small horse-shoe one. If you bring it near the saucer with a needle floating in it, the needle will obediently approach the appropriate edge of the saucer. The effect will be more noticeable, if before placing the needle on the water you pass the magnet several times along it (but only use one end of the magnet in one direction only). This turns the needle itself into a magnet, therefore it'll even approach a nonmagnetized iron object.

You can make many curious observations with the magnetic needle. Leave it alone without attracting it by a piece of iron or the magnet and it'll orient itself in the water in one direction, namely north-south, just like the needle of a compass. Turn the saucer and the needle will still point to the north with one end and to the south with the other. Bring one end (pole) of the

magnet to an end of the needle and you'll find that it won't be attracted to the magnet at that end. It may turn away from the magnet in order that the opposite end might approach. This is a case of an *interaction* between two magnets. The law of this interaction states that *unlike* ends (the north pole of one magnet and the south pole of another) are attracted and *like* ones (both north or south) are repelled.

Figure 54



Having investigated the behaviour of the magnetized needle, make a toy paper boat and hide your needle in its folds. You might astonish your uninitiated friends by controlling the motion of the boat without so much as touching it: it would obey every motion of your hand. Of course, you would be holding the magnet so that the spectators wouldn't suspect it.

Magnetic Theatre

Or rather circus, as starring in it are rope dancers cut out of paper (of course).

First of all, you have to make the circus building out of cardboard. At the bottom of it you'll stretch a wire and fix above the stage a horse-shoe magnet, as shown.

Now to the artists. They are cut out of paper, their stance being chosen to suit the purpose. The only condition is that their height be equal to the length of a needle glued on from behind along the length of the

Figure 55

figure. You could use two or three drops of sealing-wax for the glue.

If a figure like this is installed onto the "rope", it not only won't fall, but will stay upright pulled by the magnet. By slightly jerking the wire you'll animate your rope dancers. They'll swing and jump all the while keeping their balance.

Electrified Comb

Even if you're ignorant of electricity and not even acquainted with its ABC, you can still do a number of electrical experiments that would be fascinating and will, in any case, be useful when you meet this amazing force of nature in future.

The best place for these electrical experiments is a warm room in a frosty winter. These experiments are especially successful in dry air, and in winter warm air is far drier than air at the same temperature during summer.

Now, to our experiments. You may have passed a conventional comb over dry (*completely* dry) hair. If you did so in a warm room in full silence, you may have heard some slight crackling on the comb. Your comb had been electrified by friction with the hair.

The comb can also be electrified by material other than hair. If you rub it against a dry woollen fabric (a piece of flannel, say) it also acquires electrical properties and to quite a larger degree. These properties manifest themselves in a wide variety of ways, notably by attracting light objects. Bring a rubbed comb close to some pieces of paper, chaff, a ball of elder core, etc. and these small things will all stick to the comb. Make tiny ships of light paper and launch them on water. You'll be able to control the movements of your paper fleet using an electrified comb like a magic wand. You could stage the experiment in a more impressive way. Place an egg in a dry egg-support and balance a rather long ruler on it. As the electrified comb approaches one of its ends the ruler will turn fairly quickly. You can make it follow the comb obediently moving it in any direction and making it rotate.

Figure 56

An Obedient Egg

Electrical behaviour is inherent not only in the comb but in other things as well. A rod of sealing-wax rubbed against a piece of flannel or the sleeve of your coat, if it's woollen, behaves in the same way. A glass rod or tube, too, is electrified if rubbed by silk. But the experiment with silk is only a success in exceedingly dry air and only then if both the silk and the glass are well dried by heating.

Here is a further funny experiment on electrical attraction. Empty a chicken egg through a small hole,

which is best done by blowing the contents out through another hole at the opposite end. You've thus obtained an empty shell (the holes are sealed with wax). Put it on a smooth table, board or large plate and, using the electrified rod, make the empty egg roll obediently after it. An outsider, not aware that the egg is empty, would

Figure 57

be bewildered by experiment (invented by the English scientist Faraday). A paper ring or a light ball, too, follow an electrified rod.

Interaction

Mechanics teaches that one-sided attraction, or any one-sided action, in general, doesn't exist. Any action is, in fact, an interaction. In consequence, if the electrified rod attracts various things, then it itself is attracted to them. To bear this out you have only to make the comb, or rod, easily movable, e.g. by suspending it from a loop made of a piece of thread (the thread should preferably be a silk one).

Figure 58

Then, you will quickly find that any electrified thing– your hand, say–attracts the comb making it turn, and so forth.

To repeat, this is a general law of nature. It shows up always and everywhere–any action is an interaction of two bodies affecting each other in opposite directions. Nature doesn't know of an action that is one-sided and doesn't involve the interaction of another body.

Electrical Repulsion

Let's return to the experiment with the suspended electrified comb. We've seen that it is attracted by any electrified body. It would be of interest to test the way in which another, also electrified, thing affects it. An experiment will convince you that two electrified bodies can interact in different ways. If you bring an electrified glass rod to the electrified comb, the two things will attract each other. But if you bring an electrified sealing-wax rod or another comb to the comb, the interaction will be repulsive.

The physical law describing this fact of nature states: *unlike* charges attract, *like* charges repel. Like charges

will be those on plastics and sealing-wax (the so-called amber or *negative*, charge) and unlike charges are those on amber and glass which is *positive*. The ancient names "amber" and "glass" charges have now gone out to use, being completely replaced by the names "negative" and "positive" charges.

The repulsion of like-charged things lies at the basis of a simple device to detect electricity – the so-called electroscope. The word "scope" comes from Greek and means to "indicate", it enters words like "telescope", "microscope", and so forth.

Figure 59

You can make this simple device on your own. Through the middle of a cardboard circle or a cork that fits the neck of a jar or bottle, a rod is passed, part of it protruding from the top. To the end of the rod two strips of foil or tissue-paper are attached using wax. Next the neck is plugged with the cork or cardboard circle, sealing the edges with sealing wax. The electroscope is ready to use. If now you bring an electrified thing to the protruding end of the rod, the two strips will become electrified, too. They charge up simultaneously and, therefore, separate due to electrostatic repulsion. The separation of the strips is the indication that the thing that touched the electroscope rod is electrified.

Figure 60

If you are no good at handiwork, you could make a simpler version of the device. It won't be as convenient and sensitive, but will still work. Suspend two elder-core balls on a stick from pieces of string so that they hang in contact with each other. That's all. On touching a ball with a thing being tested you'll notice that the other ball deflects if the thing is charged.

Finally, in the accompanying figure you can see yet another form of a primitive electroscope. A foil strip, folded in two, is suspended from a pin stuck into a cork. Touching the pin with an electrified thing makes the strips separate.

One Characteristic of Electricity

With the help of an easily manufactured makeshift device you can observe the interesting and very important feature of electricity – to accumulate on the surface of an object only, and on protruding parts at that.

Cement a match vertically to a match box using a sealing-wax drop, then make another such support.

Figure 61

Now cut out a paper strip about a match-length wide and three match-lengths long. Turn the ends of the paper strip into a tube so that you could fix it to the supports. Glue three or four narrow ribbons of thin paper-tissue to the either side of the strip (Fig. 61) and fix the assembly on the matches.

Our device is ready for experiments. Touch the straight strip with an electrified sealing-wax rod and the paper and all the ribbons on it will charge up simultaneously. This can be judged by the ribbons sticking out on either side. Now arrange the supports so that the strip curves into an arc, and charge it up. The strips will now stick out on the convex side only, those on the concave one will dangle as before. What does this indicate? That the electric charge has only accumulated on the convex side. If you make the strip into an S-shape, you'll see that the electric charge is only present on the convex parts of the paper.

Alex put the ruler on a table so that a part of it overhung the edge.

"Touch the protruding end. It's easy to press it down, isn't it? Well, try to press it down after I've covered the other end with the newspaper."

He spread the newspaper on the table over the ruler, carefully smoothing the folds.

"Now take a stick and strike the protruding part of the ruler very hard. Strike with all you strength!"

I swung the stick back, and said, "I'll strike it so hard that the ruler will break through the paper and hit the ceiling!"

"Go ahead, don't spare your strength."

The result was astonishing: there was a crack, the ruler broke, but the newspaper remained on the table, still covering the other piece of the ruler.

Figure 63

Alex asked archly, "The newspaper appears to be heavier than you've been thinking?"

Bewildered, I shifted the eyes from the fragment of the ruler to the newspaper.

"Is it an experiment? Electric?"

"It's an experiment but not electric one. The electric ones will follow. I just wanted to show you that a newspaper can actually be a device to do physical experiments with."

"But why didn't it let the ruler go? Look, I can easily lift it from the table."

That is the kernel of the experiment. Air presses down on the ruler with a powerful force: a good solid kilogramme on each centimetre of the newspaper. When you strike the protruding end of the ruler, its other end pushes up against the newspaper from below, and so the newspaper should rise. If it's done slowly some air gets under the rising paper and compensates for the pressure from above. But your stroke was so fast that air had no time to get under the paper. Thus the edges of the paper were still sticking to the table when its middle was already being forced upwards. Therefore, you had to lift not only the paper but also

the paper with the air pressing down on it. In a nutshell, you had to lift with the ruler as many kilogrammes as there were square centimetres in the newspaper. If it were an area of only 16 square centimetres (a square with a side of 4 centimetres), then the air pressure would be 16 kilogrammes. But the area you lifted was notably larger, and accordingly, you had to lift a substantial weight, perhaps something near 50 kilogrammes. The ruler couldn't bare this load and broke. Now do you believe that a newspaper can be used for physical experiments? After dark, we'll make the experiments."

Sparks from Fingers ● Obedient Stick ● Electricity in Mountains

My brother took a clothes-brush in one hand and held the newspaper against the warm stove with the other. He then began to rub the newspaper with the brush like a decorator smoothing wall-paper on the wall for the paper stick perfectly.

"Look!" Alex said and took both hands away from the paper.

I had expected that the paper would slide down onto the floor. This, however, didn't happen: strange as it was, the paper stuck to the smooth tiles as if glued.

I asked, "How does it keep on? It's not smeared with glue."

"The paper is held by electricity. It's now electrified and attracted to the stove."

"Why didn't you tell me that the newspaper in the bag was electrified?"

"It wasn't. I did it right now, before your eyes, by rubbing it with the brush. The friction electrified it."

"So, it's a real electric experiment?"

"Yes, but we're just beginning... Turn off the lights please."

In the dark the black figure of my brother and the greyish spot of the stove looked blurred.

"Now watch my hand."

I guessed, rather than saw what he did. He took the paper down from the stove and, holding it with one hand, moved his spread fingers of the other hand to it.

And then – I could hardly believe my eyes – sparks flew out from his fingers, bluish-white sparks!

"The sparks were electricity. Want to try for yourself?"

Figure 64

I promptly hid my hands behind the back. Not for the world!

My brother again applied the paper to the stove, brushed it and again produced an avalanche of long sparks from his fingers. I managed to notice that he didn't touch the newspaper at all, but held his fingers about 10 centimetres away from it.

"Don't be scared, just try, it doesn't hurt. Give me your hand," he took hold of my hand and led me to the stove. "Spread your fingers!.. Well! Does it hurt?"

In a twinkling of an eye a bundle of bluish sparks shot out from my fingers. In their light I saw that my brother had only partially detached the newspaper from the stove, the lower part being still "glued". Simultaneously with the sparks I felt a slight prick but the pain was trifling. Indeed, nothing to be scared of.

"Again!" I asked.

Alex applied the newspaper to the stove and began to rub, this time only with his palms.

"What are you doing? Have you forgotten the brush?"

"It's all the same. Now, are you ready?"

"Nothing doing! You've rubbed it with bare hands without using the brush."

"It's possible without the brush too if only your hands are dry. You just have to rub."

And it was true, this time also sparks rained from my fingers.

After I had the sparks to my heart's content my brother proclaimed: "That'll do. Now I am going to show you a flow of electricity, just like the one Columbus and Magellan saw at the tops of the masts of their ships... Pass me the scissors."

In the dark Alex brought the points of the spread scissors near to the newspaper, which was half-separated from the stove. I expected to see sparks but saw something new, the points of the scissors were crowned by glowing bundles of short bluish and reddish threads although the scissors were still far away from the paper. This was accompanied by faint prolonged hissing.

Figure 65

"Sailors often see the same sort of fire brushes, only far larger ones, at the mastheads and yardarms. They called them St. Elm's fires."

"Where do they come from?"

"You mean who holds an electrified newspaper above the masts? True, there's no newspaper there, but a low

90–91

electrified cloud. It's a substitute for the newspaper. You shouldn't think, however, that this sort of electric glow on pointed structures only occurs at sea, it's also observed on land, especially in the mountains. So, Julius Caesar wrote that on a night in a cloudy weather the spear heads of his legionnaires glowed this way. Sailors and soldiers are not afraid of these electric lights – on the contrary they view them as a good omen. Of course, without any reasonable grounds. In the mountains electric glows even occur on people at times, on their hair, caps, ears – that is, on all the protruding parts. In the process, they often hear a buzz, like the one produced by our scissors."

Figure 66

"Does this fire burn strongly?"

"Not at all. After all, this isn't a fire, but a glow, just a cold glow. So cold and harmless that it cannot even ignite a match. Look: instead of the scissors I use a match. And you see: the head is surrounded by the electric glow, but it doesn't go off."

"But I think it is burning because flames are coming out straight from the head."

"Turn on the light and inspect the match."

I made sure that the match not only hadn't charred but it hadn't even blacken. It was thus indeed surrounded by a cold light, and not fire.

"Leave the light on. We'll carry out the next experiment in the light."

Alex shifted a chair to the middle of the room and put a stick across its back.

After several tries he managed to balance the stick at one point.

"I didn't know that a stick could be supported in this way," I said, "it's so long."

"It works for exactly that reason. A short one wouldn't. A pencil, for example."

I agreed, "A pencil, no means."

"Now, to business. Can you make the stick turn towards you without touching it?"

I thought about it.

"If we loop a rope onto its end..." I began.

"No ropes, it must touch nothing. Can you?"

"Aha, insight!"

I put my face close to the stick and began sucking air into my mouth to attract the stick to me. It didn't stir, however.

"Any progress?"

"None. It's impossible!"

"Impossible? Let's see."

He took the newspaper, down from the stove, where that had been sticking to the tiles, and began slowly to move it sidewards towards the stick. At about half a metre away the stick "felt" the attraction of the electrified newspaper and obediently turned in its direction. By moving the newspaper Alex made the stick follow it rotating at the back of the chair, first in one direction, then in the other.

Figure 67

"The electrified newspaper, you see, attracts the stick so strongly that it follows and will follow the paper until all the electricity has flowed from the newspaper into the air."

"It's a pity that these experiments cannot be performed in the summer – the stove will be cold."

"The stove is only necessary to dry up the paper since these experiments are only a success with an absolutely dry newspaper. You may have noticed that newspapers absorb moisture from the air and therefore are always somewhat damp, that's why it has to be dried. You shouldn't think, however, that in summer our experiments are impossible. They can be done but not so well as in winter when the air in a heated-up room drier than in summer – that's the reason. Dryness is crucial for these experiments. In summer, a newspaper can be dried with a kitchen stove when it's not too hot for the paper not to ignite on it. After the paper has been dried adequately, it's brought onto a dry table and rubbed hard with a brush. The paper electrifies, but not as with a tile stove... Well, let's call it a day. Tomorrow we'll do some new experiments."

"Also electric?"

"Yes, and with the same electric machine, our newspaper. Meanwhile I'll give you an interesting account of Elm's fires by the famous French naturalist Saussure. In 1867 he with several companions climbed the Sarley Mountain, which is more than 3 kilometres high. And here's what they experienced there.

Alex took down the book *The Atmosphere* by Flammarion from the bookcase, thumbed through it and gave me the following passage to read:

"The climbers leaned their alpenstocks against a cliff and were preparing for their dinner when Saussure felt a pain in his shoulders and back as if a needle were being driven slowly into his body. 'Thinking some pins had got into my canvas cape,' recounted Saussure, 'I threw it off but there was no relief, on the contrary, the

pain became more accute and embraced the whole of my back from shoulder to shoulder. It was as if a wasp was walking all over my skin stinging it everywhere. I hastily threw off another coat but I could find nothing that could hurt so badly. The pain continued and came to feel like a burn. It seemed to me that my woollen sweater had caught fire and I was about to undress when my attention was attracted by a noise, a sort of hum. It came from the alpenstocks we had leaned against the cliff and resembled the rumbling of heated water about to boil. The noise continued for five minutes or so.

'I then understood that the painful sensation was caused by an electrical flux released by the mountain. In the broad daylight I didn't see any glow on the alpenstocks. They produced the same sharp noise whether they were held vertically with the tip pointed up and down, or horizontally. No sound came from the soil.

'In several minutes I felt that the hair on my head and beard were rising as if a dry razor was being passed over a stiff beard. My young companion cried out that the hair of his moustache was rising and the tops of his ears were giving off strong currents. Having raised my hands, I felt currents emanating from my fingers. In short, electricity was being liberated from sticks, clothing, ears, hair, in fact everything that was protruding.

'We left the summit hastily and descended about a hundred metres. As we were climbing down, our alpenstocks were producing ever lower noise and finally, the sound became so soft that we could only hear it by bringing them close to our ears.'"

In the same book I read about other cases of Elm's fires.

"The liberation of electricity by protruding rocks is often observed when the sky is covered by low clouds gliding just over summits.

"On June 10, 1863, Watson and several tourists climbed up the Jungfrau pass in Switzerland. It was a fine morning but the travellers got into a strong hail storm in the pass. A terrible clap of thunder came and soon Watson heard a hissing sound coming from his stick that resembled the sound of a kettle about to boil. The people stopped and found that their sticks and axes produced the same sound, and didn't stop making the sound even when stuck with one end into the

Miniature Lightning ● Experiment with a Water Stream ● Herculean Breath

On the next night my brother Alex made some unusual preparations.

He took three glasses, warmed them at the stove, put them on the table and covered with a tray that he had also preheated.

"What is it going to be?" I inquired. "Shouldn't the glasses be placed on the tray and not vice versa?"

"Just wait, take your time. It's going to be an experiment with miniature lightning."

Alex used the "electric machine" again – that is, he simply rubbed the newspaper on the stove. He then folded the newspaper in two and resumed the rubbing. Next he "unglued" it from the stove tiles and swiftly put on the tray.

"Feel the tray... Is it cold?"

Suspecting nothing, I light-heartedly stretched out my hand... and promptly jerked it back: something had cracked and pricked my finger painfully.

Alex laughed, "How did you like it? You were struck by a lightning. Heard the crack? That was miniature thunder."

"I felt a strong prick, but I didn't see any lightning."

"You will when we repeat it in the dark."

"But I won't touch that tray any more," I proclaimed decisively.

"That's not necessary, we can produce sparks, say, with a key or a tea-spoon. You'll feel nothing, but the sparks will be as long as they were earlier. The first sparks I'll extract myself while your eyes adapt to the dark."

He turned off the light.

"Silence now. Keep your eyes open!" a voice said in the dark.

Crack! and a bright bluish-white spark about 20 centimetres long darted between the edge of the tray and the key.

"See the lightning? Hear the thunder?"

"But they were at the same time. A real thunder always comes after you see the lightning."

"True. We always hear thunder later. Still, they occur simultaneously, rather like the crack and spark in our experiment."

"Why then is the thunder later?"

"You see, lightning is light, and it travels so fast as to

Figure 72

cover terrestrial distances in almost no time. Thunder is an explosion, i.e. sound, but sound travels in air not so fast and markedly lags behind the light, thus coming to us later. That's why we see a lightning flash before we hear the accompanying thunder."

Alex passed me the key, removed the newspaper and – my eyes had now adapted – suggested extract a "lightning" from the tray.

"Without the newspaper, will there be any spark?"

"Just try."

I had hardly put the key near the tray edge when I saw a spark, long and bright.

My brother again put the newspaper on the tray and again I extracted a spark, though it was weaker this time. He did so dozens of times (without rubbing the newspaper again on the stove), and each time I made a spark, which was getting ever weaker.

"The sparks would continue for a longer time if I held the newspaper silk strings or ribbons rather than with my bare hands. When you study physics you'll understand what occurred. Meanwhile it only remains for you to look with your eyes not head. Now one more experiment, with a water stream. We'll make it in the kitchen at the water tap. Let the newspaper stay on the stove."

We make a thin stream of water from the tap so that it hits the basin bottom loudly.

"Now, without touching the stream, I'll make it fall somewhere else. Which way do you want it to be deflected, left, right, or forward?"

I replied at random, "To the left."

"All right. Don't touch the tap while I fetch the newspaper."

Alex was back with the newspaper, trying to hold it with his arms outstretched so as not to lose too much electricity. He brought the newspaper close to the stream from the left and I clearly saw it bend to the left. Having transferred the newspaper on the other side, he made the stream deflect to the right. Finally, he drew it forward so much that the water poured over the basin edge.

"You see how strongly the attractive force of electricity manifests itself. By the way, this experiment can also be easily performed without a stove or oven. If, instead of the charged newspaper you take a conventional plastic comb like this one," my brother produced a comb and passed it through his thick hair.

Figure 73

"I have charged it up this way."

"But your hair is not electrical."

"No, it's just like yours or anybody else's. But if you rub plastic on your hair, it gets charged in the same way the newspaper does by the brush. Look."

When the comb was brought to the water stream, it made it deflect noticeably.

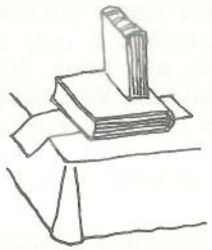
"The comb is unsuitable for other our experiments since it accumulates too little electricity. It gets far less than the 'electric machine' that can be made from a simple newspaper. I'd like to make one more experiment with the newspaper, the last one. This time it's not an electric experiment, but again one with air pressure, rather like the experiment with the ruler."

We returned to the sitting room and Alex began to cut and glue a long bag out of a newspaper.

"While it dries, get several books, large and heavy."

On the bookshelf I found three massive volumes of some medical atlas and placed them on the table. My brother asked, "Can you inflate this bag with your mouth?"

Figure 74



"Of course."

"A simple business, isn't it? But what if I put a couple of these volumes on the bag?"

"Oh, then the bag won't inflate no matter how hard you try."

Alex silently put the bag at the edge of the table, covered it with one of the volumes and stood another one upright on it.

"Just watch. I'll inflate the bag."

"Perhaps you want to blow those books away?" I asked laughing.

"Exactly."

Alex started to blow into the bag. Just imagine: as the bag swelled the lower book sloped up and overturned the top one. But the two books weighed about five kilograms!

Figure 75

Without allowing me to recover from my surprise, Alex prepared to repeat the trick. This time he loaded the bag with all three tomes. He blew–a Herculian breath!–and the three tomes overturned.

The amazing thing is that this experiment had nothing miraculous in it. When I dared to repeat it for myself, I managed to overturn the books as easily as Alex did. You need to have neither an elephant's lungs nor the muscles of an athlete–everything comes about on its own accord, nearly without effort.

My brother later explained the reason to me. When we inflated the paper bag, we forced some air into it that is more compressed than the air around us, otherwise the bag wouldn't expand. The air outside presses down with about 1,000 grammes on each square centimetre. If you express the area under the books in square centimetres, you can readily work out that even if the excess pressure in the bag is only a tenth of that outside of it, i.e. a hundred grammes per square centimetre, then the total force from the air pressure inside the part of the bag under the books may be as high as 10 kilogrammes. Clearly this force is sufficient to overturn the books.

Thus ended our physics tests with the newspaper.

Seventy-Five More Questions and Experiments on Physics

4

How to Weigh Accurately with An Inaccurate Balance

Which is the more important possession, a precise pair of weighing scales or a precise set of weights? Many people believe that the scales are more important, but in fact – the weights, since it's impossible to weigh anything accurately with inaccurate weights. If the set of weights is true, then you can still weigh quite accurately with inaccurate scales.

For example, suppose you have beam scales with pans. Place a weight that is heavier than your object on one pan. Then on the other pan put as many weights as will be required to make the beam balance. Next put your object onto the pan with the weights and, of course, this pan will sink. In order to balance the beam again you will need to remove some of the weights and the weights removed will show the correct weight of your object. It should be clear why: your object now pulls down its pan with the same force with which the weights you took off did before. Hence your object and the total of the weights you took off weigh the same.

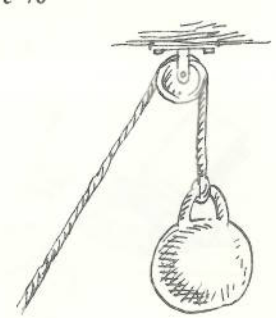
This excellent way of weighing accurately using inaccurate scales was discovered by the great Russian chemist Dmitri Mendeleyev.

On the Platform of a Weighing Machine

A man stands on the platform of a weighing machine and suddenly he squats down. Which way will the platform move, up or down?

The platform will move upwards. Why? Because as he is squatting the muscles pulling the man's body down also pull the legs up, thus reducing the force with which the body presses on the platform with the result that it goes up.

Figure 76



Weight on Pulley

Suppose a man is able to lift a mass of 100 kilogrammes from the floor. Wanting to lift more he passed a rope tied to the load through a pulley fixed in the ceiling (Fig. 76). What load will he be able to lift using this rig?

Such a pulley could help him lift no more than what

he could do with his own hands, perhaps even less. If he pulled the rope passed through a fixed pulley, he could not lift a mass exceeding his own. If his mass is less than 100 kilogrammes, he would be unable to handle a 100-kilogramme load with the pulley.

Two Harrows

People often confuse *weight* and *pressure*. However, they are by no means the same. An object may have a marked weight but still exert a negligible pressure on its support. By contrast something else may have a small weight but exert a large pressure on its support.

The following example will clarify the difference between weight and pressure and at the same time give you an idea of how to work out the pressure a body exerts on its support.

Let two harrows of the same type work in field, one with 20 teeth, the other with 60, the first one weighing 60 kilogrammes, the second 120 kilogrammes.

Which one penetrates more deeply into the soil?

It's easy to figure out that the greater the force acting on a harrow's teeth, the deeper they penetrate the soil. With the first harrow the total load of 60 kilogrammes is evenly distributed among the 20 teeth, hence 3 kilogrammes per tooth. With the second harrow, 120/60, i.e. 2 kilogrammes per tooth. Consequently, though in general the second harrow is heavier, its teeth penetrate less deeply than the first harrow's. The pressure per tooth with the first harrow is larger than with the second.

Pickled Cabbage

Consider another simple calculation of the pressure.

Two barrels of pickled cabbage are each covered with a wooden disk held down by stones. One disk is 24 centimetres across and the stones on it weigh 10 kilogrammes while the other is 32 centimetres across and its stones weigh 16 kilogrammes.

In which is the pressure larger?

Clearly, the pressure will be higher in the barrel where the load per square centimetre is larger. In the first case the 10 kilogrammes are distributed over an area* of $3.14 \times 12 \times 12 = 452$ square centimetres.

* The area of a circle is about 3.14 times the circle's radius (half the diameter) times the circle's radius.

Hence the pressure is 10,000/452, i.e. about 22 grammes per square centimetre. In the second barrel the pressure will be 16, 000/804, i.e. less than 20 grammes per square centimetre. The pickled cabbage is thus more compressed in the first barrel.

Awl and Chisel

Why does an awl penetrates deeper than a chisel does if both are acted upon by an equal force?

The point is that when thrusting the awl all the force is concentrated at an extremely small area at its point. With the chisel the force is distributed over a much larger surface. For instance, let the awl's surface area at the point be 1 square millimetre and the chisel's be 1 square centimetre. If the force on each tool is one kilogramme, then the material under the chisel blade is subjected to a pressure of 1 kilogramme per square centimetre, and under the awl $1/0.01 = 100$ or 100 kilogrammes per square centimetre (since 1 square millimetre = 0.01 square centimetre). The pressure of the awl is one hundred times larger than of the chisel. Now it is clear why the awl penetrates deeper than the chisel.

You'll now understand that when you are pressing with your finger on a needle when you are sewing you produce a very great pressure, not smaller than the steam pressure in a boiler. This is also the principle behind the cutting action of the razor. The slight force of hand creates a pressure of hundreds of kilogrammes per square centimetre on the thin edge of the razor that can cut through hair.

Horse and Tractor

A heavy crawler tractor is well supported by loose ground into which the legs of horses and people are mired. This is inconceivable to many people since the tractor is far heavier than the horse and very much heavier than man. Why then do the horse's legs are mired in loose ground, and the tractor doesn't?

To grasp this, you'll have to remember once again the difference between *weight* and *pressure*.

An object does not penetrate deeper because it is heavier but because it exerts a higher pressure (or force per square centimetre) on its support. The enormous weight of a crawler tractor is distributed over the larger

surface area of its tracks. Therefore, each square centimetre of the tractor's support carries a load as low as several grammes. On the other hand, the horse's weight is distributed over the small area under its hooves, thus giving more than 1,000 grammes per square centimetre or ten times more than the tractor. No wonder then that a horse's feet sink more deeply into mud than does a heavy crawler tractor. Some of you may have seen that to ride over marshes and bogs horses are shod with wide "shoes", which increase the supporting area of horses' hooves with the result that they are mired much less.

Crawling Over Ice

If ice on a river or lake is insecure, experienced people crawl rather than walk over it. Why?

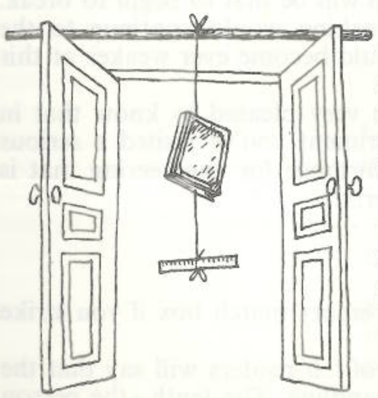
When a man lies down, his weight, of course, doesn't change, but the supporting area increases, each square centimetre of it thus carries less load. In other words, the man's pressure on his support is reduced.

It's now clear why it's safer to move over thin ice by crawling – this decreases the pressure on the ice. Some people also use a wide board and lie on it as they move about thin ice.

What load can ice support without breaking? The answer is dependent on the thickness of the ice. Ice 4 cm thick can support a walking man.

It is of interest to know the thickness of ice required for a skating rink on a river or lake. For this purpose 10-12 centimetres would be sufficient.

Figure 77



Where Will the String Break?

You'll need an arrangement shown in Fig. 77. Put a stick on top of the open doors, tie a string to the stick and tie a heavy book in the middle. If now you pull a ruler tied to the bottom of the string, where will the string break: above or below the book?

The string can break both above and below, according as you pull. It's up to you, you can break it either way. If you pull carefully, the *upper* part of the string will break, but if you jerk it, the *lower* part breaks.

Why does this happen? Careful pulling breaks the upper part of the string because the string is being pulled down both by the force of your hand and by the

weight of the book, whilst the lower part of the string is only acted upon by the force of your hand. Whereas during the short instant of the jerk the book doesn't acquire very much motion and therefore the upper part of the string doesn't stretch. The entire force is thus "consumed" by the lower part, which breaks even if it's thicker than the upper part.

Torn Strip

Figure 78

A strip of paper that is about 30 cm long and one centimetre wide can be material for a funny trick. Partly cut or tear the strip in two places (Fig. 78) and ask your friend what will happen to it if it's pulled by the ends in the opposite directions.

He will answer that it'll break in the places where it's been torn.

"Into how many parts?" you ask then.

Generally the answer is: "Into three parts, of course." If you receive this answer, ask your friend to test his hunch by an experiment.

Much to his surprise he will see that he was mistaken, for the strip will only separate into two parts. You can repeat the experiment many times taking strips of various length and making little tears of various depth and you'll never get more than two pieces. The strip breaks where it's weaker which goes to prove the proverb: "The chain is only as strong as its weakest link". The reason is that of the two tears or cuts, however hard you strive to make them identical, one is bound to be deeper than the other. Even if it's imperceptible to your eyes, one will still be deeper. The weakest place of the strip will be first to begin to break. And once begun the breaking would continue to the end because the strip would become ever weaker at this place.

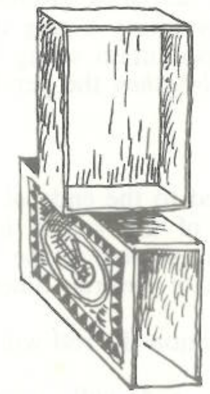
You might perhaps be very pleased to know that in making this trifling experiment you've visited a serious branch of science of importance for engineering that is called "strength of materials".

A Strong Match Box

What will happen to an empty match box if you strike it with all your might?

I'm sure that nine out of ten readers will say that the box won't survive such handling. The tenth – the person

Figure 79



who has actually performed the experiment or heard about it from somebody – will maintain that the box will survive.

The experiment should be staged as follows. Put the parts of an empty box one on top of the other, as shown in Fig. 79. Strike this assembly sharply with your fist. What will occur will surprise you: both parts will fly apart but, having collected them, you'll find that each one is intact. The box behaves like a spring and this saves it because it bends but doesn't break.

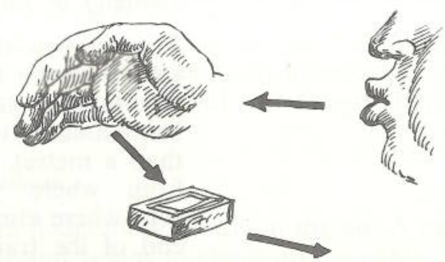
Bringing Something Closer by Blowing

Place an empty match on a table and ask somebody to move it away by blowing. Clearly this is no problem. Then ask him or her to do the opposite, i.e. make the box *approach* without leaning forward to blow the box from behind.

There'll hardly be many who'll twig. Some will try to move the box nearer by sucking in air, but that won't work of course. The answer, however, is very simple. What is it?

Ask somebody to put the hand vertically behind the box. Begin to blow and the air that reflects from the hand will strike the box and shift it towards you (Fig. 80).

Figure 80



The experiment is, so to speak, "failsafe". You'll only have to make sure it's on a sufficiently smooth table (even unpolished) which is not of course covered with a table-cloth.

Grandfather's Clock

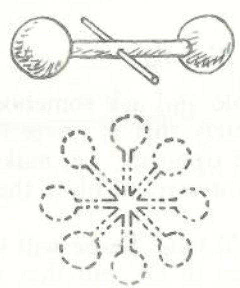
Suppose a grandfather's clock that uses weights to wind it up is fast or slow. What should be done with the pendulum to correct it?

The shorter a pendulum the quicker it swings. You

can easily prove this by an experiment with a weight suspended from a piece of rope. This suggests the solution of our problem: when the clock is *slow* you *shorten* the pendulum a little by lifting a ring on the pendulum rod, so making the pendulum swing faster; and when the clock is *fast* you *lengthen* the pendulum.

How Will Rod Settle Down?

Figure 81



Two balls of equal mass are fixed to the ends of a rod (Fig. 81). Right in the middle of the rod a hole is drilled through which a spoke is passed. If the rod is spun about the spoke, it'll rotate several times and settle down.

Could you predict in what position the rod will come to rest?

Those who think that the rod will invariably settle down in a horizontal position are mistaken. It can remain balanced in any position (see Fig. 81)–horizontal, vertical, or at an angle–since it's supported at the centre of mass. Any body supported or suspended at the centre of mass be in equilibrium at any position. Therefore, it's impossible to predict the final position of the rod.

Jumping in Railway Carriage

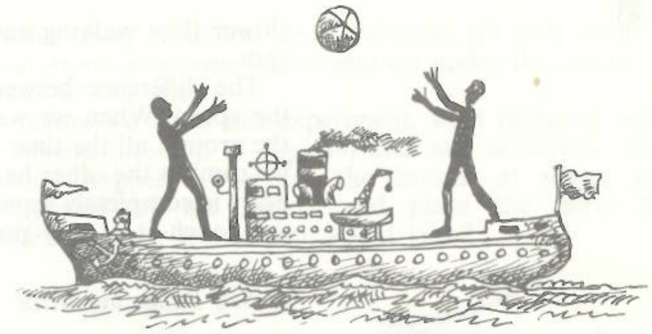
Imagine you are travelling in a train at a speed of 36 kilometres an hour and you jump up. Supposing that you manage to spend a whole second in the air (a brave assumption because you'll need to jump up more than a metre), where will you land, at the same place from where you started or somewhere else? If somewhere else, where then–closer to the beginning or end of the train?

You'll land at the same place. You shouldn't think that while you've been in the air the floor (together with the carriage) has shifted forward. To be sure, the carriage was tearing along but you also were travelling in the same direction and at the same speed carried by inertia. All the time you were directly above the place from which you jumped up.

Aboard a Ship

Two people are playing ball on the deck of a steaming vessel (Fig. 82). One stands nearer the aft and the other

Figure 82



nearer the bows. Which one can throw the ball easier to his partner?

If the ship is travelling with a steady speed and in a straight line neither has any advantage, just as if they were on a stationary ship. You should not suppose that the man standing nearer the bows recedes from the ball after it's been thrown or that the other man moves to meet it. By inertia the ball has the ship's speed which is equally possessed by both partners and the ball. Therefore, the motion of the ship (uniform and rectilinear) gives neither player an advantage.

Flags

A balloon is being carried away due north. In which direction will flags on its car fly?

The balloon carried by an air flow is at rest with respect to the surrounding air, therefore the flags won't be blown by the wind, but will dangle limply like they do in still weather.

On a Balloon

A balloon floats motionlessly in the air. A man gets out of the car and begins to climb up the cable. Which way will the balloon move in the process, upwards or downwards?

The balloon will shift *downwards*, since the man pushes the cable (and the balloon) in the opposite direction as he is climbing. The situation is similar to what happens if someone walks forward over the bottom of a small rowing boat: the boat shifts backwards.

Walking and Running

What is the difference between running and walking?

Before answering remember that running can be

slower than walking and that you can even run on the spot.

The difference between running and walking is not the speed. When we walk our body is in contact with the ground all the time at some point in our feet. When we run, on the other hand, there are moments when the body is completely separated from the ground and does not touch it at any point.

A Self-Balancing Stick

Put a smooth stick on the index fingers of both of your hands, as shown in Fig. 83. Now move your fingers together to meet each other half-way. Strangely, in the final position the stick doesn't fall off but keeps its balance. Make the experiment several times varying the initial position of your fingers, the result will invariably be the same: the stick will be balanced each time. Replace the stick by a ruler, a billiards cue, or a broom, and you'll notice the same behaviour.

Figure 83

What is the secret?

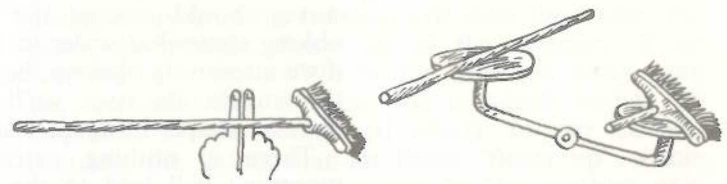
The following is clear: if the stick is balanced on your fingers brought together, this suggests that your fingers have closed up under the centre of mass (a body is in equilibrium if the centre of mass is over an area confined by the support's boundaries).

When your fingers are spread apart, the larger load is on the finger that is closer to the stick's centre of mass. Friction increases as the load grows and the finger closer to the centre of mass is subject to larger friction than the other one. Therefore, the finger that is closer to the centre of mass doesn't slide under the stick and at all times the only finger that moves is the one farther away from this point. Once the moving finger is closer to the centre of mass than the other, the fingers change their roles, the change taking place several times until the fingers come together. Since only one finger is moving at each instant of time, namely the one that is

farther away from the centre of mass, it's only natural that eventually both fingers end up under the centre of mass of the stick.

Before we leave this experiment, we'll repeat it with a broom (the top of Fig. 84) and ask ourselves what would happen if we cut the broom at where it's supported by the fingers and place the parts on different pans, which pan would sink?

Figure 84



It would seem that if both parts of the broom balance each other on your fingers these should also do so on the pans of the scales. Actually, the part with the brush will outweigh. The clue is not difficult to find, if we take into account that when the broom was balanced on your fingers the gravity forces of both parts were applied to unequal arms of the lever. On the pans of the scales, by contrast, the same forces are applied to the ends of an equal-arm lever.

Rowing in the River

A rowing boat and wooden chip alongside it are floating in a river. What is it easier for the rower: to get ahead of the chip by 10 metres or to lag behind it by 10 metres?

Even those practising water sports often give the wrong answer. It's more difficult, they argue, to row upstream than downstream, accordingly, to pass the chip is easier than to lag behind it.

No doubt, to reach a point rowing upstream is more difficult than rowing downstream. But if the point you are going to reach is floating alongside, just like our chip, the situation is quite different.

One should take into account that the boat carried by the current is *at rest* with respect to the water. The situation is the same as what it would be on the still water of a lake.

Thus, in both cases the rower needs exactly the same effort whether he wishes to pass or lag behind the boat.

Circles on Water

A stone thrown into still water produces concentric waves.

What form will the waves have if the stone is thrown into the flowing water of a river?

If you fail, from the very beginning, to follow the right track, you'll be easily lost in the argument and come to the conclusion that in the flowing water the waves should assume the form of an ellipse or an oblong somewhat wider in the upstream direction. But if we attentively observe the waves produced by a stone thrown into the river, we'll find no deviation from the circular shape, however fast the stream is.

There is nothing extraordinary in that. Simple reasoning will lead to the conclusion that the waves should be circular both in still and in flowing water. Let's treat the motion of particles of the waves as a combination of two movements: radial (from the centre of oscillations) and translational (downstream). A body participating in several motions eventually comes to the same point it would come to, if it performed all the component motions in succession.

We'll therefore assume that the stone is thrown into still water. In that case, the waves will clearly be circular.

Now suppose that the water is moving – no matter with what velocity, uniformly or not – the motion has only to be translational. What will happen to the circular waves? They'll undergo a translation without any distortion, i.e. will remain circular.

Deflection of Candle Flame

If you carry a candle about a room you will have noticed that initially the flame deflects backwards. Which way will it deflect if the candle is carried about in a closed casing? Which way will the flame in the casing deflect if it's uniformly rotated, horizontally in an outstretched hand?

If you think that in the casing there'll be no deflection, you are mistaken. Experiment with a burning match and you'll see that if it's protected by the hand as it's moved, the flame will deflect, but forwards – quite unexpectedly! – not backwards. This is because the flame is thinner than the surrounding air. A force imparts to a body with a small mass a larger velocity than to a body with a larger mass. Therefore,

the flame moving faster than the air in the casing deflects forwards.

The same reason (the smaller density of the flame than that of the surrounding air) also accounts for the unexpected behaviour of the flame when we move the casing in a circle, the flame deflects *inwards*, not outwards as might be expected. This would be clear if we remember how mercury and water are arranged in a ball rotated in a centrifuge. The mercury tries to be farther away from the rotation axis than the water. The latter, as it were, floats up in the mercury, if we consider the "bottom" to be the direction away from the rotational axis (i.e. the one in which bodies are displaced by the centrifugal effect). In our circular rotation, the lighter-than-air flame "floats up" within the casing, i.e. in the direction to the rotation axis.

A Sagging Rope

With what force must one pull at a rope for the latter not to sag?

However taut the rope is, it is bound to sag. Gravity that causes the sagging acts normally, whereas the stretching force on the rope has no vertical component. Two such forces can never balance each other out, i.e. their resultant force cannot be zero. And this resultant force is responsible for the sagging.

No force, however strong, can stretch a rope strictly horizontally (except when the rope is upright). The sagging is unavoidable, you can reduce it to a desired degree but cannot make it zero. Consequently, any nonvertically stretched rope or driving belt will sag.

For the same reason it is impossible, by the way, to stretch a hammock so that its ropes are horizontal. The taut net of a bed sags under the weight of a man. And the hammock, whose ropes are not so taut, turns into a dangling bag when a man lies on it.

How to Drop a Bottle?

In which direction with respect to a moving railway carriage should you throw a bottle so that the danger that it gets broken when hitting the ground is the least?

As it is safer to jump forwards from a moving carriage, it would appear that the bottle would not hit the ground so strongly if you throw it forwards. This is not so: objects should be thrown *backwards*. In that

case the velocity imparted to the bottle by throwing it will be *subtracted* from the one due to inertia with the result that the bottle will strike the ground with a smaller velocity. Throwing the bottle forwards will cause the reverse, the velocities would add up and the collision would be stronger.

That it is safer for a man to jump forward is accounted for by quite a different reason: he is hurt less by jumping this way.

Cork

A piece of cork has got into a bottle with water. The cork is small enough to pass freely through the neck, but try as you can, shaking or upending the bottle, the outpouring water will not for some reason bring the cork out. It's only when the bottle is completely empty that the cork leaves the bottle with the last bit of the water. Why?

The water doesn't bring the cork out for the simple reason that cork is lighter than water and therefore is always on its surface. The cork can only come to the opening when almost all of the water has come out. That's why it is the last to leave the bottle.

Floods

During a spring flood the surface of a river becomes convex–higher in the middle than near the banks. If loose logs float along such a swollen river, they will slide down to the banks leaving the mainstream free (the top of Fig. 85). In midsummer, when the water is low, the river surface becomes concave–lower in the middle than near the banks. In this case logs will accumulate in the middle (the bottom of Fig. 85).

What's the reason?

This is explained by the fact that in the middle water flows quicker than near the banks because the friction

Figure 85

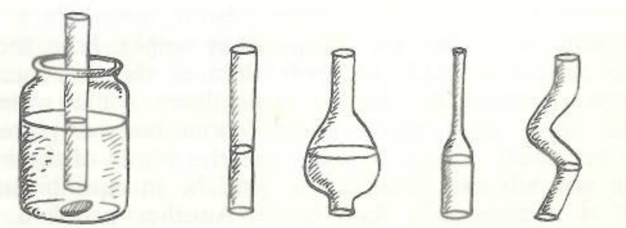
of the water along the bank slows the current down. During a flood, water comes from the upper reaches faster along the middle than near the banks because the current's speed is faster in the middle. Understandably, if more water comes to the middle, then the river should swell here. The situation changes in midsummer when water subsides. Now, owing to the swifter current the water run-off in the middle is higher than near the banks with the result that the river becomes concave.

Liquids... Press Upwards!

That liquids exert a pressure downwards, on the bottom of a vessel, and sidewards, on its walls, is known even to those who have never studied physics. But many people don't suspect that liquids press *upwards* as well. A glass tube will help you to make sure such a pressure does exist. Cut out a disk of a strong cardboard, its size being sufficient to cover the hole of the tube. Put it over the hole and dip both into some water. For the disk not to drop off when you are dipping it, it can be held by a piece of string passed through its centre or pressed on with a finger. When the tube has sunk to a certain depth, you will notice that the disk holds securely on its own without being pressed on with the finger or held with the string, being supported by the water that presses it up.

You can even measure the amount of this upward pressure. Carefully pour some water into the tube. Once the water level approaches the level outside the tube, the disk will fall off. The water pressure on the disk from below is thus balanced out by the pressure of the water column within the tube, its height being equal to the depth of the disk in the water. This is a law about the pressure of liquids on a submerged body. By the way, this also causes the weight "loss" in liquids and was formulated as the famous Archimedes principle.

Figure 86

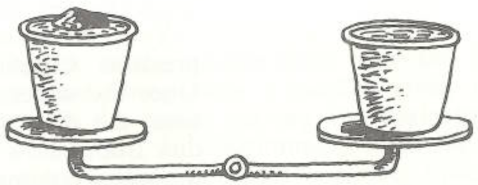


If you have several glass tubes of various shapes but with the same opening (e.g. as shown in Fig. 86) you can also test another law relating to liquids, namely the pressure of a liquid on the bottom of a vessel is only dependent on the bottom area and the level and is independent of the vessel shape. An experiment with the various glass tubes is described below. Dip them into the water to the same depth (for which purpose you'll have to glue paper strips onto them at the same height) and you'll notice that the disk will always fall off at the same level of the water within the tube (Fig. 86). In consequence, the pressure due to water columns of various shapes is the same if only their base areas and heights are the same. Notice that it's the *height*, not the *length*, that matters because a long inclined column exerts exactly the same pressure on the bottom as a short upright column of the same height (the base areas being equal).

Which is Heavier?

On one pan of scales is placed a pail that is filled to the brim with water. On the other pan, exactly the same sized pail is placed, also brimful, but with a piece of wood floating in it (Fig. 87). Which pail will be heavier?

Figure 87



I asked various people this question and got conflicting answers. Some answered that the pail with the wood would be heavier because "the pail has the water and the wood." Others held that, on the contrary, the first pail would be heavier "since water is heavier than wood."

Both views are a mistake for both pails have the *same weight*. True, there is less water in the second pail than in the first because the floating piece of wood displaces some water. The immersed part of every *floating* body displaces exactly the *same weight of water* as the whole of the body *weighs*. That's why the scales will be in equilibrium.

Another problem. Suppose I place on the scales

a glass of water and put a weight near it. When the system is *balanced* by the weights on the other pan, I drop a weight into the glass. What will happen with the balance?

According to the Archimedes principle the weight in the water becomes lighter than before. It might be expected that the pan with the glass would rise but in actual fact the scales will remain in equilibrium. Explain.

The weight in the glass has displaced some water, which has risen above the initial level, with the result that the pressure on the bottom of the vessel has increased so that the bottom is acted upon by an added force equal to the weight lost by the weight.

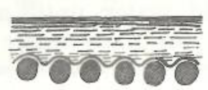
Water on a Screen

It turns out that water can be carried on a screen in real life and not only in fairy tales. A knowledge of physics will help to make this proverbially impossible thing possible. You'll need a wire screen about 15 centimetres across with a mesh size of about 1 millimetre. Immerse the network into melted wax and when it is taken out of the wax the wire will be covered with a thin layer of wax hardly noticeable for the naked eye.

The screen will still remain a screen – a pin will freely pass through its mesh – but now you will be able literally to carry water on it. The screen will hold a fairly high level of water without any seepage through the mesh. You need only to pour the water carefully and see to it that the screen is not jerked.

Why then doesn't the water seep? Because it doesn't wet wax and thus forms thin films between the meshes and it is the films' downward convexity that holds the water (Fig. 88).

Figure 88



Such a waxed screen may be placed on water and it will remain on the surface. It is thus possible not only to carry water on a screen but to float on it too.

This paradoxical experiment accounts for a number of the everyday phenomena we take for granted because we get used to them so. Tarring barrels and boats, painting with oil paints and, in general, coating things we want to render water-tight with oily materials, and the rubberizing of fabrics, these are all nothing but the making of "screens" like the one just described. The idea behind each phenomenon is the

same. Only in the case of the screen it appears in a somewhat unusual disguise.

Soap Bubbles

Can you make soap bubbles? This is not as simple as it might seem. I also once thought that it didn't take much dexterity until I found out practically that blowing large and beautiful bubbles is an art that requires much exercise.

But is it worthwhile to occupy yourself with such a trifling business as blowing soap bubbles? Used as a figure of speech the notion of soap bubbles is not complimentary. But the physicist has another view of them. The great English scientist Lord Kelvin wrote, "Blow a soap bubble and observe it, it may take a lifetime to investigate it, incessantly deriving lessons of physics from it."

Indeed, the fabulous play of colours on the surface of thin soap films enables physicists to measure the wavelengths of light, and the study of the tension in these frail films gives an insight into the laws governing the interaction between particles, those cohesion forces without which there would be nothing in the world but fine dust.

The several experiments that follow do not pursue such serious objectives, they are just amusements that will only acquaint you with the art of blowing soap bubbles. In his book *Soap Bubbles* the English physicist Charles Boys gave a detailed account of a number of experiments involving them. Those interested are referred to this fascinating book, but we'll only describe the simplest of the experiments here.

These can be performed using a solution of a conventional soap*, but for best results olive- or almond-oil soaps are recommended. A piece of soap is carefully dissolved in pure cold water until a fairly thick solution is obtained. Rain or thaw water is the best but if it's unavailable cooled boiled water will do. For bubbles to have a long life it is recommended to add one third of the volume of glycerin. Using a spoon, remove foam and bubbles from the surface and insert into it a long clay tube whose end on the outside and inside has already been smeared with soap. Good results are also achieved with straws about 10

* Toilet soaps are unsuitable.

centimetres long that are split across the end.

The bubbles are blown thus: dip the tube into the solution holding it upright so that a liquid film be formed at the end and carefully blow into it. Since the bubble is filled with warm air from your lungs, which is lighter than the surrounding air in the room, the bubble just blown will rise into the air.

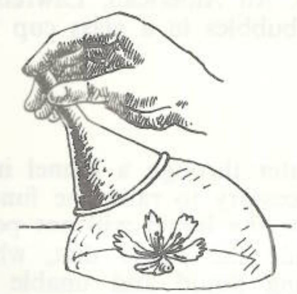
If from the very beginning you can produce a bubble 10 centimetres in diameter, the solution is good, otherwise some more soap will have to be added to the liquid until bubbles of the above-mentioned size are obtained. But this test is not sufficient. When a bubble is produced, dip a finger into the soap solution and try to punch the bubble. If it doesn't burst you may proceed to the experiments, but if the bubble doesn't survive the test, add some more soap.

Experiments should be carried out carefully, slowly, and quietly. If possible, the illumination should be bright, otherwise the bubbles will not show their iridescent play.

The following are a number of entertaining experiments with soap bubbles.

A Bubble Around a Flower. Pour some soap solution onto a plate or a tray so that the bottom is covered with a layer 2-3 mm thick. Place a flower or small vase in the middle and cover it with a glass funnel. Then, slowly lifting the funnel, blow into the narrow tube to form a soap bubble. Once the bubble has reached a largish size, tip the funnel over as shown in Fig. 89, and liberate the bubble from under it. The flower will then be under a transparent hemispherical hood of soap film which will show all the colours of the rainbow.

Figure 89



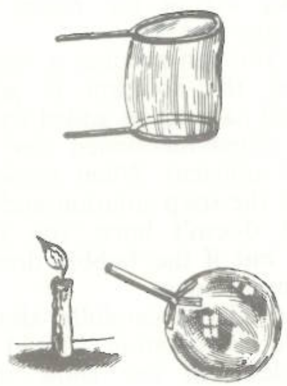
Instead of a flower you can take a small statue and crown its head with a soap bubble. First you need to drop some solution onto the head of the statue and then, after blowing the large bubble, pierce it and blow a smaller one inside it.

Bubbles Inside One Another. Blow a large bubble using the funnel, then immerse a straw into the soap solution so that only the end you will put into your mouth is dry and poke it carefully through the wall of the first bubble to the centre. By carefully drawing the straw back, a second bubble can be blown inside the first one, then a third, fourth, and so on.

A Cylinder of Soap Film (Fig. 90) can be blown between two wire rings. In order to do this lower

a conventional ball-shaped bubble onto the bottom ring. Then put a wetted second ring over the top of the bubble and by raising it the bubble will extend until it becomes cylindrical. Curiously enough, if you raise the upper ring higher than the length of the ring's circumference, the cylinder will become narrower at one end and wider at the other, and then it will disintegrate into two bubbles.

Figure 90



The film of a soap bubble is always in tension and exerts a pressure on the air inside it. By directing the funnel at the flame of a candle you can make sure that the force of the thin film is not all that negligible since the flame will be deflected quite a bit (Fig. 90).

It is interesting just to observe a bubble when it is taken from a warm room into a cold one. It will shrink appreciably, and conversely it will expand when brought from the cold room into the warm one. Clearly, the reason is that the air in the bubble expands and contracts. If, for example, the volume of a bubble at $-15\,^{\circ}C$ is 1,000 cubic centimetres and the bubble is brought into a room at $+15\,^{\circ}C$, its volume will increase by about $1{,}000 \times 30 \times 1/273$ or about 110 cubic centimetres.

Furthermore, it should be noted that the common idea that soap bubbles are short-lived is wrong since with adequate handling a soap bubble can survive for weeks. The English physicist Dewar (famous for his works on air liquefaction) kept soap bubbles in special bottles that were protected from dust, drying and jerks. Under these conditions he managed to keep some bubbles for a month or so. An American, Lawrence, succeeded in keeping soap bubbles in a glass cup for years.

An Improved Funnel

Those who have poured water through a funnel into a bottle know that it is necessary to raise the funnel from time to time, otherwise the liquid will not pour out of it. It's the air inside the bottle that, when compressed by the incoming liquid and unable to escape, stops more liquid from coming in from the funnel. Understandably, by raising the funnel we let the compressed air out, thus again enabling some more liquid to go in.

It would perhaps be quite practical to design a funnel so that it has longitudinal crests on its outer surface that would keep the funnel from sticking to the bottleneck. But I haven't ever seen such a funnel in everyday life, only in laboratories they use a filter designed after this fashion.

Seventy-Five More Questions
and Experiments on Physics

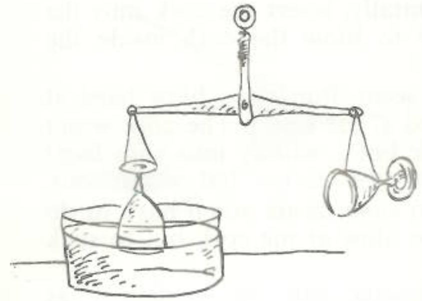
How Much Does Water Weigh in a Glass Held Upside-Down?

You'd say, "Nothing, of course. Water won't stay in the glass."

I'd ask, "And if it does stay, what then?"

Actually, it is possible to keep water in a glass held upside-down so that it doesn't pour out. The method is shown in Fig. 91. An upturned goblet tied at the bottom on one side of a balance is filled with water so that it doesn't pour out because the goblet's edges are immersed in water. On the other side of the balance tie an empty goblet, exactly the same sort.

Figure 91



Which side will go down?

That to which the upturned goblet with water is tied. This goblet is exposed to atmospheric pressure from above, and from below – to the atmospheric pressure minus the weight of the water contained in the goblet. For the system to be in balance you'd have to fill the other goblet with water. Accordingly, the water in the upturned glass weighs in these circumstances as much as it would in a normally held glass.

How Much Does the Air in a Room Weigh?

Can you say, however inaccurately, how much the air in a small room weighs? Several grammes or several kilogrammes? Would you be able to lift such a load with a finger or would it be difficult to hold it on your back.

Perhaps these days there is no one who believes air is weightless as was widely held in ancient times. But even today many wouldn't estimate its weight.

Remember that a litre jar of the warm summer air near the ground (not in the mountains) weighs 1.2 grammes. A cubic metre holds 1,000 litres and therefore weighs 1,000 times as much, i.e. 1.2 kilogrammes.

Now we can easily work out the weight of the air in a room. To do so, we'll only need to know how many cubic metres there are in it. If, say, the area of the room is 15 square metres, the height is 3 metres, then it

contains $15 \times 3 = 45$ cubic metres. The air thus weighs 45 kilogrammes plus 1/5 of 45, i.e. 9 kilogrammes, which makes 54 kilogrammes in all. You could not move this load with a finger or carry it about on your back with ease.

An Unruly Cork

This experiment will vividly demonstrate that compressed air has a force and an appreciable one at that.

For the experiment we'll only need a common bottle and a cork that's somewhat smaller than the bottleneck.

Hold the bottle horizontally, insert the cork into the neck and ask somebody to blow the cork inside the bottle.

No problem, it would seem. But try it, blow hard at the cork, you'll be amazed at the result. The cork won't be driven inside the bottle but... will fly into your face! The harder you blow the faster it'll shoot out. If you want the cork to slide inside you'll have to do quite the opposite-not to blow at the cork but to suck the air from the hole.

These strange phenomena can be explained as follows. When you blow into the bottleneck you drive some air through the gap between the cork and the wall of the neck. This increases the pressure inside the bottle and throws the cork out. If then you suck the air out, the air inside the bottle becomes thinner and the cork is pushed inside by the pressure of the air outside. The trick works out well only when the neck is absolutely dry as a wet cork sticks.

The Fate of a Balloon

Balloons sometimes go astray. But where? How high can they fly?

A balloon that escapes is carried always not to the boundaries of the atmosphere, but only to its "ceiling", i.e. to a height where the air is thin and the weight of the balloon equals that of the air displaced by it. But it does not always reach its ceiling. Since it swells (due to the reduction in the external pressure) it may burst before it reaches the ceiling.

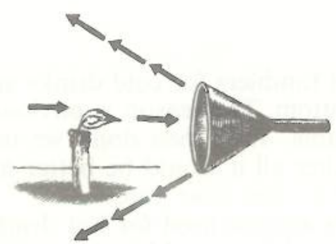
How to Blow Out a Candle?

It's child's play, you might think, to blow out a candle. But occasionally an attempt is a failure. Try and blow a candle out through a funnel and you'll see that this requires especial dexterity.

Place the funnel against the flame of a candle and blow at it through the thin end. The flame won't so much as stir, although the stream of air from the funnel would seem to be striking the flame directly.

Perhaps you now think the funnel is too far away from the flame, and so you bring it nearer and again begin to blow hard. You might be shocked by the result: the flame deflects not away from you but towards you, against the stream of the air coming from the funnel.

Figure 92



What is to be done then to kill the candle flame? It is necessary to locate the funnel so that the flame is not on the axis of the funnel but in the line of its cone part. Now by blowing into the funnel you'll easily extinguish the candle.

This is explained by the fact that the air stream leaving the narrow part of the funnel does not propagate along its axis but spreads along the walls of the cone, thus forming a sort of an air vortex. But the air along the funnel axis is rarefied with the result that a return air flow sets in near it. It is now clear why a flame located on the axis of the funnel leans towards the funnel, and when the flame is on the periphery of the cone, it bends the other way and goes out.

Tyre

A car wheel with a tyre is rolling to the right, its rim rotating clockwise. The question is: in what direction does the air inside the tyre move-against the direction of rotation or in the same direction?

The air moves away from the place of compression in both directions-forwards and backwards.

Why Are There Gaps Between the Rails?

Railway builders always leave gaps between the butts of adjacent rails on purpose. Without the gaps the railway would soon fall into disrepair. The reason is that all things expand on heating. A steel rail, too, elongates in summer, heated by the sun. If no space were allowed for the rails to expand, these would push against

adjacent rails with an enormous force and bend sideways wrenching out the spikes and destroying the track.

The gaps are designed with due account of winter temperatures. In winter the rails shrink from cold, thereby additionally increasing the gaps. Therefore, they are calculated very carefully considering the local climate.

An example of the use of the property of a body to shrink on cooling is the old procedure of shoeing cart wheels. A heated iron shoe is slided onto the rim of a cart wheel. When the shoe is allowed to cool down, it shrinks and squeezes tightly onto the rim.

A Glass and Tumbler

You may have noticed that tumblers for cold drinks are often made with a thick bottom. The reason is obvious: such a tumbler is more stable. Why then don't we use tumblers for hot drinks? After all it would be better for glasses to be more stable in that case too.

Thick-bottomed tumblers are not used for hot drinks because the walls of such tumblers would be heated by the hot liquid and expand more than the thick bottom. The thinner the glassware and the less difference there is between the thickness of the wall and the bottom, the more uniform will be the heating and the less the risk of cracking.

The Hole in the Cap of a Tea-Kettle

The cap of a metallic tea-kettle has a hole. What for? To let some vapour out, otherwise it will pop the cap off. But the cap expands on heating in all directions. What happens to the hole in the process? Does it become narrower or wider?

It becomes wider. In general the volume of holes and cavities becomes larger on heating in exactly the same way as an equal piece of surrounding material does. For that reason, by the way, the capacity of vessels increases on heating, not decreases as is widely believed.

Smoke

Why does smoke go up in still weather?

The smoke from a chimney ascends because it's

carried by hot air that expands on heating, thus becoming lighter than the air around the chimney. When the air supporting the smoke particles cools down, the smoke descends and spreads out over the ground.

Incombustible Paper

We can perform an experiment in which a paper strip doesn't burn in the flame of a candle.

Wind a narrow paper strip tightly around an iron rod. If now you introduce the rod with the wound strip into the flame of a candle, the paper won't catch fire. The fire will lick the paper, the latter will char but not burn down until the rod becomes hot.

Why? Because iron, just like any metal, is a good heat conductor; it leads away the heat obtained by the paper from the flames. Replace the metal rod by a wooden stick and the paper will burn because wood is a poor heat conductor. With a copper rod the experiment is even more successful.

Instead of the paper strip you could also use a piece of string wound tightly around a key.

How to Seal Window Frames for Winter

An adequately sealed window frame keeps out cold. But to seal it properly you should get it right why the frame "heats" a room.

Many believe that a second frame is used in winter because two windows are better than one. That is not so. It's not the second window that matters here but the air confined between the windows.

Air is a very poor heat conductor. Therefore, some air adequately confined for it not to carry any heat away prevents the room from cooling.

But for best results the air must be sealed tightly inside. Some people wrongly think that when a frame is sealed for the winter the upper gap in the external frame should be left unsealed. Should you do so the air within the cavity would be displaced by outside cold air, thus chilling the room. On the contrary, both frames should be treated painstakingly and not even the tiniest chink should be left.

Alternatively, you can with good results glue frames over with strips of strong paper. Well sealed or glued windows cut down your heating expenses.

Draught from a Closed Window

It might seem unusual that in a cold weather there is often a draught from a window that is tightly closed, carefully sealed and does not have the smallest hole. There is nothing surprising about that.

The air inside a room is almost never at rest. There are invisible flows caused by the heating and cooling of the air. Heating makes air thinner, and hence lighter. Conversely, cooling makes it denser and heavier. The light, heated air over a lamp or stove is displaced by cold air up to the ceiling because the heavy air that has cooled near the windows or cold walls, flows down to the floor.

These currents in a room are readily discovered using a balloon with a small weight attached to it for it not to strike the ceiling and float freely in the air. Let the balloon go near a warm stove and it'll travel about the room pulled around by the invisible air currents: from the stove to the window under the ceiling, then down to the floor and back to the stove for a new cycle.

That's why in winter we feel a draught from a window, especially at the bottom, even though the frame is securely sealed and keeps the outside air out.

How to Chill with Ice

If you want to chill a bottle of drink, where should you place it, on or under the ice?

Many put a bottle on the ice without a moment's hesitation, just like they put a tea kettle on a fire. That's not the way to do it. Heating should be done from below, but chilling, on the contrary, is better from the top.

Explain why. You know that colder substances are denser than warm ones. Thus a chilled beverage is denser than a warm one. When you place the ice over the top of the bottle, the upper portions of the drink (adjacent to the ice) sink on cooling being replaced by another amount of the liquid that in turn cools down and descends as well. In a short while all the drink in the bottle will have been in contact with the ice and chilled. But if the bottle is placed over the ice, the lowest portion cools first, its density increases and it stays at the bottom making no room for the rest of the liquid that is warmer. No mixing occurs here and the chilling is extremely poor.

Seventy-Five More Questions and Experiments on Physics

It pays to chill everything from the top and not just drinks–meat, vegetables and fish should be placed under ice as well. They are chilled not so much by the ice itself as by the surrounding air because the cold air comes down. If you need to cool a room with ice don't place it on the floor but put it up high on a shelf or suspend from the ceiling.

The Colour of Water Vapour

Have you ever seen water vapour? Could you say what colour it is?

Strictly speaking, water vapour is absolutely transparent and colourless. It is invisible, just like air. The white fog that is popularly known as "vapour" is really a multitude of water droplets, it is a suspension of fine water particles, not vapour.

Why Does a Boiler "Sing"?

A boiler or a kettle produces a singing sound when the water is about to boil. The water adjacent to the heater vaporizes to form small bubbles. Being much lighter these are expelled upwards by the surrounding water and as they go up the bubbles pass through water that has a temperature of less than 100 °C. The vapour in the bubbles cools, contracts and the bubbles collapse under pressure. Thus, just before boiling sets in, more and more bubbles go up but fail to reach the surface collapsing on the way to produce a cracking sound. It is these numerous cracking that produce the sound we hear at the outset of boiling.

When the water eventually heats to boiling temperature, the bubbles cease to collapse on their way up and the "singing" discontinues. However, once the water starts to cool down, again the earlier conditions occur and the "singing" resumes.

A Miraculous Top

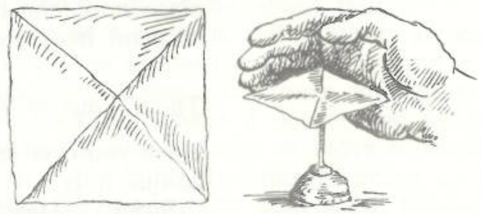
Cut a small square out of thin tissue-paper. Fold it diagonally twice and smooth it out again. You'll thus know where the centre of mass of the square is. Now place the paper on the point of an upright needle so that the latter supports it at the middle.

The paper will balance since it's supported at the centre of mass. A slightest flow of air will make it

rotate on the needle.

So far there is nothing miraculous about it. But bring your hand close to the paper as shown in Fig. 93. Do it carefully so that the paper is not swept away by the air flow. A strange thing will happen, the paper will start

Figure 93



rotating, slowly at first but then faster. Remove your hand and the rotation will stop. Bring it close again and it will start again.

This miraculous rotation at one time–in the 1870s–made many people think that our bodies possess some supernatural power. Mystics looked at this experiment as a support for some hazy teaching about the miraculous force emanating from the human body. Meanwhile the reason is natural and simple enough: the air heated from below by the hand moves upwards, pushing the paper and making it rotate, like the paper "snake" above a lamp discussed earlier. When you folded the paper, you made some of it slightly inclined.

An acute observer will have noticed that the paper rotates in a definite direction, from the wrist along the palm to the fingers. This is explained by the difference in temperatures between parts of the hand: the finger tips are invariably colder than the palm, therefore the palm produces a larger upward current that strikes the paper stronger than that caused by the fingers.

Does a Fur Coat Heat?

What would you say if told that a fur coat *doesn't heat* a bit? To be sure, you'd think the speaker is pulling your leg. And if this were proved by a number of experiments? For example, like the following.

Record the reading of a thermometer and wrap it in a fur coat. Get it out several hours later. You'll see that it hasn't heated up by even a fraction of a degree, the reading will be as it was before. This proves that the fur coat doesn't heat. You might suspect that fur coats even *cool*. Take two bottles with ice in them, wrap one

in the coat and allow the other to stand in the room uncovered. When the ice in the uncovered bottle has melted, unfold the fur coat and you'll see that the ice is nearly intact. In consequence, the fur coat not only didn't heat the ice, but, as it were, cooled it, thus hampering the melting!...

What objections could be raised here? How could you refute the arguments?

There is no objecting or refuting. In fact, fur coats don't heat things up if by "heat up" we mean to *impart heat*. A lamp heats, a stove heats, a human body heats, too, because all of these bodies are sources of heat. But a fur coat is not. *It generates no heat, but only stops the heat of our body from going astray.* That's why a warm-blooded animal whose body itself is a source of heat will be warmer with a fur coat than without it. But the thermometer generates no heat of its own and its temperature won't change in the coat. The ice in the coat retains its low temperature longer because the fur coat–a fairly poor heat conductor–hinders the passage of heat from the outside.

Snow "heats" the earth in the same way a fur coat does. A loose powder substance, snow is a poor heat conductor and helps to keep cold out. Not infrequently a thermometer in snow-covered soil indicates it is as much as ten degrees hotter than is exposed soil. Farmers are well aware of this heating effect of a snow cover.

Thus, the answer to the question of whether a fur coat heats or not is that it only helps us to heat ourselves. Or rather we heat the fur coat, not vice versa.

How to Air Rooms in Winter

The best way to air a room is to open a window when a fire is burning. Fresh, cold outside air will then force out the warm, lighter air from the room into the fire-place and out through the chimney into the atmosphere.

However, do not think that the same thing will occur when the window is closed, for the outside air will leak into the room through gaps in the window, walls, etc. True, some of it will really get into the room but not enough to sustain the fire. Therefore, apart from the outside air some air must come from other rooms where it might be neither pure nor fresh.

Figure 94

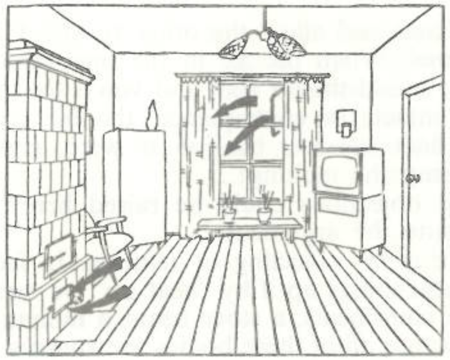
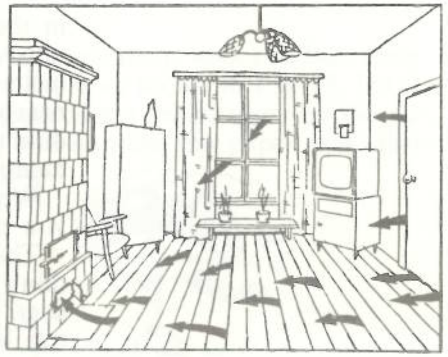


Figure 95



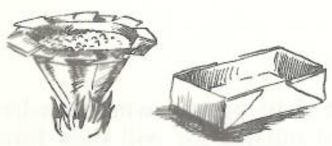
The two accompanying figures demonstrate the difference between the two cases. The arrows indicate the flow of air.

Where to Arrange a Ventilation Pane

Where? At the top or bottom of a window? In some homes ventilation panes are at the bottom. Admittedly, these are convenient to open and close, but they are inefficient. Let's consider the physics of the air exchange through the ventilation pane. Outside air is colder than that inside and displaces the latter. However, it occupies the part of the room below the ventilation pane. The air above the pane doesn't contribute to the exchange, i.e. is not ventilated.

Paper Saucepan

Figure 96



Look at Fig. 96. An egg is being boiled in a paper vessel!

You'd say, "Oh, but the paper'll now catch fire and the water will pour out!"

Try the experiment on your own. Make the "saucepan" from parchment paper and attach it to a wire holder. The paper won't be destroyed by the fire. The reason is that water in an open vessel can only be heated to boiling temperature, i.e. 100 °C. Therefore, the water, which has a large thermal capacity, absorbs excess heat from the paper and so does not allow it to heat up more above 100° to a point when it might ignite. (Perhaps it would be more convenient to make use of a small paper box as shown in the figure.) So the paper does not catch fire although flames touch it.

A similar kind, but disastrous, "experiment" is at

times performed by absent-minded people who put an empty kettle onto a fire with the pitiful result that the latter gets unsoldered. The reason is clear now: solder is relatively low-melting and it is only its close contact with water that saves it from its temperature rising dangerously. This applies to all sorts of soldered things.

Further, you could melt a piece of lead in a small box made of a playing card. You'll only need to expose to flames the place that is in direct contact with the lead. Being a good heat conductor, the metal will quickly take away heat from the paper. The temperature of the paper will thus be maintained at about 335 °C (melting point for lead), which is insufficient to ignite the paper.

What is the Lamp Glass for?

Few people know what a long history the lamp glass went through before it appeared in its present-day form. For millennia people had used flames for lighting without resorting to the services of glass. It took the genius of Leonardo da Vinci (1452–1519) to introduce this important improvement of the lamp. But Leonardo used a metal tube, rather than a glass one, to surround the flame. Three more centuries passed before the metal tube was replaced by the transparent cylinder. You see thus that the lamp glass is an invention developed by scores of generations.

What's its purpose?

Not all of you will come up with the right answer to this natural question.

To protect the flames from wind is only a secondary role of the glass. Its main effect is to increase the *brightness* of the flames, to boost the combustion process. The role of the glass here is like that of a chimney or stack; it intensifies the inflow of air to the flames, i.e. improves the "draught".

Let's take a closer look at this. The column of air inside the glass is heated by the flames much faster than the air surrounding the lamp. After it has heated and thereby become lighter, the air is displaced upwards by the heavier cold air arriving from below through holes in the burner. This results in a steady flow of air upwards, a flow that continually takes the combustion products out and brings fresh air in. The higher the glass, the more difference there is between the heated and unheated air columns and the more

intensive is the inflow of fresh air, and hence the burning. The situation is like that in industrial chimney stacks which is why they are made so high.

Interestingly, even Leonardo had understood these phenomena. In his manuscripts he says, "Where fire appears, an air flow forms around it, the flow supports and intensifies it."

Why Doesn't a Flame Go Out by Itself?

A closer examination of the process of combustion inevitably leads to the above question. After all, the combustion products are carbon dioxide and water vapour, *noncombustible* substances incapable of supporting the process. Accordingly, once started a flame must be surrounded by noncombustibles that hinder the inflow of air. Combustion cannot occur without air and the flame would be bound to die out.

Why then this is not the case? Why does the process of combustion carry on as long as there is a supply of combustibles? For the only reason that gases expand on heating and *become lighter*. It's owing to this that heated combustion products don't stay where they've been formed, i.e. in the immediate neighbourhood of the flames, but are at once forced upwards by fresh air. If the principle of Archimedes didn't apply to gases (or there were no gravity), any flame would go out after a while on its own.

You can easily verify that combustion products kill a flame. At times you make use of this unawares to extinguish the fire in a lamp. How do you blow out a kerosene lamp? Blow into it from above, i.e. do not let the combustion products out. The flames go out deprived of the supply of fresh air.

Why Does Water Kill Fire?

A seemingly simple question... that is not always correctly answered.

Let's briefly explain the phenomenon.

First, on touching a hot body water turns into vapour, so taking heat away from the burning body. To convert boiling water into vapour takes more than five times as much heat as is required to heat the same amount of cold water to 100 °C.

Second, the resulting vapour occupies hundreds of times more space than the source water. The vapour

envelopes the body, cutting off the air that is indispensable for its burning.

To improve the fire-extinguishing power of water they sometimes add ... gunpowder to it. Strange as it might seem, the measure is quite reasonable because the powder burns down quickly evolving a great amount of noncombustible gases that cover the burning material to hinder combustion.

Heating with Ice and Boiling Water

Is it possible to use a piece of ice to heat another? Or, to cool?

Is it possible to heat one quantity of boiling water with another?

If some ice at a low temperature, −20 °C say, is brought into contact with a piece of ice at a higher temperature, −5 °C say, then the first piece of ice will heat up (become less cold), and the second will cool down. Therefore, it is quite possible to cool or heat ice with ice.

But one body of boiling water cannot heat another body of boiling water (at the same pressure), for at a given pressure boiling water is always at the same temperature.

Can You Bring Some Water to the Boil Using Other Boiling Water?

Pour some water into a small bottle (jar or phial) and place it in a saucepan with pure water so that it doesn't touch the bottom. Of course, you'll have to suspend the bottle from a piece of wire. Put the saucepan on a fire. When the water in the saucepan boils, it would seem that the water in the bottle should also boil shortly. Only you will never see this, however long you wait. The water in the bottle will get hot, very hot, but boil will it not. The boiling water appears to be too cool to bring another body of water to the boil.

Quite an unexpected result, it seems, but let's analyse it more closely. To bring water to the boil it is not sufficient only to heat it to 100 °C – it also needs a substantial supply of so-called latent heat. Pure water boils at 100 °C, and under standard conditions its temperature never exceeds this, however long you heat it. In our case the source of heat used to heat the water in the bottle has a temperature of 100 °C. It, too, is

only able to heat the water in the bottle up to 100 °C. Once the 100 °C mark is reached in the bottle, *any further transfer of heat from the water in the saucepan to that in the bottle will cease.* No amount of heating can then supply the water in the bottle with the latent heat required for the water to vapourize (each gramme of water heated to 100 °C requires upwards of 500 calories* to vapourize). That's why the water in the bottle, although it gets hot, doesn't boil.

A guestion may arise: what is the difference between the water in the bottle from that in the saucepan? After all, the bottle contains the same water except that it is only separated from the rest of the water by a glass partition. Now, why isn't it involved in the processes occurring beyond the partition?

Because the partition interrupts the currents that mix the water in the pan where each particle of water can directly touch the hot bottom. But the water in the bottle only contacts boiling water through the partition.

Thus, pure boiling water cannot boil another amount of water. But just add a handful of salt to the saucepan... and everything changes. Salt water boils at somewhat higher than 100 °C and can thus bring the pure water in the bottle to the boil.

Can You Bring Water to the Boil with Snow?

The reader might answer, "If boiling water won't do it how can snow!" Take your time to answer. You'd better carry out an experiment, say, with the bottle that you were just using before.

Fill it halfway with water and insert it into boiling *salt water.* Wait for the water in the bottle to boil, take it out of the saucepan and plug quickly with a prepared well-fitting cork. Now turn the bottle upside-down and wait for the boiling inside to cease. Then pour boiling water over the bottle and the water inside won't boil. But place some snow on its bottom or simply pour cold water over it, as shown in Fig. 97, and you'll see that the water in the bottle will boil...

The snow has done what the boiling water couldn't. This is all the more mysterious because the bottle won't be especially hot, only warm. But you will have

Figure 97

* The *calorie* is a unit of heat. The *small calorie* is the amount of heat required to increase the temperature of 1 gramme of water by 1°C.

seen with your own eyes that the water in the bottle has boiled.

The secret is that the snow cooled the walls of the bottle with the result that the vapour inside it condensed to form water droplets. Since the air has been expelled from the bottle before, the water inside the bottle is now under much lower pressure. But it's well known that lowering the pressure of a liquid reduces its boiling point. Although in our bottle we have boiling water it's *not hot.*

If the walls of the bottle are very thin, the sudden condensation of vapour inside it can cause something like an explosion: the external pressure can squeeze the bottle (you see that the word "explosion" is unsuitable here and in fact what took place was an 'implosion'). Instead of the bottle you'd better use a round vessel (a bulb with a convex bottom) so that external air would exert its pressure on the arch.

Figure 98

It is safer to perform this experiment with a tin can used to contain kerosene, oil, etc. Boil some water in it, screw its plug on tightly and pour cold water over it. The can will at once collapse under the pressure of the external air as the vapour inside will condense into water on cooling.

The can will be crumpled by atmospheric pressure as if hit by a heavy hammer (Fig. 98).

A Hot Egg in Your Hand

Why doesn't an egg just out of boiling water hurt your hand?

The egg is wet and hot. Water cools the shell on evaporating from the surface and the hand is not burned. But this only occurs in the first instants, until the egg has dried, and then its high temperature will hurt you.

Removal of Fat Stains by an Iron

The removal of fat stains from fabrics is based on the fact that the surface tension of liquids decreases with increasing temperature. "Therefore," Maxwell wrote in *The Theory of Heat*, "if the temperature in various parts of the fat stain is different, the fat seeks to move from heated places to colder ones. Apply to one side of the fabric a heated iron, and to the other, cotton paper, the fat will then transfer to the cotton paper."

Accordingly, material absorbing the fat should be placed at the side opposite to the iron.

How Far Can You See From High Places?

From a flat place we only see the group up to a certain boundary. This boundary of view is called the "horizon line". Trees, houses and other high structures lying beyond the horizon line are seen not in full, because their lower parts are blotted by the convexity of the earth. Even plains or the sea, although apparently flat, are in fact convex, for they are parts of the curved surface of the globe.

How far then does an average-sized man see over a plain?

He can only see up to 5 kilometres. To see beyond that he'll have to climb up higher. A man on horseback on a plainland would see up to 6 kilometres and a sailor on a mast 20 metres high would see the sea around him up to 16 kilometres away. From the top of a lighthouse towering above water at 60 metres the sea is seen for nearly 30 kilometres.

But, of course, the widest panoramas open up before airmen. From an altitude of 1 kilometre they can see almost for 120 kilometres in all directions, if not hindered by clouds or fog. At twice the height an airman will see for 160 kilometres using a perfect optical device. Further, from 10 kilometres one can see within 380 kilometres, and astronauts orbiting the Earth see the whole of one side of the globe.

Where Does a Chirring Grass Hopper Sit?

Sit somebody in the middle of a room, with his eyes blindfolded, and ask him to sit still and not turn his head. Take then two coins and tap one on the other at various places in the room but at about the same distance from your friend's ears. Ask your friend to guess the place whence the sound comes. It will be difficult to do and your friend will point in some other direction.

If you step aside, the errors won't be as bad because now the sound in the nearest ear of your friend will be heard somewhat louder, so enabling him to determine the location of the source.

The experiment makes it clear why it's impossible to spot a grass hopper chirring in the grass. The sharp

sound is heard two paces away from you. You look there and see nothing, but now the sound is distinctly heard from the left. You turn your head in that direction, but no sooner have you done that than the sound already comes from some other direction. The speed of the grass hopper stuns you and the quicker you turn to the direction of the singing insect the quicker the invisible musician hops about. But in reality the insect is sitting placidly in place and his "hops" are just an illusion. Your problem is that when you turn your head you put it exactly so that the grass hopper becomes equally separated from both of your ears. This condition (as you should know it from the experiment just described) is conducive to an error. If the chirring comes from ahead of you, you place it, erroneously, in the opposite direction.

In consequence, if you want to determine where a sound comes from, you should not turn your head towards the sound, but conversely, turn it away. Which is exactly what we do when we, as it were, "prick up our ears".

Echo

When a sound we have produced is reflected from a wall or another obstacle and returns to our ears, we hear an *echo*. It is only heard distinctly if the time-lag between the sound generation and its return is not too short. Otherwise the reflected sound would melt with the initial one and amplify it, the sound will then reverberate, e.g. in large empty halls.

Imagine that you are standing in an open place and there is a house in front of you 33 metres away. Clap your hands. The sound will travel through the 33 metres, reflect from the walls and come back. How long will that take? Since the sound covered 33 metres there and the same distance back, it'll return in 66/330 or 1/5 of a second. Our sharp sound was so short that it terminated in less than 1/5 second, i.e. before the echo arrived. The two sounds didn't merge and were heard separately. A *monosyllabic* word ("yes", "no", etc.) is pronounced in about 1/5 second and we can hear such words echoed at a distance of only 33 metres from an obstacle. But for *bisyllabic* words the echo merges with the initial sound intensifying it but rendering it obscure, we don't hear it separately.

At what distance must the obstacle be then so that

we could hear a bisyllabic echo distinctly, "halloo", say? Such words take 1/5 second to pronounce, during which time the sound should cover the distance to the obstacle and back, i.e. double the separation from the obstacle. But in 2/5 second sound covers $330 \times 2/5 =$ = about 132 metres.

A half of it – 66 metres – is precisely the least distance to the obstacle capable of producing the bisyllabic echo.

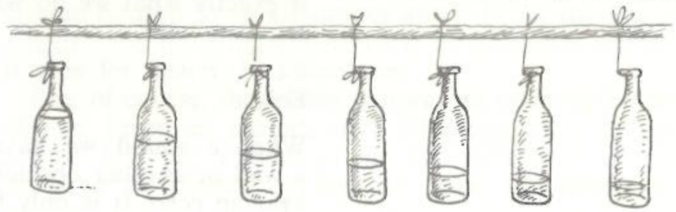
Now you'll be able to work out that a trisyllabic echo requires a distance of about one hundred metres.

Musical Bottles

If you have ear for music, you could contrive a sort of a jazz band from conventional bottles and play simple tunes.

What and how is to be done is evident from Fig. 99. Suspend 7 bottles with water from a pole fixed

Figure 99



horizontally between two chairs. The first bottle should be nearly full, each successive bottle contains a little bit less water than the previous one, the last bottle having virtually none.

By striking the bottles with a dry wooden stick you'll produce various notes of the octave. The less water in a bottle the higher its pitch. Therefore, by adding or removing some water you'll be able to achieve the tones that make up a scale.

With an octave you could play some simple melodies on this bottle instrument.

The Murmur in a Shell

If you put a cup or a large shell next to your ear you'll hear a murmur that occurs because the shell is a *resonator* that amplifies the numerous noises in the surrounding world that are not normally noticed because they are too weak. This mixed sound reminds people of the murmur of the sea and has led to numerous legends.

To See Through a Palm

Fold a sheet of paper into a tube, bring it up to your left eye with your left hand and look through it at some distant object. Now bring your right palm near to your right eye so that it nearly touches the tube. Both hands should be about 15–20 centimetres away from the eyes. You'll then make sure that your right eye sees perfectly through your palm as if there were a round hole in it. Why?

The reason of this unexpected phenomenon was as follows. Your left eye prepared to view a distant object through the tube and the crystalline lens adapted accordingly. The eyes function in such a way that they always adapt in sympathy.

In the experiment described the right eye, too, adapted to distant sight with the result that the near palm appeared blurred to it. In short, the left eye clearly sees the distant object, the right one, sees the palm unclearly. The net result is that it seems to you that the distant object is seen through the shielding palm.

Through Binoculars

At a seaside you are watching a boat approaching the shore through a pair of binoculars that magnifies three times. How many times will the speed be increased with which the boat is approaching the shore?

Assume that the boat is sighted 600 metres away and is approaching the observer with a speed of 5 metres per second. Through binoculars with triple magnification the boat at 600 metres appears to be at 200 metres. A minute later it will be $5 \times 60 = 300$ metres closer and will then be 300 metres away from the observer. In the binoculars its apparent size would indicate it were 100 metres away. Consequently, an observer looking through the binoculars would think the boat has travelled $200 - 100 = 100$ metres, whereas in actual fact it has actually covered 300 metres. It follows that the speed at which the boat approaches when observed through the binoculars has decreased not increased by three times.

The reader can arrive at the same result by another argument, i.e. by taking the initial distance, speed and period.

The speed with which the boat approaches has thus reduced by as many times as the binocular magnifies.

From the Front or the Back?

There are many things in each household that are used inefficiently. I've already mentioned that some people cannot use ice properly to chill drinks – they place them *on* the ice instead of *under* it. It appears that some people cannot use a conventional mirror either. Quite frequently, if they want to see themselves better in the mirror they turn the light on *behind* themselves in order to "illuminate the reflection", instead of illuminating themselves *from the front*.

Drawing Before the Mirror

That a mirror reflection is not identical with the original may be demonstrated by the following experiment.

Stand or hang an upright mirror in front of you on the table, place a sheet of paper on it and try to draw something, for example, a rectangle with diagonals. But in doing so don't look directly at your hand, but follow the movements of its reflection in the mirror.

You'll find that this seemingly simple problem is almost intractable. Over the years our visual perceptions and motions have been correlated but the mirror violates this and represents our motions to our eyes in an inverted form. Long-term habits rebel against each our motion: you want to draw a line to the right, say, but the hand draws to the left, and so on.

Stranger things will occur if instead of a simple figure you attempt to draw more intricate designs or write something whilst looking in the mirror. The result will be a funny confusion.

The impressions left on carbon paper are inverted lettering, too. Just try to read the text on it. Quite a challenge! But bring it to a mirror and the text will appear in its habitual form. The mirror gives the reflection of what is itself an inverted image of normal writing.

Black Velvet and White Snow

Which is the lighter – black velvet on a sunny day or pure snow on a moonlit night?

Nothing, it seems, surpasses black velvet in blackness or white snow in whiteness. These age-old metaphors of white and black appear, however, quite different when viewed by a physical instrument – a photometer. It then

turns out that the blackest velvet in sunlight is *lighter* than the purest snow in moonlight.

This is because a black surface, however dark it might be, doesn't completely absorb all the visible incident light. Even soot and platinum black – the blackest substances known – scatter about 1–2 per cent of the incident light. We take 1 per cent for argument's sake and suppose that snow scatters 100 per cent of the incident light (which is undoubtedly an overstatement)*. It is known that the illumination provided by the sun is 400,000 times that of the moon. Therefore, the 1 per cent of sunlight scattered by the black velvet is thousands of times more intense than the 100 per cent of moonlight scattered by snow. In other words, sunlit black velvet is many times lighter than moonlit snow.

To be sure, this is true not only of snow but also of the best white pigments (the whitest of them all – lithopone – scatters 91 per cent of light). Since no surface, unless it's hot, can beam out more light than strikes it, and the sun sends out 400,000 times as much light as the moon, it's impossible to have a white pigment that would in moonlight be lighter than the blackest pigment on a sunny day.

Why is Snow White?

Why, indeed? It consists of transparent ice crystals.

For exactly the same reason that ground glass and all ground transparent substances in general are white. Grind some ice up in a mortar or chip it with a knife and you'll get white powder. The colour is due to the fact that light, when penetrating into tiny pieces of transparent ice, doesn't pass through them but reflects inside them at the boundaries between the ice and the air (total internal reflection). But a randomly scattering surface is perceived by the eye as white.

Thus, snow is white because it consists of tiny particles. If the gaps between the snow flakes are filled with water, the snow becomes transparent. Such an experiment is easy. Put some snow into a jar and pour some water into it, and before your very eyes the snow will become colourless, transparent.

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* Fresh snow only scatters about 80 per cent of light.

The Shine on a Blackened Shoe

Why does a blackened shoe shine?

Neither the sticky black shoe polish, nor the brush seem to have anything to impart the gloss to shoes. Therefore, it's a mystery for many.

We'll first clear up the difference between the glossy polished surface and the dull one. It's widely believed that the polished surface is smooth and the dull one is irregular. This is not so: both are irregular. There are no absolutely smooth surfaces. Examined under the microscope polished surfaces would be like razor blades and for a man reduced 10,000,000 times the surface of a smoothly polished blade would appear to be a hilly terrain. There are irregularities, depressions and scratches on any surface, both dull and polished. What matters is the *size* of these irregularities. If they are smaller than the wavelength of the incident light, then the rays are reflected correctly, i.e. at the angle of incidence. Such a surface gives mirror reflections, it shines and we call it polished. If, on the other hand, the irregularities are larger than the wavelength of the incident light, the surface scatters the ray randomly and does not follow the reflection law. Such scattered light gives no mirror reflections and highlights, and the surface is called dull.

This suggests, by the way, that a surface may be polished for some rays and dull for others. For visible light with a mean wavelength of about half a micrometre (0.0005 mm) a surface with irregularities of about that size will be polished; for infrared light, which has longer wavelength it's polished, too. But for ultraviolet light, which has shorter wavelength, it's dull.

But back to the pedestrian subject of our problem. Why do polished shoes shine, after all? The unblackened surface of leather has a highly irregular microstructure with "peaks" larger than the mean wavelength of visible light, it's dull. By blackening it we smooth out the surface and lay the hairs that stick out down. Brushing removes any excess polish at projections and fills the troughs, reducing the irregularities down to a size at which the peaks become smaller than the wavelengths of visible rays and the surface turns into a glossy one.

Through Stained Glass

What colour are red or blue flowers viewed through green glass?

Green glass will only transmit green light and catch all the rest. Red flowers send out mostly red light. If we look through green glass at a red flower we'll receive no light from its petals as the only rays it emits are retained by the glass. The red flower will therefore appear to be *black* through such glass.

Now you should easily see that the blue flower viewed through green glass will be black as well.

Professor M. Yu. Piotrovsky, a physicist, artist and acute observer of nature, made a number of interesting observations in his book *Physics on Summer Outings*.

"Observing flowerbeds through a *red* glass we see that purely red flowers, say, geranium, appear to us as bright as purely white one; green foliage appears as absolutely black with a metallic lustre; blue flowers (aconite, etc.) are so black as to be next to impossible to make them out against the black background of the leaves and yellow, pink, and lilac flowers appear more or less dull.

"Through a *green* glass we see the unusually bright green of the foliage and white flowers come out still more distinctly against it; somewhat more pale are yellow and blue ones; red flowers are jet black; lilac and light-pink colours appear as dull and grey so that, for example, the light-pink petals of a wild rose are darker than its richly coloured leaves.

"Finally a *blue* glass will again make red flowers look black, white flowers will look bright, yellow–absolutely black, and blue and dark-blue–almost as bright as the white ones.

"It's easily seen from this that red flowers really do emit much more red light than any other colour, yellow flowers emit about an equal amount of red and green, but very little blue, whilst pink and purple flowers emit a lot of red and blue, but very little green light."

A Red Signal

On the railways the stop signal is a red colour. Why?

Red light has the longest wavelength in the visible spectrum and is thus less scattered by any particles suspended in the air than are other colours. Therefore, red light penetrates farther. It is of paramount importance to obtain the greatest visibility possible for

a transport signal since to be able to stop his train the engine-driver should begin breaking long before reaching an obstacle.

The greater transparency of the atmosphere to longer waves, by the way, explains why astronomers use infrared filters to photograph planets (especially Mars). Fine details blurred in a conventional picture come out distinctly on a photograph taken through a glass that only transmits infrared light. In the case of Mars it's possible to photograph the surface of the planet, while a conventional picture only shows its atmospheric envelope.

A further reason for selecting the red light for the stop signal is that our eyes are more sensitive to this colour than to blue or green.

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Optical Illusions

Tricks of Vision

The optical, or visual, illusions to which this section is devoted are not accidental companions of our vision–they occur in definite circumstances, are governed by physical laws and affect any normal human eye. That human beings are subject to visual illusions and can be mistaken as to the source of their visual perceptions, should by no means be considered an undesirable disadvantage or an unqualified flaw in our constitution, whose removal would benefit us in many respects. The artist would rebel against such an "infallible" vision. For him our ability, under certain conditions, to see what really is not is a blessing enriching enormously the potentialities of the fine arts. The 18th century mathematician Euler wrote: "Artists are especially skilled at using this common illusory experience. The whole of the art of painting is based on this. If we were used to judge about things as these are in reality then this art would be impossible, it would be as if we were all blind. In vain the artist would exhaust his skill in colour blending, for we would merely say: there is a red spot on this board, here a blue one, here a black and there, several whitish lines. Everything is in the same plane and there is no difference in distance. It would thus be impossible to represent anything. No matter what were painted in the picture, it could seem to be like writing on a paper, and perhaps we would, in addition, try to make out the *signification* of all the coloured spots. *For all the perfection, weren't we to be pitied greatly, being devoid of the pleasure we derive every day from such pleasant and useful arts!*"

Since the subject is of such lively interest for the artist, physicist, physiologist, physician, psychologist, philosopher, and for any inquisitive mind, many books and articles have been published in this country and elsewhere.*

We'll here consider several types of tricks played by our unaided eye, i.e. without any appliances such as stereoscopes, punched cards, and so on.

As to the causes of one or another visual illusion, only a relatively small number have well established,

* See, e.g. *The Nature of Experience* (1959) by R. Brain; *Optical Illusions* (1964) by S. Tolansky; *The Neurophysiological Aspects of Hallucinations and Illusory Experience* (1960) by W. Grey.

unquestionable explanations. These include those due to the structure of our eyes, irradiation, Mariotte's illusion (blind spot), astigmatism illusions, and so forth.

As an instructive example we'll consider the optical illusion of Fig. 141: white circles arranged in a certain way on a black background are perceived as hexahedrons. It seems to be well established that this kind of illusion is totally caused by so-called irradiation, i.e. the apparent expansion of light areas (which can be given a simple, clear physical explanation). "White circles expand due to irradiation and reduce the black gaps between them", Professor Paul Bert writes in his *Lectures on Zoology*. He goes on to say that, "as each circle is surrounded by six other, it pushes adjacent ones on expanding and appears to be confined by a hexagon".

Suffice it to glance at the neighbouring figure (Fig. 141) where the same effect is observed for *black* circles against a *white* background for this explanation to be rejected: here irradiation only could reduce the size of black spots but could not change them into hexagons. For the two cases to be covered by the same principle the following interpretation might be suggested. When viewing from a certain distance, the angle of vision of the gaps between the circles becomes smaller than a limit, so enabling their forms to be distinguished. Each of the six neighbouring gaps then appears to be a straight line of a uniform thickness and the circles are thus bounded by hexagons. This interpretation also covers the paradoxical fact that at some distances white circles continue to appear to be round, whereas the black fringes around them have already assumed hexagonal forms. It's only at larger distances that the hexagonal configuration is transferred from the fringe to the white spots. However, this explanation, too, is only a plausible assumption and, perhaps, there are several other possible explanations. It is necessary to prove that the *possible* cause is here the *actual* one.

Most of attempts to explain individual illusions (except for the few mentioned above) are as unreliable and uncertain. Some tricks of vision still await their explanation. By contrast, others have too many explanations each of which would perhaps be sufficient in itself were there not so many additional ones that make it less convincing. Remember the famous illusion discussed since the time of Ptolemy—that of the increasing of the size of celestial bodies near horizon.

No less than six possible theories, it seems, have been suggested, each of which has the only drawback that there are five more equally adequate explanations... Obviously, the entire domain of visual illusions is still in the pre-scientific stage of treatment and in need of establishing the basic methodology of its investigation.

For want of any solid foundation in the form of relevant theories I have confined our discussion to the demonstration of unquestionable facts providing no explanations of what caused them and seeking only to present all the major types of visual illusion.* Only those involving portraits are explained at the end of the section since these are quite clear and incontestable... I also wanted to do away with some of superstitious notions that developed around this unique optical illusion.

The series of illustrations opens with samples of illusions caused by clearly anatomical and physiological peculiarities of the eye. These are illusions due to the blind spot, irradiation, astigmatism, the retention of light impressions, and retina fatigue (see Figs. 100-107). In the blind spot experiment some of your field of vision may disappear in another way as well—as Mariotte did for the first time in the 18th century. The effect perhaps is even more striking. So Mariotte writes: "Against a dark background approximately at the level of my eyes I attached a small circle of white paper and at the same time asked someone to hold another circle beside the first one about 2 feet to its right but somewhat lower so that the image would strike the optical nerve of my right eye when I closed my left one. I stood next to the first circle and stepped back gradually without taking my right eye off it. When I was about 9 feet away, the second circle, which was about 4 inches across, *completely disappeared* from my field of vision.

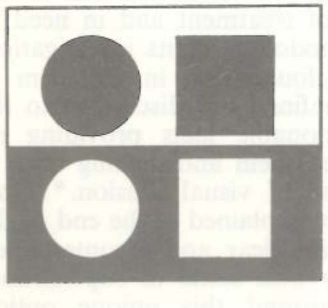
"I couldn't ascribe it to its lateral position, as I could discern other things further to the side than it. I'd have thought it removed had I not been able to find it again with the slightest movement of my eye...".

These "physiological" tricks of vision are followed by a much larger class of illusions that are due to psychological reasons, which have not yet been sufficiently

* The selection of illusions here is the result of many years of collecting. I've excluded, however, all those published illusions that have effect not on anybody's eye or are not perceptible enough.

clarified. It may perhaps be established that illusions of this kind are only the consequence of some preconceived erroneous judgement that is involuntary and subconscious in nature. The source of the misperception here is the mind, not the sensor. Kant aptly remarked, "Our senses deceive us not because they do not judge correctly, but because they do not judge at all".

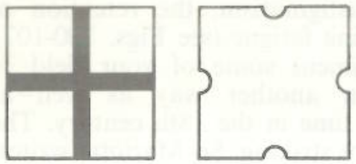
Figure 100



Irradiation. When viewed from a distance the figures below – the circle and square – seem to be larger than those above, although they are equal in size. The larger the distance the more pronounced is the illusion. The phenomenon is called *irradiation* (see below).

Irradiation. When viewed from a distance the figure with the black cross seems, owing to irradiation, to be distorted as shown in the accompanying figure on the right.

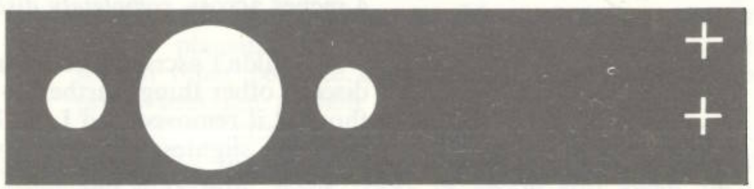
Figure 101



Irradiation is due to the fact that each light point of an object produces on the retina of an eye not a point but a small circle because of so-called spherical aberration. Therefore a light surface on the retina is fringed by a light band that *increases* the place occupied by the surface. On the other hand, black surfaces produce reduced images because of the light band.

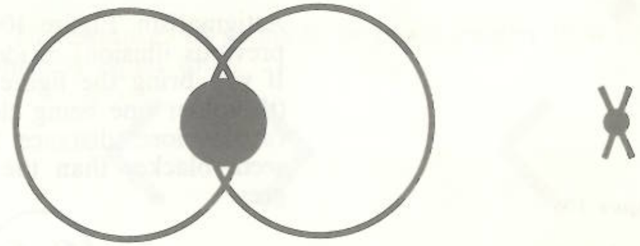
The Mariotte Experiment. Close the right eye and look with the left one at the *upper* cross from a distance of 20-25 centimetres. You'll notice that the middle, large white circle disappears completely, although the two smaller circles on either side are seen distinctly. If, with the same arrangement, you look at the *lower* cross, the circle only disappears in part.

Figure 102



The phenomenon is caused by the fact that with this arrangement of the eye with respect to the figure the image of the circle falls on the so-called *blind spot* – the place insensitive to photic stimulation where the optic nerve is connected.

Figure 103



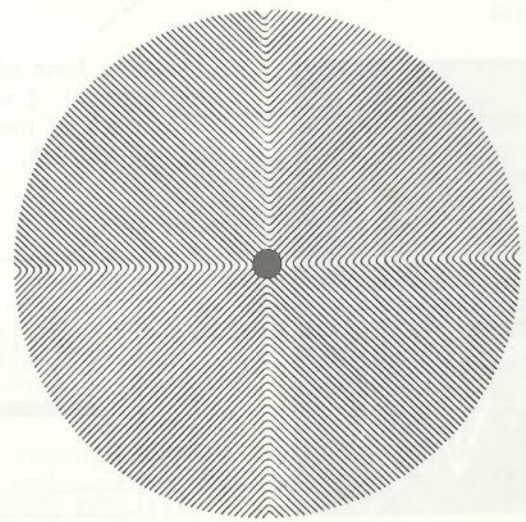
The Blind Spot. This experiment is a modification of the previous one. If you look at the cross at the right of the figure with your left eye at a certain distance you won't see the black circle at all, although the two circumferences will be seen.

Figure 104

Astigmatism. Look at the lettering with one eye. Do all of the letters appear equally black? Normally one of the letters appears blacker than the rest of them. You need only to turn the page by 45° or 90° and some other letter will seem to be blacker.

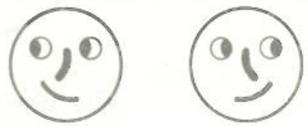
The phenomenon is explained by so-called *astigmatism*, i.e. different curvatures of the retina in different directions (vertical, horizontal, etc.). It's only rarely that an eye is free of this imperfection.

Figure 105



Astigmatism. Figure 105 furnishes another way (cf. the previous illusion) of identifying astigmatism in an eye. If you bring the figure to the eye under examination (the other one being closed) you'll notice at a certain, rather close, distance that two opposite sectors will seem blacker than the other two, which will appear grey.

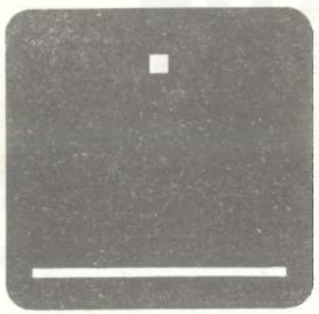
Figure 106



When viewing this figure, move it to the right and left and it'll seem to you that the eyes in the figure swing horizontally.

The illusion is accounted for by the eye's property to retain visual perceptions for a short time after the stimulus has disappeared (cinema is based on this).

Figure 107



Having concentrated on the white square at the top you'll notice about *half a minute* later that the lower white line will have disappeared (owing to retina fatigue).

The Müller-Lyer Illusion. The segment *bc* seems to be longer than *ab*, although they are in fact equal.

Figure 108

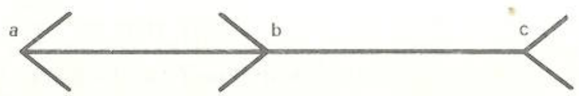
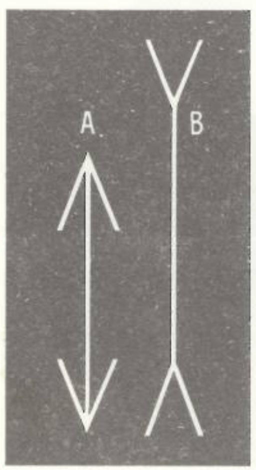


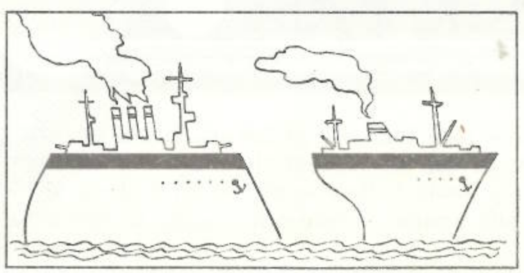
Figure 109



Another form of the previous illusion is Fig. 109 and here segment *A* seems to be shorter than *B*.

The deck of the right ship seems to be shorter than that of the left, actually both are the same length.

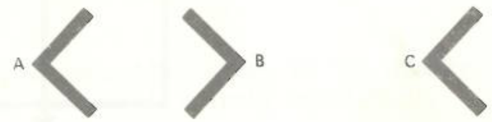
Figure 110



148–149

The distance *AB* seems to be much smaller than *BC*, which is equal to it.

Figure 111



The distance *AB* seems to be larger than *CD*, which is equal to it.

Figure 112

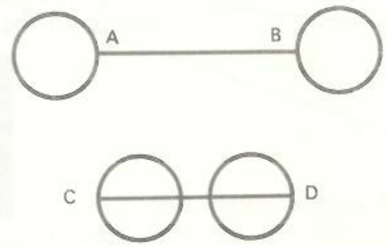
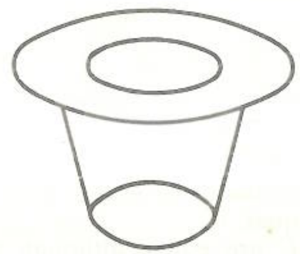


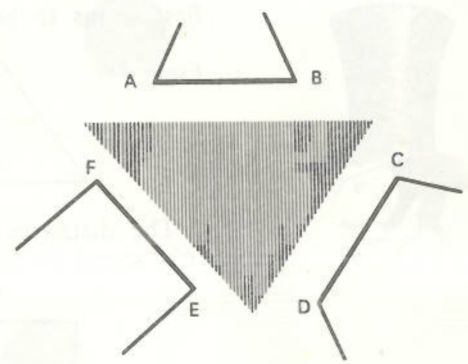
Figure 113



The lower oblong seems to be larger than the internal one, although these are equal (the influence of the arrangement).

The equal distances *AB*, *CD* and *EF* seem to be unequal (the influence of the arrangement).

Figure 114



The rectangle crossed longitudinally seems to be longer and narrower than the equal rectangle crossed transversely (Fig. 115).

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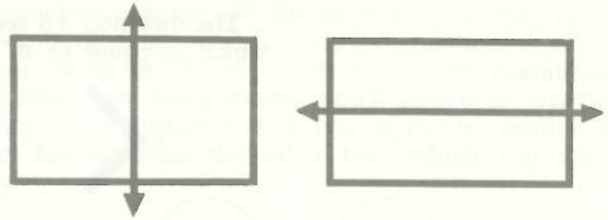
Figure 115



Figures A and B are equal squares, although the first seems, to be higher and narrower than the second.

Figure 116

The height of this figure seems to be larger than its width, although both are equal.

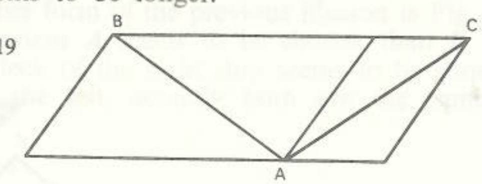
Figure 117

The height of the top hat seems to be longer than its width, although these are equal.

Figure 118

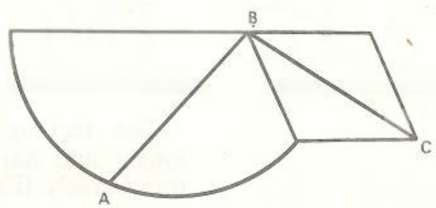
The distances AB and AC are equal, although the first seems to be longer.

Figure 119



The distances BA and BC are equal, although the first seems to be longer.

Figure 120



The upright plank seems to be longer than those below, in fact these are all equal.

Figure 121

The distance MN seems to be smaller than the distance AB, which is really equal to it.

Figure 122

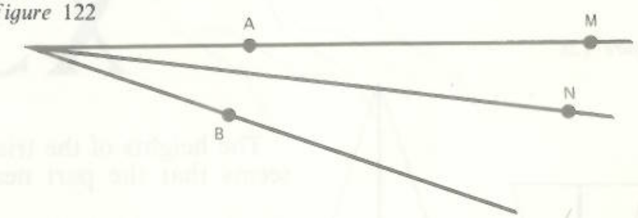
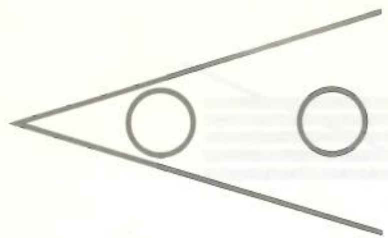


Figure 123



The right circle in this figure seems to be smaller than the equal-sized circle on the left.

The distance AB seems to be smaller than the equal distance CD. This illusion becomes more pronounced with distance (Fig. 124).

The empty gap between the lower circle and each of the upper ones seems to be larger than the distance between the outer edges of the upper circles. In actual fact they are equal (Fig. 125).

Figure 124

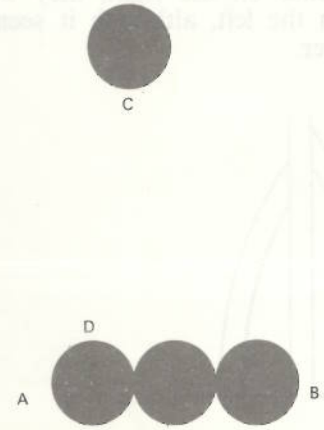


Figure 125

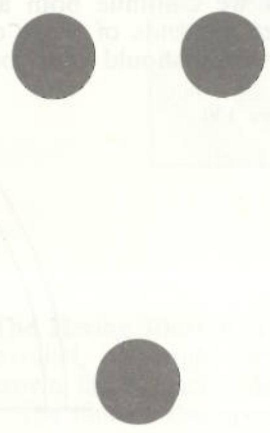


Figure 126

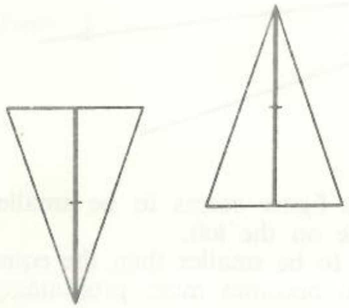
The "Smoking Pipe" Illusion. The dashes on the right in this figure seem to be shorter than those on the left.

The "Print Type" Illusion. The upper and lower parts of each of these characters seem to be equal to each other. But turn the page over and you'll immediately see that the upper parts are smaller.

Figure 127

X38S

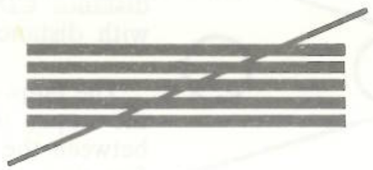
Figure 128



The heights of the triangles are divided in two, but it seems that the part near the vertex is shorter.

The Poggendorf Illusion. The oblique *straight* line intersecting the black and white strips seems to be broken from a distance.

Figure 129



If we continue both arches on the right, they will meet the ends of those on the left, although it seems that they should pass lower.

Figure 130

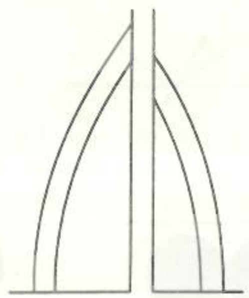
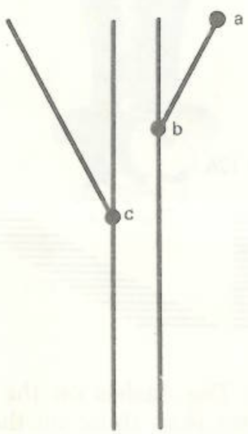
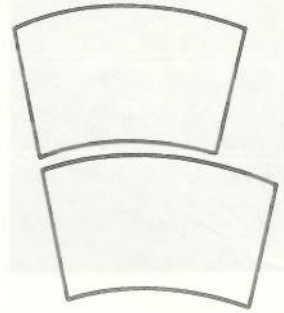


Figure 131



Point *c* lying on the continuation of line *ab*, seems to lie below it.

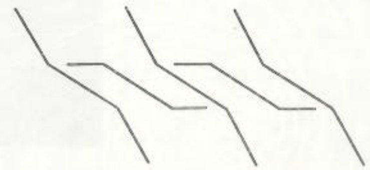
Figure 132



Both figures are identical, although the upper one seems to be the shorter and wider.

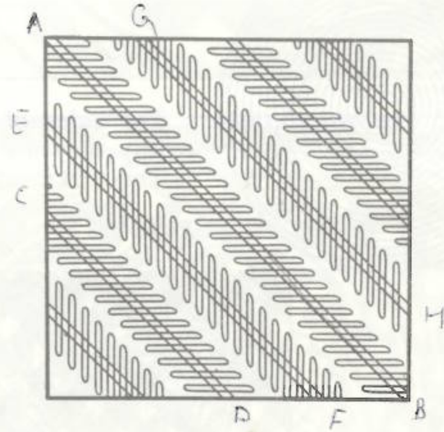
The middle parts of those lines do not seem to be parallel, although they are so.

Figure 133



The Zöllner Illusion. The long oblique lines of this figure are parallel, although these seem to be diverging.

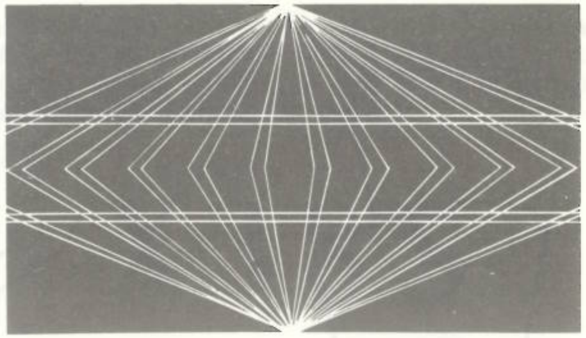
Figure 134



The Hering Illusion. The two double parallel lines are parallel, although they seem to be arches with the crown facing each other (Fig. 135).

The illusion disappears if (1) you hold the figure level to your eyes and view it so that you are glancing along the lines, or (2) you point the end of a pencil at some point and concentrate at this point.

Figure 135



The lower arch seems to be more convex and shorter than the upper one. The arches are similar, though.

Figure 136

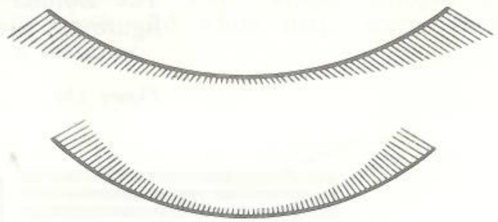
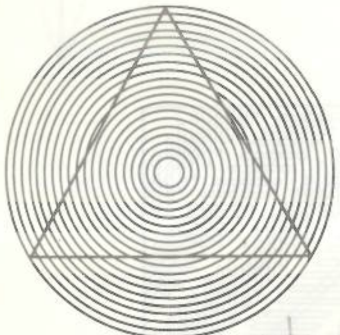


Figure 137



The sides of the triangle seem to be concave, in reality they are straight.

The letters are upright type.

Figure 138

The curves in Fig. 139 seem to be a spiral, although they are circles, which is readily found by following anyone of them with a pencil.

Figure 139

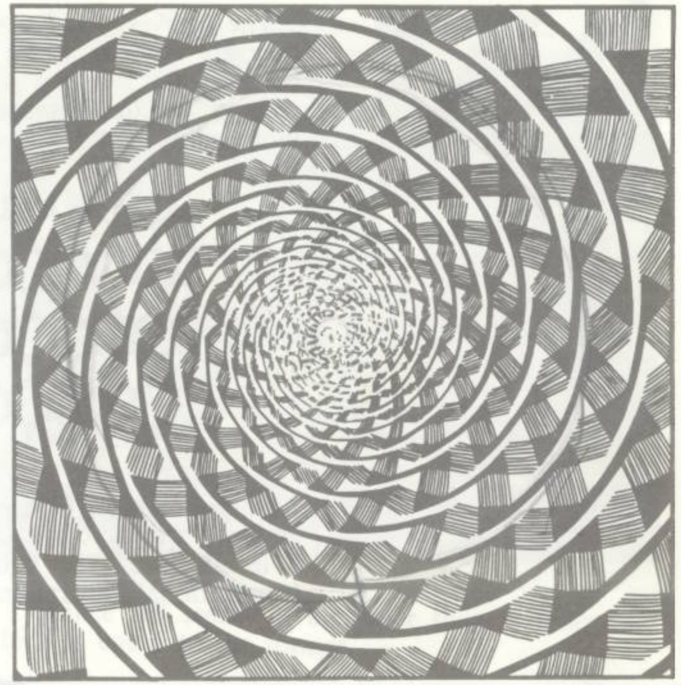
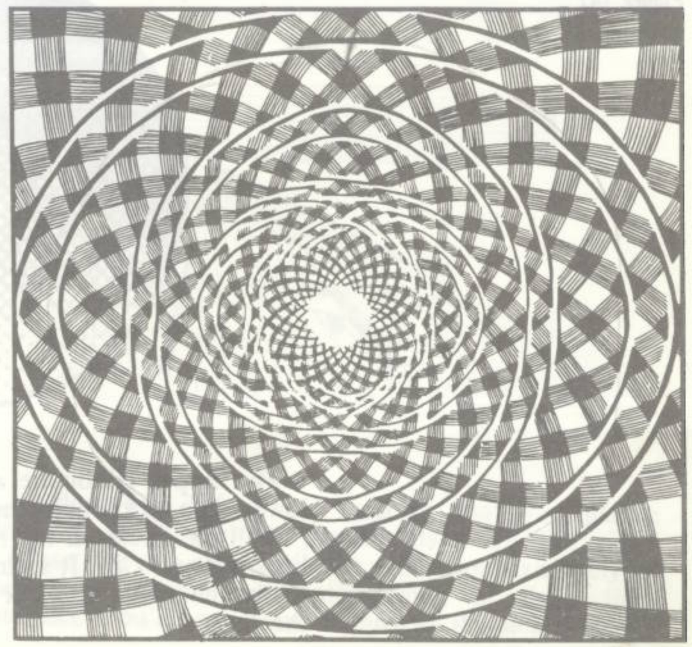


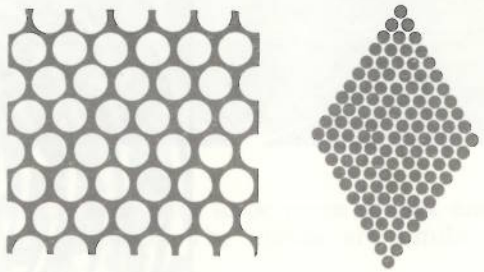
Figure 140



The curves in Fig. 140 seem to be oval, but they are circular, which can be tested with a pair of compasses.

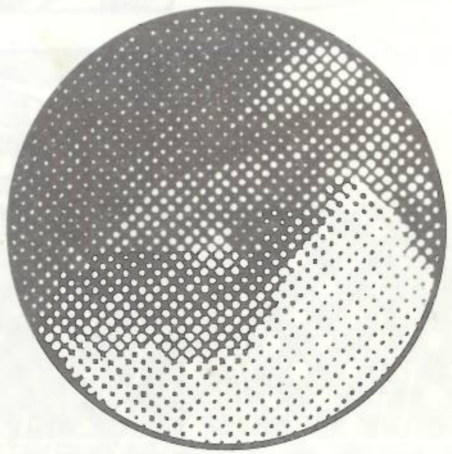
At a certain distance the circles in these figures (both white and black) seem to be hexagons.

Figure 141



Autotype Illusion. Consider the pattern from a distance, and you will perceive an eye and part of the nose of a female face. The figure is a part of an autotype (conventional illustration in a book) multiplied tenfold.

Figure 142

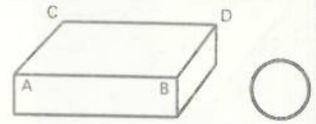


The upper silhouette seems to be longer than the lower one, although they are both the same size.

Will the circle on the right of the figure get between lines *AB* and *CD*? It seems at first sight that it will. But really the circle is wider than the separation between the lines.

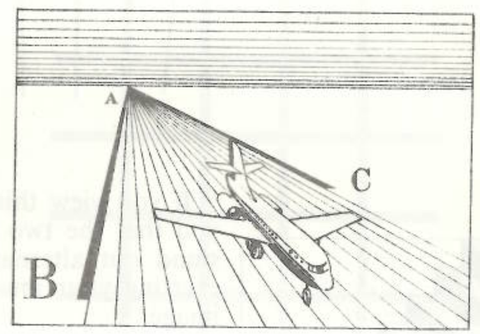
Figure 143

Figure 144



Distance *AB* seems to be wider than distance *AC*, which is equal to the former.

Figure 145



Holding Fig. 146 at eye level so that you glance along it, you'll see the picture given on the right.

Close one eye and place the other approximately at the point where the continuations of these lines intersect. You'll see a row of pins as if stuck into the

Figure 146

paper. Shift the figure slightly sidewards and the pins will swing.

Figure 147

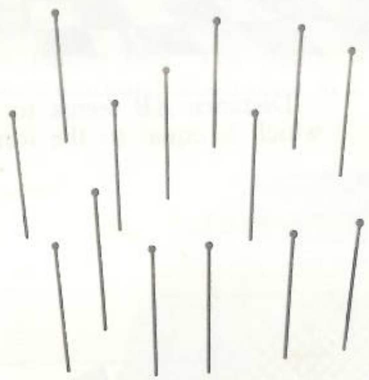
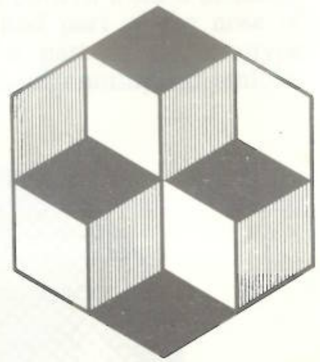
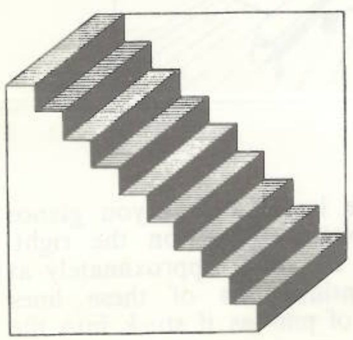


Figure 148



If you view this figure for a long time, it'll seem to you that the two cubes at the top and at the bottom stand out alternately. Also, you can intentionally, by exerting your imagination, call forth one or the other image.

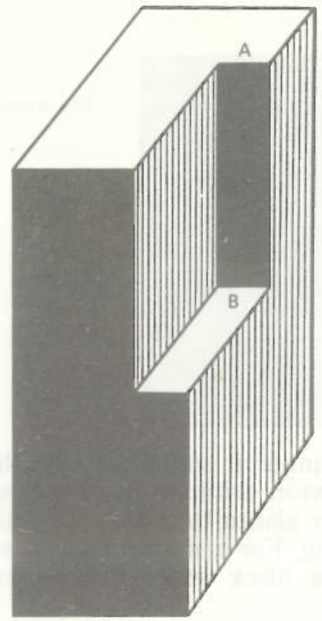
Figure 149



The Schröder Stairs. This figure might be perceived in three ways: (1) as stairs, (2) as a step-wise niche, or (3) as a pleated paper strip stretched out. The perceptions may interchange intentionally or unintentionally.

The figure may represent, as you like it, either a block with a recess (the back side of the recess is the plane *AB*), or a part of an empty box with a block

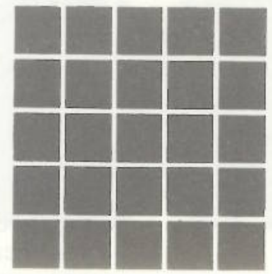
Figure 150



touching the walls from the inside, the box being open at the bottom.

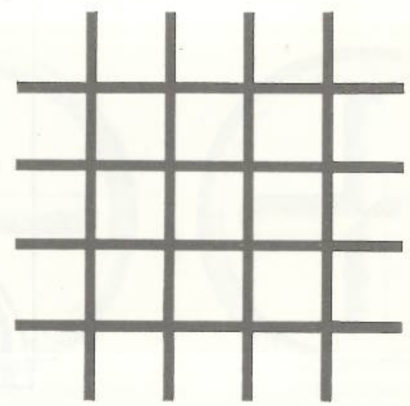
The intersections of the white lines in this figure seem to have yellowish square spots that appear and disappear, as if flashing. In actuality, the lines are absolutely white throughout, which can be seen if you cover adjacent rows of black squares with paper. The effect is because of the contrast.

Figure 151



A modification of the illusion of Fig. 151, in which white spots appear at the intersections.

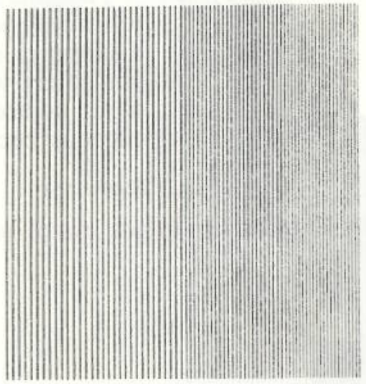
Figure 152



Look at this figure from a distance. Its four strips each seem to be a concave stripe that is lighter at the edge and adjacent to a neighbouring, darker strip. But by masking neighbouring strips to exclude the influence

of contrast you can see that each strip is uniformly darkened.

Figure 153

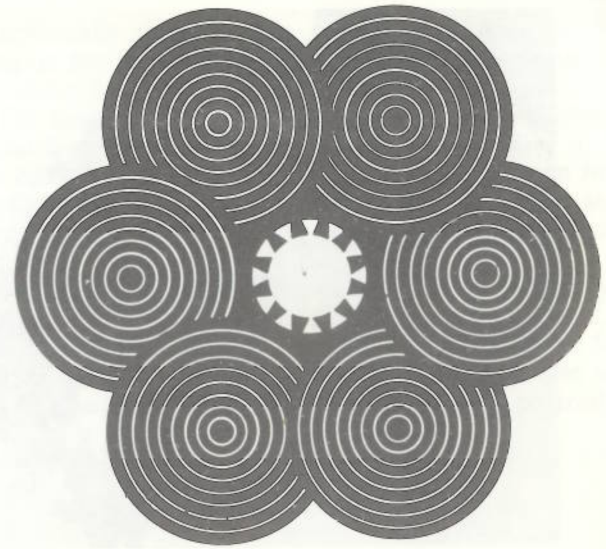


Look attentively for a minute at some point on this "negative" portrait of Newton without moving your eyes, then quickly shift your glance to a piece of white paper, greyish wall or ceiling. For a moment you'll see the same portrait, but the black spots will become white, and vice versa.

Figure 154

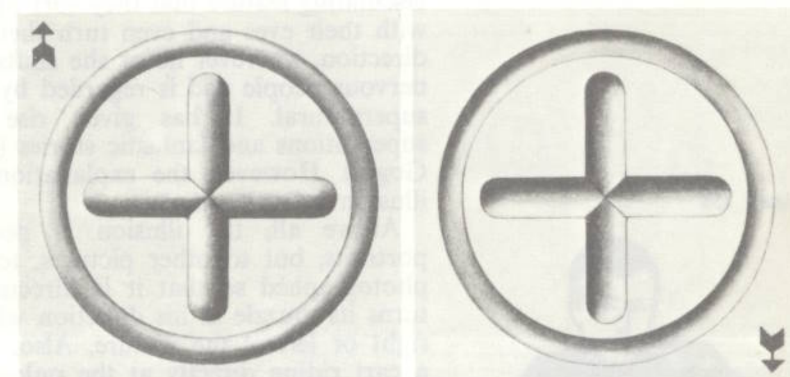
The Silvanus Thompson Illusion. If you rotate this figure (by turning the book) all the rings and the white toothed wheel will seem to be rotating, each about its own centre, in the same direction and with the same speed.

Figure 155



On the left you see a convex cross, on the right – a concave one. But turn the book upside down and the figures will change their places. Actually the figures are identical, only they're shown at different angles.

Figure 156



Look at the photograph in Fig. 157 with one eye 14-16 centimetres away from the centre of the picture. With this arrangement your eye will see the picture from the same point the objective of the camera "saw"

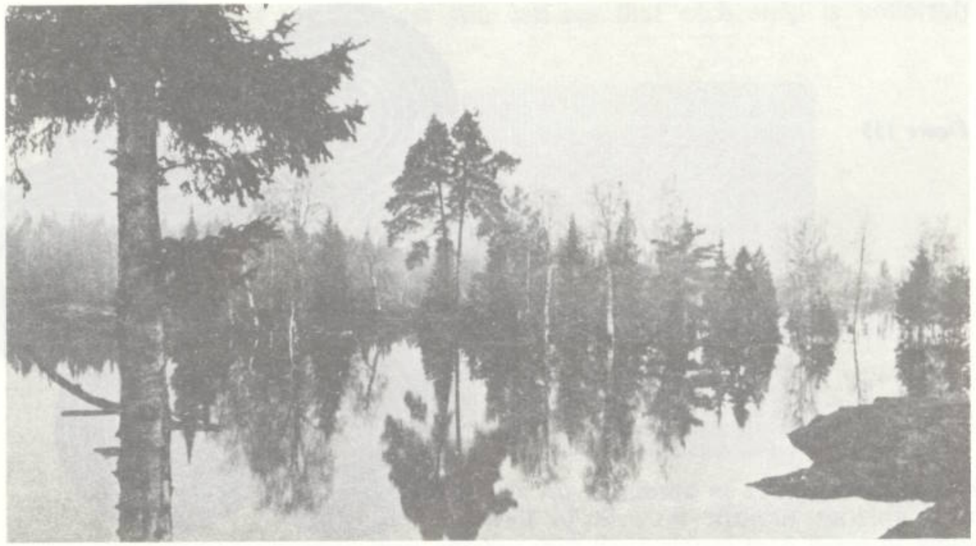


Figure 157

the scene. It's this that accounts for the liveliness of the impression. The landscape acquires depth, the water glimmer.

The eyes and the finger seem to point directly at you and follow you when you shift to the right or left.

It has long been known that some portraits have the fascinating feature that they sort of follow the onlooker with their eyes and even turn their faces in his or her direction, wherever he or she shifts. This feature scares nervous people and is regarded by many as something supernatural. It has given rise to a number of superstitions and fantastic stories (e.g. *The Portrait* by Gogol). However, the explanation of this interesting illusion is very simple.

Above all, the illusion is peculiar not only to portraits, but to other pictures, too. A gun drawn or photographed so that it is directed at the onlooker* turns its muzzle in his direction when he moves to the right or left of the picture. Also, there is no evading a cart riding directly at the onlooker.

All of these phenomena have one common and

Figure 158

* Such a photograph is obtained if, in photographing, the muzzle of the gun is directed at the objective. In exactly the same way if the person being photographed looks into the objective, then his eyes in the picture will be directed at the onlooker, at whatever angle he views the picture.

exceptionally simple cause. If we view the picture we imagine the things shown in it, and it seems to us that the thing has changed its position.

The same applies to the portraits. When we observe a real face from the side, we see another part of it. We can only see the same part as before if the person turns his face to us, but in a portrait we always see the same view. When a portrait is perfectly executed the effect is striking.

Clearly, there is nothing surprising in this property of portraits. Conversely, it would be more unusual if, as we shift sidewards, we would see the side of the face. But, this, in essence, is what is expected by those who regard the apparent turn of the face in a portrait as something supernatural!

Brain-Twisting Arrangements and Permutations

6

In Six Rows

You may have heard the funny story that nine horses have been put into 10 boxes, one in each. The problem that is now posed is formally similar to this famous joke, but it has a real solution *. You must arrange 24 people in six rows with five in each.

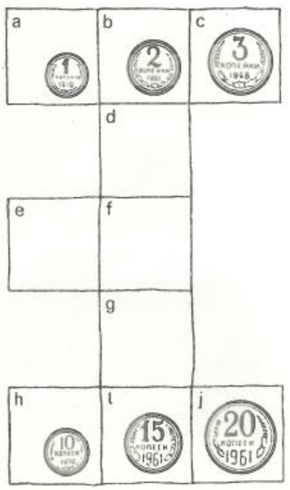
In Nine Squares

Figure 159

This is a trick question – half a problem, half a trick.
Using matches make a square with nine small square cells and place a coin in each so that each row and column contain 6 kopecks (Fig. 159).
The figure shows the arrangement of the coins. Place a match on one coin.
Now ask your friends to change the arrangement without moving the coin with the match so that the rows and columns each still contain 6 kopecks.
They'll say it's impossible. However, a small trick will help you to perform this "impossible" task. Which one?

Coin Exchange

Figure 160



Make a large drawing of the arrangement in Fig. 160 and denote each of the small squares by a letter in the top left corner as shown. Put 1 kopeck, 2 kopeck, and 3 kopeck coins into the three squares of the upper row. Now put 10 kopeck, 15 kopeck, and 20 kopeck coins into the three squares of the lower row. The rest of the squares are empty.
By shifting the coins on vacant squares you make the coins exchange their places so that the 1 kopeck changes with the 10 kopecks, the 2 kopecks changes with the 15 kopecks, and the 3 kopecks with the 20 kopecks. You may occupy any vacant place of the figure but you are not permitted to place two coins into one square. Also, it isn't allowed to skip an occupied square or go beyond the figure.
The problem is solved by a long series of moves. Which moves?

* In what follows the answers to problems are given at the end of each section.

Nine Zeros

Nine zeros are arranged as shown below:
0 0 0
0 0 0
0 0 0

You must cross all the zeros with four lines only.
To simplify the solution I will add that the nine zeros are to be crossed without the pencil leaving the paper.

Thirty Six Zeros

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

You see that 36 zeros are arranged in the cells of this network.
You must cross out 12 zeros so that each row and column retain an equal number of uncrossed zeros.
Which zeros are to be crossed?

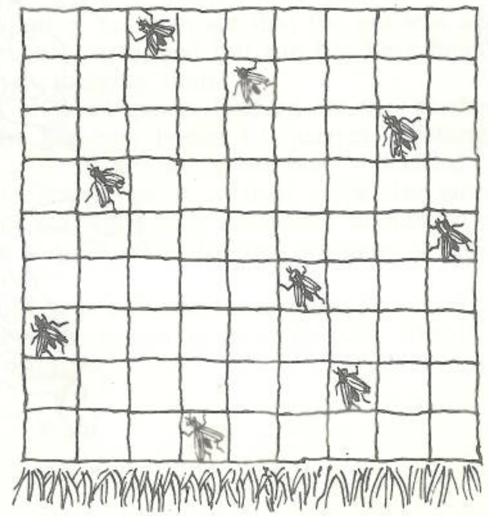
Two Draughtsmen

Put two different draughtsmen on a draughts board. How many different arrangements are possible?

Flies on a Curtain

Nine flies are sitting on a chequered window curtain. They happened to have arranged themselves so that no two flies are in the same row, column, or diagonal (Fig. 161).

Figure 161

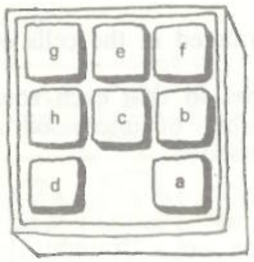


After a while three flies shifted into neighbouring, unoccupied cells and the other six stayed in the same place. Curiously enough, the nine flies still continued to be arranged so that not a single pair appeared in the same direct or oblique line.

Which three flies shifted and which cells did they choose?

Eight Letters

Figure 162

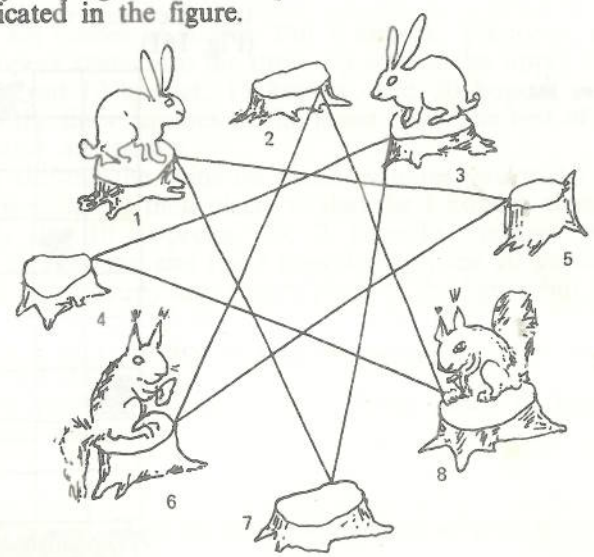


The eight letters arranged in the cells of the square shown in Fig. 162 are to be arranged in alphabetical order by shifting them into a vacant cell, as in the two previous problems. This is not difficult if the number of moves is not limited, but you are required to achieve the result using a minimum number of moves. You must find out for yourself what the minimum number is.

Squirrels and Rabbits

Figure 163 shows eight numbered stumps. On stumps 1 and 3 sit two rabbits, and on stumps 6 and 8, two squirrels. But both the squirrels and the rabbits are not happy with their seats and want to exchange them, the squirrels want to take the places of the rabbits, and the rabbits the places of the squirrels. They can only make it by leaping from a stump to the other along the lines indicated in the figure.

Figure 163



166–167

How could they make it?

Observe the following rules:

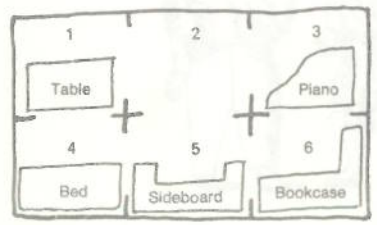
(1) each animal may make several leaps at once;

(2) two animals may not seat on the same stump, therefore they must only leap on a vacant stump.

Further, you should take into account that the animals want to reach their goal using the least possible number of leaps, although it's impossible to make less than 16 leaps.

Cottage Problem

Figure 164



The accompanying figure shows the plan of a small cottage whose poky rooms house the following furniture: a desk, a piano, a bed, a sideboard, and a bookcase. Only room 2 is free of furniture.

The tenant wanted to change around the piano and the bookcase. This appeared to be a difficult problem because the rooms are so small that no two of the above pieces could be in the same room. The free room 2 was of help. By shifting the things from one room to another the desired arrangement was eventually achieved.

What is the least number of changes required to achieve the goal?

Three Paths

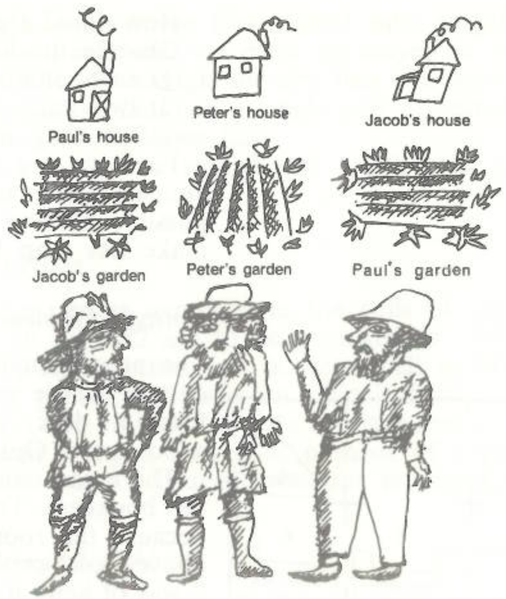
Three brothers – Peter, Paul, and Jacob – got three vegetable gardens located near their houses, as shown in the figure. You can see that the gardens are not very conveniently arranged but the brothers failed to agree about exchanging them.

The shortest paths leading to the gardens crossed and the brothers began to quarrel. Wishing to avoid future conflict the brothers decided to find nonintersecting paths to their respective gardens. After a lot of searching they succeeded in finding such paths and now they come to their gardens without meeting each other.

Could you indicate these paths?

One requirement is that no path should go round Peter's house.

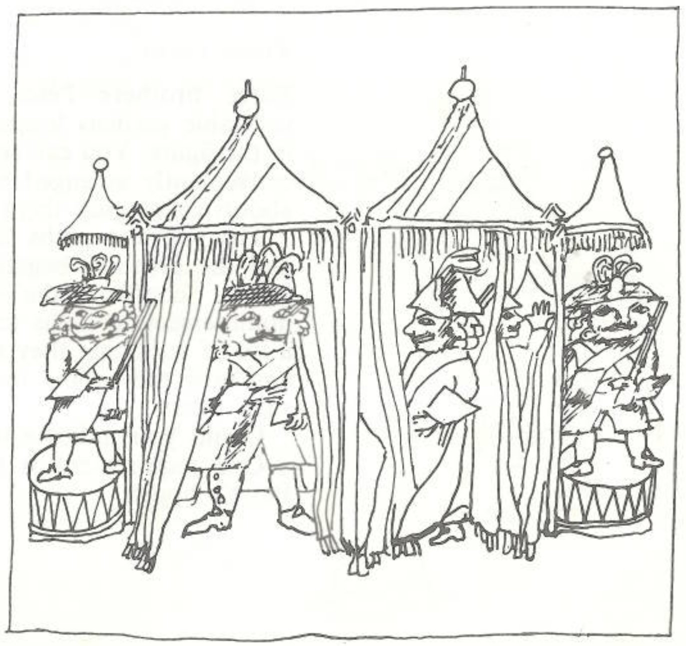
Figure 165



Pranks of Guards

The following is an ancient problem having many modifications. We'll discuss one of them.

Figure 166



The commander's tent is guarded by sentries housed in eight other tents (Fig. 166). Initially in each of the tents there were three sentries. Later the sentries were allowed to visit each other and their chief didn't punish them when, having come to a tent, he found more than three soldiers in it and less than three in the others. He only checked the total number of soldiers *in each row* of tents, thus if the total number of soldiers in the three tents of each row was nine, the chief thought that all of the guards were present.

Having noticed this the soldiers found a way to outwit their chief. One night four guards left and this passed unnoticed. On the next night six left and got away with that. On later night the guards began to invite guests: at one time four, at another eight, and at yet another, a full dozen guests. And all of these pranks passed unnoticed as the chief always found nine soldiers in the three tents of each row.

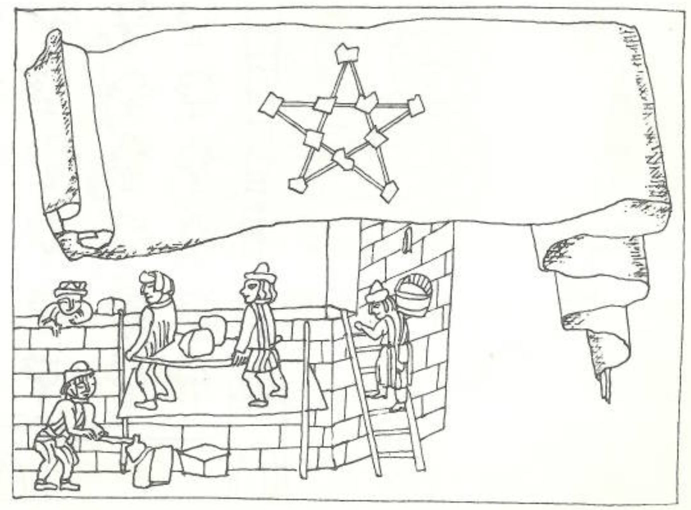
How did they manage to do so?

Ten Castles

In olden days a prince desired to have 10 castles built. They should be connected by walls arranged on five straight lines with four castles on each. The architect submitted the plan given in Fig. 167.

But the prince wasn't satisfied with the plan because the arrangement made all the castles vulnerable to outside attack, but he wished there to be at least one or

Figure 167



two castles protected within the walls. The architect objected that it was impossible to satisfy the condition whilst the 10 castles had to be arranged four in each of the five walls. But the prince insisted.

After a lot of head-scratching the architect in the long run came up with an answer.

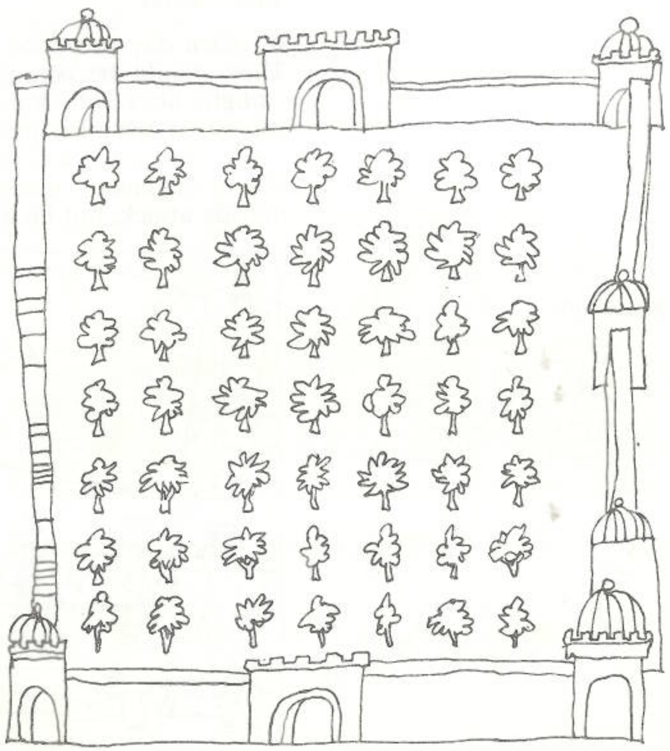
Maybe you'll be happy enough, too, to arrange the 10 castles and the five interconnecting walls so as to meet the above conditions?

An Orchard

There were 49 trees in an orchard, arranged as shown in Fig. 168. The gardener decided that the orchard was too crowded, so he wanted to clear the garden of excess trees to make flowerbeds. He called in a workman and ordered: "Leave only five rows of trees, with four trees in each row. Cut down the rest and take them home for firewood as your payment for the work".

When the tree felling had finished the gardener came

Figure 168



to see the result. Much to his dismay he found the orchard almost devastated: instead of the 20 trees the workman had left only 10 and cut 39.

"Why have you cut so many? You were told to leave 20 trees!" the gardener was enraged.

"No. You only told me to leave five rows with four trees in each. I did so. Just look."

The gardener was amazed to find that the 10 remaining trees formed five rows with four trees in each. His order had been fulfilled literally, and still... 39 trees had been cut down instead of 29.

How had the workman managed it?

The White Mouse

All of the 13 mice in the figure are doomed to be eaten by the cat. But the cat wants to consume them in a certain order. The cat eats one mouse and then counts around the circle in the direction in which the mice are looking. When it gets to 13 it eats the mouse and starts counting again, missing out the eaten mice. Which mouse must it start from for the white mouse to be eaten last?

Figure 169

6

In Six Rows

The requirement of the problem is easily met if the people are arranged in the form of a hexagon as shown in the figure.

Figure 170

In Nine Squares

You don't touch the forbidden coin but shift the whole of the lower row upwards (Fig. 171). The arrangement has changed but the requirement of the problem is satisfied: the coin with the match hasn't been shifted.

Figure 171

Coin Exchange

The following is the series of moves required (the number is the coin, the letter is the cell to which the coin is shifted):

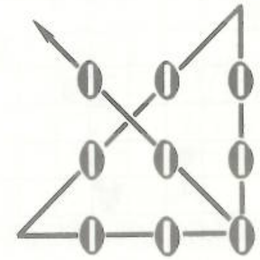
2–e	15–i	2–d	10–a
15–b	3–g	1–h	3–e
10–d	20–c	10–e	15–b
2–h	1–e	2–j	2–d
20–e	3–a	15–i	3–j
10–j	15–b	3–g	2–i

It's impossible to solve the problem in less than 24 moves.

Nine Zeros

The problem is solved as shown in Fig. 172.

Figure 172



Thirty Six Zeros

As it's required to cross out 12 of the 36 zeros, we'll have 36 – 12, i.e. 24 zeros with four zeros in each row.

The remaining zeros will be arranged as follows:

0		0	0	0	
		0	0	0	0
0	0	0			0
0	0		0		0
0	0			0	0
	0	0	0	0	

Two Draughtsmen

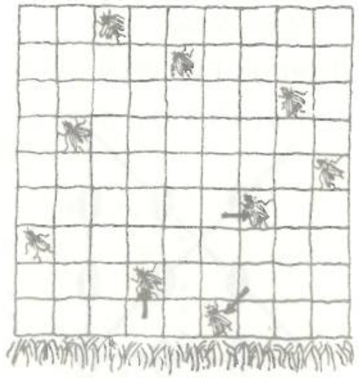
One draughtsman may be placed at any of the 64 squares of the board, i.e. in 64 ways, then the second one can occupy any of the 63 remaining squares. Hence for each of the 64 positions of the first draughtsman you can find 63 positions for the second one. Consequently, the total number of the various permutations of the two draughtsmen is:

$$64 \times 63 = 4{,}032.$$

Flies on a Curtain

The arrows in Fig. 173 indicate which flies must be shifted and in which direction.

Figure 173



Eight Letters

The least number of moves is 23. These are as follows:

A B F E C A B F E C A B D H G A B D H G D E F

Squirrels and Rabbits

Shown below is the shortest way of the rearrangement. The first number in each pair indicates from which stump an animal should leap and the second number the destination stump (for example, 1-5 means that a squirrel has leapt from the first stump to the fifth). The total number of leaps required is 16, namely:

1–5; 3–7; 7–1; 5–6; 3–7; 6–2; 8–4; 7–1;
8–4; 4–3; 6–2; 2–8; 1–5; 5–6; 2–8; 4–3.

Cottage Problem

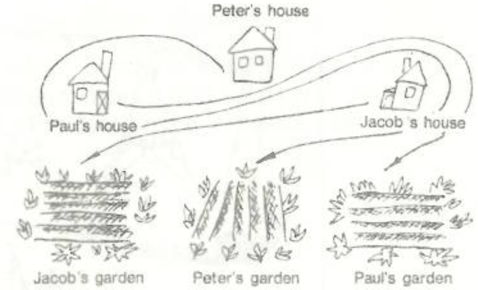
The exchange can be achieved in no less than 17 moves. The pieces of furniture are moved in the following sequence:

1. Piano	7. Piano	13. Bed
2. Bookcase	8. Sideboard	14. Sideboard
3. Sideboard	9. Bookcase	15. Table
4. Piano	10. Table	16. Bookcase
5. Table	11. Sideboard	17. Piano
6. Bed	12. Piano	

Three Paths

The three nonintersecting paths are shown in Fig. 174.

Figure 174



Pranks of Guards

The problem is easily solved by the following reasoning. For four guards to be able to be absent unnoticed by the chief it's necessary that in rows *I* and *III* (Fig. 175*a*) there are nine soldiers in each. As the total number is $24 - 4 = 20$, then in row *II* there will be $20 - 18 = 2$, i.e. one soldier in the left tent of this row and one in the right. In the same way we find that there must be one soldier in the upper tent of row *V* and one in the lower. It is now clear that the corner tents must house four guards. Accordingly, the required arrangement for four soldiers to be absent is as shown in Fig. 175*b*.

A similar argument yields the desired arrangement for six soldiers to be absent (Fig. 175*c*).

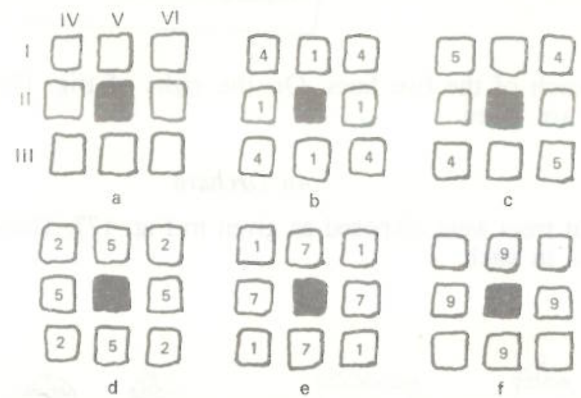
For four guests the arrangement is shown in Fig. 175*d*;

For eight guests in Fig. 175*e*;

And finally, Fig. 175*f* shows the arrangement for 12 guests.

It is easy to see that under these conditions no more than six soldiers can be absent with impunity and no more than 12 guests can visit the guards.

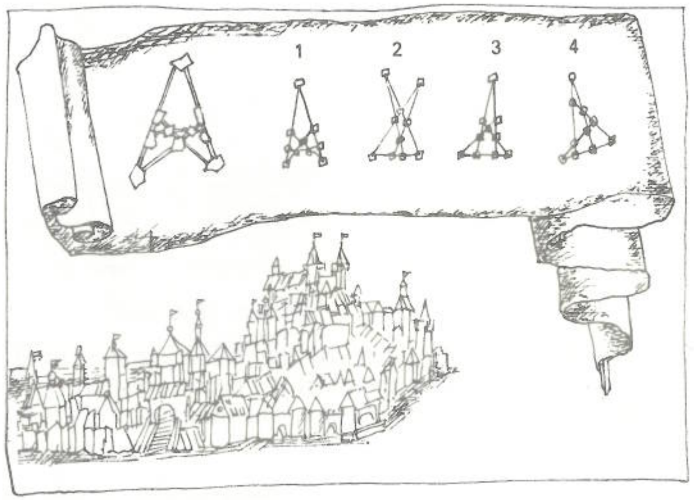
Figure 175



Ten Castles

Figure 176 (on the left) shows the arrangement with two castles protected from the external attack. You see that the 10 castles are disposed as required in the problem:

Figure 176

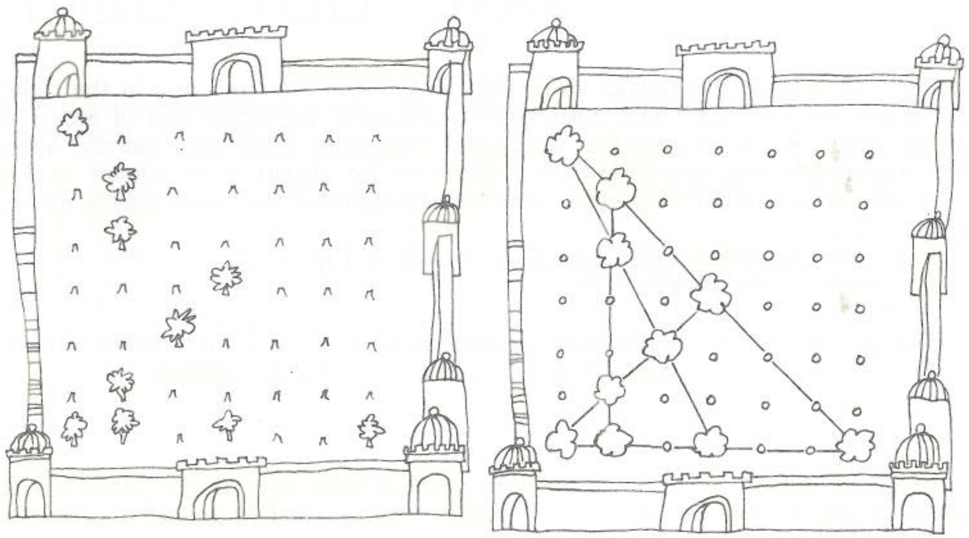


four on each of the five lines. On the right of Fig. 176, four more solutions to the problem are given.

An Orchard

The uncut trees were disposed as given in Fig. 177. These form five straight rows with four trees in each.

Figure 177



The White Mouse

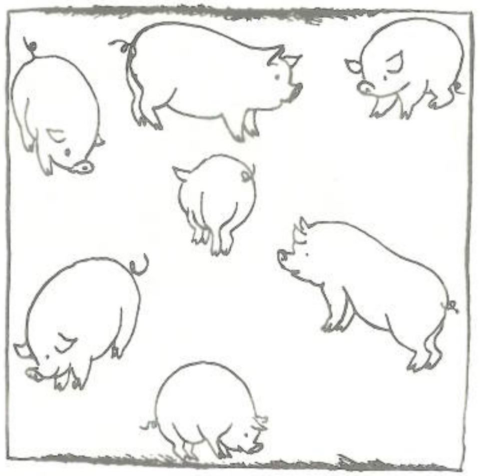
The cat should first eat the mouse at which it is looking, i.e. the sixth one from the white. Try it by beginning with this mouse and cross out every 13th mouse. You'll see that the white mouse will be the last to be crossed out.

Skilful Cutting and Connecting

With Three Straight Lines

Cut Fig. 178 into seven sections with three straight lines so that there is one animal in each section.

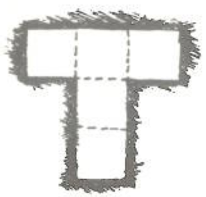
Figure 178



Into Four Parts

This ground area (Fig. 179) consists of five equal squares. Draw the area on a sheet of paper. Can you cut it into four identical areas, not five?

Figure 179



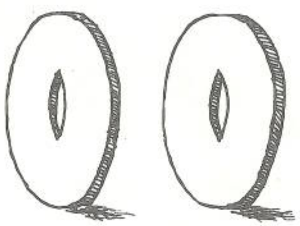
To Make a Circle

A joiner was given two pieces of rare wood with holes in them (as shown) and was asked to make them into a perfectly circular solid board for a table so that no scraps of the expensive wood would be left over. All the wood must be used.

The joiner was a master craftsman but the order was not easy. He scratched his head for a long time, tried one way and then another, and eventually hit upon an idea as to how to execute his order.

Perhaps you'll twig it, too? Cut out two paper figures, exactly like the ones in Fig. 180 (only larger) and use them to arrive at the solution.

Figure 180



A Clock Dial

The clock dial in Fig. 181 must be cut into six parts of any shape so that the sum of numbers in each section

would be the same.

The aim of the problem is not so much to test your resourcefulness but the quickness of your thought.

Figure 181

Crescent

The crescent (Fig. 182) must be divided into six parts by only two straight lines.

Figure 182

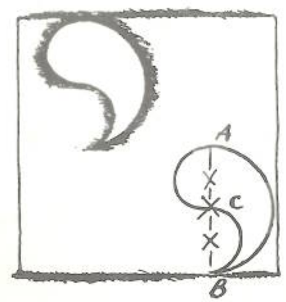
To Divide a Comma

In the accompanying figure you will see a wide comma. It's constructed very simply: a semicircle is drawn on the straight line *AB* around point *C*, then two semicircles are drawn around the middles of the segments *AC* and *CB*, one on the right, the other on the left.

You must cut the figure into two identical parts by a single curved line.

The figure is also interesting in that two such figures make up a circle. How?

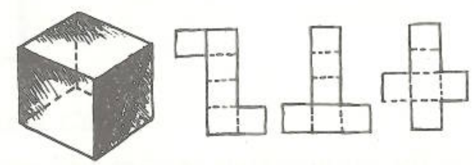
Figure 183



To Develop a Cube

If you cut a cardboard cube along edges so that it could be unfolded and placed with all six squares on a table, you'll get a figure like one of those shown in Fig. 184.

Figure 184



It's curious, but how many *different* figures can be obtained in this way? In other words, in how many ways can a cube be developed?

I warn the impatient reader that there are no less than 10 different ways.

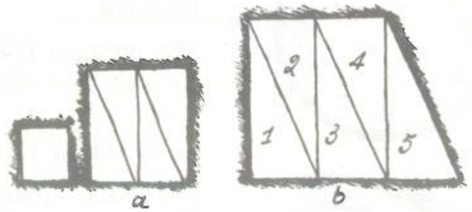
To Make Up a Square

Can you make up a square from five pieces of paper like the ones shown in Fig. 185*a*?

If you've already found the solution, try and make up

a square from five identical triangles like the ones you have just used (the base is twice as long as the height). You may cut one of the triangles into two parts but the other four must be used as they are (Fig. 185*b*).

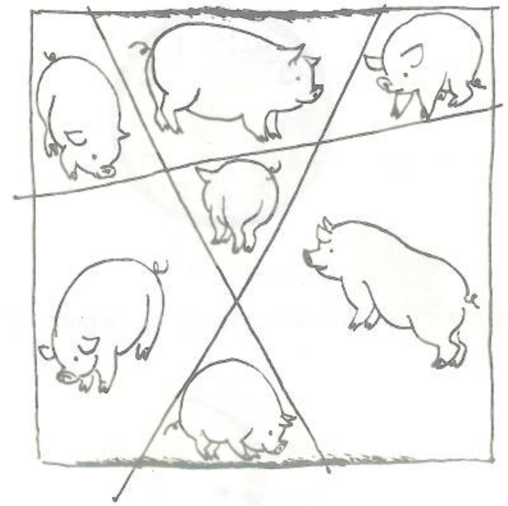
Figure 185



With Three Straight Lines

The problem is solved as follows:

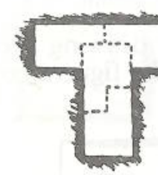
Figure 186



Into Four Parts

The dash lines show the way in which the ground must be divided (Fig. 187).

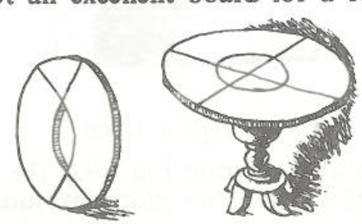
Figure 187



To Make a Circle

The joiner has cut each of the boards into four parts as shown on the left of Fig. 188. From the four smaller parts he makes up a smaller inner circle to which he glues the other four parts. He thus got an excellent board for a round table.

Figure 188



A Clock Dial

As the sum of all the numbers on the face of the dial is 78, the sum of each of the six sections must be $78 \div 6 = 13$. This facilitates finding the solution that is shown in Fig. 189.

Figure 189

Crescent

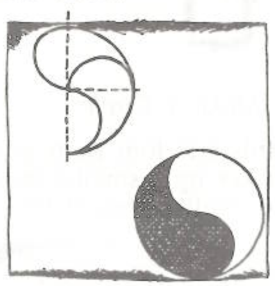
The answer is shown in the accompanying figure. The resultant six parts are numbered.

Figure 190

To Divide a Comma

The solution is seen in the accompanying drawing. Both parts of the comma are equal, as they are made up of equal parts. The figure shows how the circle is made of two commas, one white and one black.

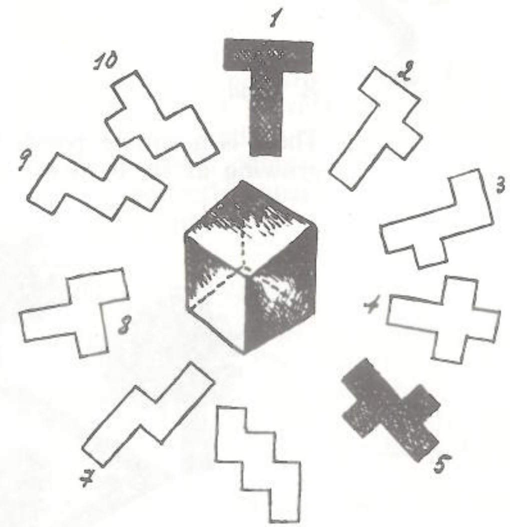
Figure 191



To Develop a Cube

All the 10 possible solutions are shown in Fig. 192. The first and fifth figures can be turned upside down and this will add two more involutes, increasing the total to 12.

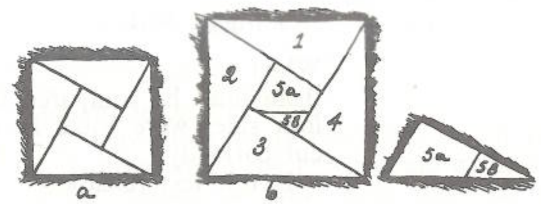
Figure 192



To Make Up a Square

The solution of the first problem is shown in Fig. 193*a*. The case of triangles is given in Fig. 193*b*. One triangle is first cut up as shown.

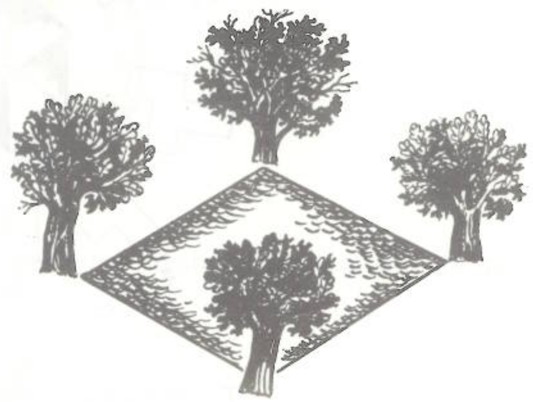
Figure 193



A Pond

There is a square pond (Fig. 194) with four old oaks growing at its corners. It is required to expand the

Figure 194



pond so that its surface area be doubled, the square shape being retained and the old oaks not destroyed or swamped.

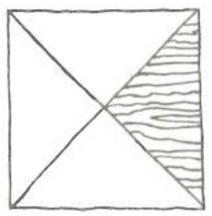
A Parquet Maker

When cutting wooden squares a parquet maker tested them thus: he compared the lengths of sides and if all four sides were equal he considered the square to be cut correctly.
Is this test reliable?

Another Parquet Maker

Another parquet maker checked his work otherwise: he measured diagonals not the sides. If both diagonals were equal, he considered the square to be true.
Are you of the same opinion?

Figure 195



Yet Another Parquet Maker

Yet another worker checked his squares by seeing if all the four sections into which the diagonals divide each other (Fig. 195) are equal to each other. In his opinion it proved that the rectangle cut was square.
What do you make of that?

A Seamstress

A seamstress wants to cut out a piece of linen in the form of a square. Having cut several pieces she checks her work by bending each piece along its diagonal to see if the edges coincide. If they do, she thinks, each piece is perfectly square.
Is she right?

Another Seamstress

Another seamstress wasn't satisfied with the check her companion used. She bent her piece first along one diagonal and then after smoothing the linen she bent it along the other. It was only if the edges of the piece coincided in both cases that she thought the square was correct.
What would you say about this test?

A Joiner's Problem

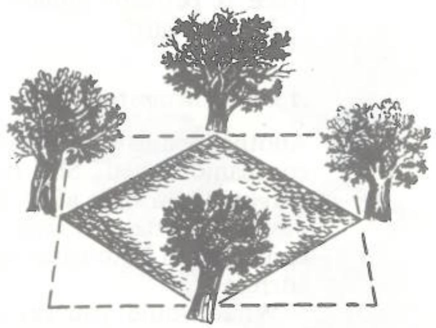
Figure 196

A young joiner has the five-sided board shown in Fig. 196. You see that it seems to be composed of a square glued to a triangle that is four times smaller than the square. The joiner is asked to make the board into a square, taking nothing away from the board and adding nothing to it. This, of course, involves cutting it into sections. Our young joiner is just going to do so, but he wants to cut the board along no more than two straight lines.
Is it possible, using two lines, to cut the figure into parts from which the joiner could make a square? And if the answer is "yes" how does he go about it?

A Pond

It is possible to double the surface area of the pond with the square shape retained and the oaks intact. The accompanying figure shows how this can be done. You can

Figure 197

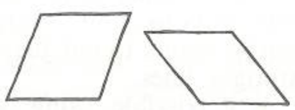


easily see that the new area is twice the earlier, just draw in the diagonals of the earlier pond and count the resultant triangles.

A Parquet Maker

The test is not sufficient. Some quadrilaterals that are by no means squares will pass. Figure 198 gives examples of quadrilaterals whose sides are equal but whose angles are not right (rhombs).

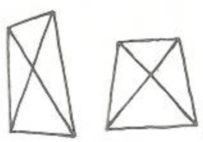
Figure 198



Another Parquet Maker

This test is as unreliable as the first one. To be sure, a square's diagonals are equal but not every quadrilateral with equal diagonals is a square. It is clearly seen from the examples in Fig. 199.

Figure 199

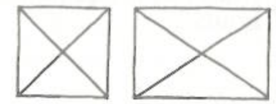


The parquet makers should apply both tests to each quadrilateral produced. One could then be sure that the work has been done correctly. Any rhomb with equal diagonals is bound to be a square.

Yet Another Parquet Maker

The test might only show that the quadrilateral in question has right angles, i.e. that it is a rectangle. But it fails to verify that all its sides are equal, as is seen in Fig. 200.

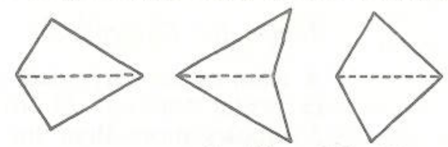
Figure 200



A Seamstress

The test is far from adequate. Figure 201 presents several quadrilaterals whose edges coincide when bent along the diagonals, yet they are not squares. You see how far

Figure 201



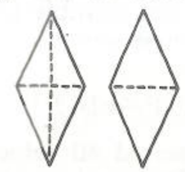
a quadrilateral may differ from a square and still satisfy this test.

The test only shows that the figure is symmetrical, no more.

Another Seamstress

This test is no better than the previous one. You could cut any number of quadrilaterals out of paper that would pass this test, although they are by no means squares. The examples in Fig. 202 all have equal sides (these are rhombs) but the

Figure 202



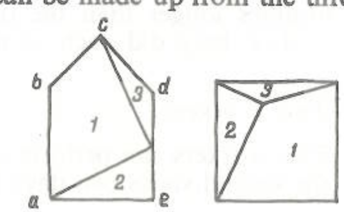
angles are not right–hence these are not squares.

In order to make really sure that the pieces cut out are squares, the seamstress should additionally check if the diagonals (or angles) were equal.

A Joiner's Problem

One line should go from the vertex *c* to the middle of side *de*, the other, from the last point to vertex *a*. A square can be made up from the three pieces *1*, *2*, and *3* as shown in Figure 203.

Figure 203



9

Problems on Manual Work

Navvies

Five navvies excavate a 5-metre ditch in 5 hours. How many navvies are required to dig 100 metres of ditch in 100 hours?

Lumberjacks

A lumberjack cuts a 5-metre log into 1-metre lengths. If each cut takes 1.5 minutes, how long will it take to cut the log?

Joiner and Carpenters

A team of six carpenters and a joiner did a job. Each carpenter earned 20 roubles, but the joiner got 3 roubles more than the average earnings of all the seven team members.

How much did the joiner earn?

Five Pieces of Chain

A blacksmith was given five pieces of chain with three links in each (Fig. 204) and asked to connect them.

The blacksmith opened and reclosed four links.

But is it not possible to do the same job with fewer links tampered with?

Figure 204

How Many Vehicles?

A shop repaired 40 vehicles (cars and motocycles) in a month. The total number of wheels on the vehicles was 100.

How many cars and motocycles were repaired?

Potato Peeling

Two people peeled 400 potatoes. One completed three pieces a minute, the other two. The second worked 25 minutes longer than the first.

How long did each work?

Two Workers

Two workers can perform a job in seven days provided the second starts two days later than the first. If the job

were done by each of them separately, then the first would take four days more than the second.

How many days would each of them take to perform the job individually?

The problem permits of a purely arithmetic solution without any need to manipulate fractions.

Typing a Report

Two typists type a report. The more experienced one could finish the work in 2 hours, the other in 3 hours.

How long will it take them to do the job if they divide it so as to spend the least time possible?

Problems of this kind are normally solved according to the procedure of the famous problem on reservoirs. Thus in our problem they would find the share of the work done by each typist, add up the fractions and divide unity by the resultant sum.

Could you think of some other procedure?

Weighing Flour

A salesman has to weigh five bags of flour. His problem was that the shop had a balance but some weights were missing so that it was impossible to weigh from 50 to 100 kilogrammes. But the bags weighed 50-60 kilogrammes each.

The man began to weigh the bags in pairs. Of the five bags it is possible to make 10 different pairs, so he had to make 10 weighings. He produced the series of numbers given below in the ascending order:

110 kg, 112 kg, 113 kg, 114 kg, 115 kg,
116 kg, 117 kg, 118 kg, 120 kg, 121 kg.

How much did each bag weigh?

Answers

Navvies

It's easy to swallow the bait and think that if five navvies dug 5 metres of the ditch in 5 hours, then it would take 100 people to dig 100 metres in 100 hours. But that argument is absolutely wrong, since the same five navvies would be required, no more.
In fact, five navvies dig 5 metres in 5 hours, so they can do 1 metre in 1 hour, and in 100 hours – 100 metres.

Lumberjacks

The common answer would be 1.5×5, i.e. 7.5 minutes. That is because many people often forget that the last cut will give *two* 1-metre lengths. Thus, it's only necessary to cut the log four times, not five, and this will take $1.5 \times 4 = 6$ minutes.

Joiner and Carpenters

We can easily find the average earnings of a member of the team by dividing the extra 3 roubles between the six carpenters. Accordingly, we should add 50 koppecks* to the 20 roubles earned by each carpenter to arrive at the average earnings of each of the seven workers.
We'll thus obtain that the joiner earned 20 roubles 50 kopecks plus 3 roubles, i.e. 23 roubles 50 kopecks.

Five Pieces of Chain

It's only necessary to open the *three links* of one of the pieces and to use the links obtained to connect the other four pieces.

How Many Vehicles?

If all the 40 vehicles were motocycles, the total number of wheels would be 80, i.e. by 20 less than in reality. Replacing a single motocycle by a car increases the total number of wheels by two and the difference decreases by two. Clearly, 10 such replacements are required for the difference to be reduced to zero. So, there were 10 cars and 30 motocycles.
In fact: $10 \times 4 + 30 \times 2 = 100$.

Potato Peeling

During the 25 extra minutes the second peeler put out $2 \times 25 = 50$ pieces. We subtract 50 from 400 to find that if the two had worked an equal time they would have yielded 350 potatoes. As their production per minute was $2 + 3 = 5$ pieces, then by dividing 350 by 5 we find that each would have worked for 70 minutes.

* 1 rouble = 100 kopecks.

This is the actual duration of work of the first peeler, the second one worked for $70 + 25 = 95$ minutes.
In fact: $3 \times 70 + 2 \times 95 = 400$.

Two Workers

If each worker performs half the job individually, the first would need two days more than the second (because the difference in duration for the *whole* job is four days). As in our case the difference is just two days when the two work together, it is then obvious that during the seven-day period the first worker performs half the job, whereas the second does his half in five days. Thus, the first worker would be able to do the whole job himself in 14 days and the second in 10 days.

Typing a Report

A nonstereotyped approach is as follows. First, we'll ask the question: if the typists are to finish the work simultaneously, how should they divide it? (Clearly, it's only under this condition, i.e. without any time wasted, that the work will be done in the shortest time possible). As the more experienced typist types 1.5 times faster it's obvious that her share should be 1.5 times larger than that of the other if both are to stop simultaneously. It follows that the first typist should take over three fifths of the report, and accordingly the second two fifths.
As a matter of fact the problem is nearly solved. It only remains to find the time taken by the first typist to do her share of the job. We know she can do the whole job in 2 hours, hence the three fifths of the job will be carried out in $2 \times 3/5 = 1 1/5$ hours. During exactly this time the second typist will finish her share of the job.
Thus, the shortest time required for both typists to type the report is 1 hour and 12 minutes.

Weighing Flour

To begin with, the salesman summed up the 10 numbers. The resultant sum (1,156 kilogrammes) is nothing but the fourfold weight of the bags: the weight of each bag enters the sum four times. If we divide by four, we'll find that the total weight of the five bags is 289 kilogrammes.
We'll now for convenience assign numbers to the bags in ascending order of their weights. The lightest bag will be No. 1, the second No. 2, etc., and the heaviest, No. 5. It will be seen that in the series of quantities: 110 kg, 112 kg, 113 kg, 114 kg, 115 kg, 116 kg, 117 kg, 118 kg, 120 kg, and 121 kg, the first quantity is the sum of the weights of the two lightest bags, No. 1 and No. 2; the second quantity, of No. 1 and No. 3. The last quantity (121) is the sum of the two heaviest bags, No. 4 and No. 5, and the penultimate, of No. 3 and No. 5. Thus:

No. 1 and No. 2 give 110 kg
No. 1 and No. 3 » 112 kg
No. 3 and No. 5 » 120 kg
No. 4 and No. 5 » 121 kg

We can thus easily find the sum of the weights of No. 1, No. 2, No. 4, and No. 5: 110 kg + 121 kg = 231 kg. Subtracting this number from the total weight of the bags (289 kg) gives the weight of No. 3, namely – 58 kg.

Further, from the sum of No. 1 and No. 3, i.e. from 112 kg, we subtract the now-known weight of No. 3 to arrive at the weight of No. 1: 112 – 58 = 54 kg.

In exactly the same way we find the weight of No. 2 by subtracting 54 kg from 110 kg, i.e. from the sum of No. 1 and No. 2. The weight of No. 2 will thus be 110 – – 54 = 56 kg.

Now from 120 kg (No. 3 + No. 5) we subtract the weight of No. 3 (58 kg) to get the weight of No. 5: 120 – 58 = 62 kg.

It remains to determine the weight of No. 4, knowing the sum of No. 4 and No. 5 (121 kg). Subtracting 62 from 121 gives that No. 4 weighs 59 kg.

The weights of the bags are thus

54 kg, 56 kg, 58 kg, 59 kg, 62 kg.

We have solved the problem without any resort to equations.

192–193 Problems on Purchases and Prices

10

How Much are the Lemons?

Three dozen lemons cost as many roubles as one can have lemons for 16 roubles.

How much does a dozen lemons cost?

Raincoat, Hat and Overshoes

A raincoat, hat and overshoes are bought for 140 roubles. The raincoat costs 90 roubles more than the hat, and the hat and the raincoat together cost 120 roubles more than the overshoes.

How much does each thing cost separately?

Use mental arithmetic only, no equations.

Purchases

When I went out shopping I had in my purse 15 roubles in 1 rouble pieces and 20 kopeck coins. When back home I had as many 1 rouble pieces as there had been 20 kopeck coins initially, and as many 20 kopeck coins as I had had 1 rouble pieces initially, my purse only containing a third of the initial sum.

How much had I spent?

Buying Fruit

One hundred pieces of various fruit can be bought for five roubles. The prices are: water-melons, 50 kopecks a piece; apples, 10 kopecks a piece; and plums, 10 kopecks a ten.

How many fruit of each kind are bought?

Prices Up and Down

The price of a product first went up 10%, and then down 10%.

When was the price lower, initially or finally?

Barrels

Six barrels of beer were shipped to a shop. The numbers in Fig. 205 show the numbers of litres in each barrel. Two customers bought five of the six barrels,

Figure 205

one bought two and the other bought three. Given that the second bought twice as much beer as the first, which barrel wasn't sold?

Selling Eggs

At first sight, this ancient problem might seem incongruous as it involves selling half an egg. Nevertheless, it's quite solvable.

A peasant woman came to a market to sell some eggs. A first buyer took half her eggs plus 1/2 of an egg. A second buyer bought half the remaining eggs plus another 1/2 of an egg. A third only bought one egg, which was the last.

How many eggs were there initially?

Benediktov's Problem

Many experts in Russian literature don't suspect that the poet V. G. Benediktov (1807-1873) was also the author of the first collection of mathematical brain-twisters in the language. The collection wasn't printed and remained in a manuscript form to be found only in 1924. I had the opportunity to get acquainted with the manuscript and even established, based on one of the problems, the year it was compiled, namely 1869 (the manuscript wasn't dated). The problem given below has been treated by the poet and named "An Ingeneous Solution of a Difficult Problem".

"An egg seller sent her three daughters to the market with ninety eggs. She gave ten to the eldest and

cleverest daughter, thirty to the second, and fifty to the third, saying:

'You should agree beforehand on the price at which you'll sell the eggs and stick to it. All of you should adhere to this price but I hope that the eldest daughter who is so bright will nevertheless be able to get as much for her ten eggs as the second daughter will receive for her thirty and she will teach the second sister how to get as much for her thirty as the youngest sister will get for her fifty eggs. Let the takings and prices be the same for the three of you. Furthermore, I'd like you to sell the eggs so that on average you will receive no less than 10 kopecks for ten, and no less than 90 kopecks for the ninety!'"

Here I interrupt Benediktov's story so that the readers could figure it out for themselves how the girls went about their business.

Answers

10

How Much are the Lemons?

We know that the 36 lemons cost as many roubles as they sell lemons for 16 roubles. But 36 lemons cost

$$36 \times (\text{price of one lemon}).$$

For 16 roubles one can have

$$16/(\text{price of one lemon}).$$

Hence,

$$36 \times (\text{price of one lemon}) = 16/(\text{price of one lemon}).$$

After some algebra we have

$$(\text{price of one lemon}) \times (\text{price of one lemon}) = 16/36.$$

Clearly, one lemon costs $4/6 = 2/3$ rouble and a dozen lemons cost $2/3 \times 12 = 8$ roubles.

Raincoat, Hat and Overshoes

If instead of the raincoat, hat, and overshoes only two pairs of overshoes were bought, the price would be not 140 roubles, but 120 roubles less. Thus, the two pairs of overshoes cost $140 - 120 = 20$ roubles, hence one pair cost 10 roubles.

Now we find that the raincoat and the hat together cost $140 - 10 = 130$ roubles, the raincoat costing 90 roubles more than the hat. We argue as earlier: instead of the raincoat and hat we could buy two hats, and we would pay not 130 roubles but 90 roubles less, i.e. $130 - 90 = 40$ roubles. Hence one hat costs 20 roubles.

Thus, the prices of the things were as follows: the overshoes – 10 roubles, the hat – 20 roubles, and the raincoat – 110 roubles.

Purchases

Denote the initial number of 1 rouble pieces by x, and the number of 20 kopeck coins by y. Then when I went out shopping I had in my purse

$$(100x + 20y) \text{ kopecks}.$$

Back from my shopping expedition I had

$$(100y + 20x) \text{ kopecks}.$$

As stated, the latter sum is three times smaller than the former, hence

$$3(100y + 20x) = 100x + 20y.$$

Rearranging the expression gives

$$x = 7y.$$

If $y = 1$, then $x = 7$. Under this assumption I initially had 7 roubles 20 kopecks

which is at variance with the statement of the problem ("about 15 roubles").

Let's try $y = 2$, this gives $x = 14$. The initial sum is thus 14 roubles 40 kopecks which checks well with the problem statement.

The assumption of $y = 3$ leads to an overestimation: 21 roubles 60 kopecks. In consequence, the only fitting answer is 14 roubles 40 kopecks. When I returned back from my shopping excursion I only had two 1 rouble pieces and fourteen 20 kopeck coin, i.e. $200 + 280 = 480$ kopecks, which actually amounts to a third of the initial sum $(1{,}440/3 = 480)$.

As I spent $1{,}440 - 480 = 960$ kopecks, my purchases had cost 9 roubles 60 kopecks.

Buying Fruit

Despite the seeming uncertainty the problem has the only solution:

	Number	Cost
Water melons	1	50 kopecks
Apples	39	3 roubles 90 kopecks
Plums	60	60 kopecks
Total	100	5 roubles 00 kopecks

Prices Up and Down

It would be erroneous to consider that the two prices are equal. It's easily shown that this is not the case. After the price went up the article cost 110%, or 1.1 of the initial price. But after the price went down it amounted to

$$1.1 \times 0.9 = 0.99,$$

i.e. 99% of the initial price. Consequently, the final price was 1% lower than the initial one.

Barrels

The first customer bought the 15 litre and 18 litre barrels and the second – the 16 litre, 19 litre and 31 litre barrels. Really,

$$15 + 18 = 33$$
$$16 + 19 + 31 = 66,$$

i.e. the second customer bought twice as much beer as the first one. The 20 litre barrel remained unsold.

This is the only possible solution as no other combination gives the relationship required.

Selling Eggs

The problem is worked out backwards from the end. After the second buyer bought half the remaining eggs plus 1/2 of an egg, there was only one egg that remained with the peasant woman. Accordingly, 1 1/2 eggs was half of what remained after the first

sale and so the full number is three eggs. We add 1/2 of an egg to obtain half of what the woman had initially. Thus, the woman had brought seven eggs for sale.
Let's check:

$$7/2 = 3\ 1/2;\ 3\ 1/2 + 1/2 = 4;\ 7 - 4 = 3$$
$$3/2 = 1\ 1/2;\ 1\ 1/2 + 1/2 = 2;\ 3 - 2 = 1,$$

which complies with the conditions of the problem.

Benediktov's Problem

We continue the interrupted story:
"The problem was a very difficult one. The sisters put their heads together on their way to the market, the two younger sisters seeking advice of the eldest. The latter gave some thought to the matter and said:
'Sisters, we'll sell the eggs not by the ten, as is the custom here, but by the seven. And we'll set a price for the seven we'll stick to as Mother said. Not a kopeck down from the set price! The first seven goes for three kopecks, agreed?
'Dirt-cheap', the second sister said.
'But', the eldest sister continued, 'we'll raise the price for those eggs that'll remain after we have sold the full sevens. I've checked beforehand that there'll be no other egg sellers in the market. No one to beat down the price. But when there is demand and the supply is dwindling the price rises. So we'll make up for our loss with the remaining eggs'.
'And what will we charge for the remaining eggs?' the youngest sister asked.
'Nine kopecks for each egg. Cash down! Those who need eggs badly will pay'.
'Rather dear,' the second sister noted again.
'What of it?' the eldest said, 'the first eggs will have been sold cheaply by the seven. One will compensate for the other!'
"Understandably, the first to go were the fifty eggs of the youngest sister. She received 21 kopecks for 7 sevens and one egg remained in her basket. The second one sold 4 sevens for 12 kopecks and two eggs remained in her basket. The eldest sister sold a seven for 3 kopecks and three eggs remained in her basket.
"The last six eggs were sold for nine kopecks each. So the eldest got 27 kopecks for her three eggs which brought her takings to 30 kopecks. The second sister got 18 kopecks for her last pair of eggs which when added to the 12 kopecks received earlier for her 4 sevens, gave her 30 kopecks as well. The youngest sister got 9 kopecks for her single egg and when she added the money to the 21 kopecks for her 7 sevens her total was 30 kopecks, too.
"Thus, the money they got for ten appeared to be equal to the money they got for fifty."

Weight and Weighing

One Million Times the Same Product

A product weighs 89.4 grammes. Figure out how many tonnes a million of them weigh.

Honey and Kerosene

A jar of honey weighs 500 grammes. The same jar filled with kerosene weighs 350 grammes. Honey is twice as heavy as kerosene.
What is the weight of the empty jar?

A Log

A round log weighs 30 kilogrammes.
How much would it weigh if it were twice as thick, but twice as short?

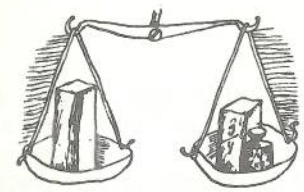
Under Water

Consider a balance on the one pan of which there is a boulder that weighs exactly 2 kilogrammes and on the other, an iron weight of 2 kilogrammes. I carefully immerse the balance in water.
Will the pans be in equilibrium?

A Decimal Balance

A decimal balance weighs 100 kilogrammes of iron nails that are balanced by iron weights.
When submerged, will the balance be in equilibrium?

Figure 206



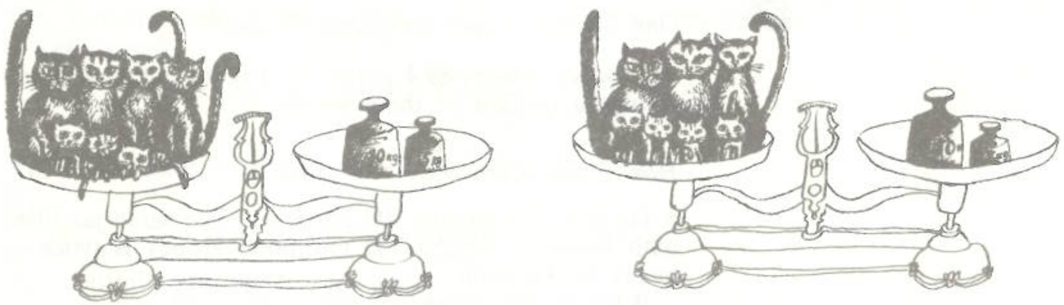
A Piece of Soap

Onto one pan of a balance a piece of soap was put, onto the other 3/4 of a same sized piece plus 3/4 kilogramme. The balance is in equilibrium.
What is the weight of a whole piece?
Try and solve the problem mentally, without a pencil and paper.

Cats and Kittens

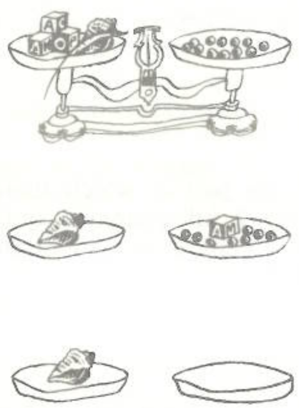
The accompanying figure shows that the four cats and three kittens together weigh 15 kilogrammes and that

Figure 207



three cats and four kittens weigh 13 kilogrammes. All the cats have the same weight, so do the kittens.
How much does one cat weigh? And a kitten? This problem, too, should be solved mentally.

Figure 208



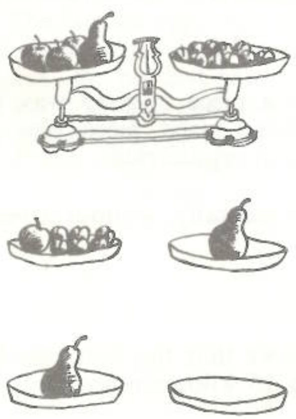
Shell and Beads

Figure 208 shows that three children's blocks and one shell are balanced by 12 beads and further that one shell is balanced by one block and eight beads.
How many beads should be placed on the vacant pan for the shell on the other pan to be balanced?

Fruit

A further problem of the same kind. It is seen in Fig. 209 that three apples and one pear weigh as much as 10 peaches, but six peaches and one apple weigh as much as one pear.
How many peaches are required to balance one pear?

Figure 209



How Many Glasses?

You see in Fig. 210 that a bottle and a glass are balanced by a jug, the bottle is balanced by a glass and a saucer, and two jugs are balanced by three saucers.
How many glasses should be placed on the vacant pan for the bottle to be balanced?

With a Weight and a Hammer

It's required to weigh out 2 kilogrammes of sugar into 200-gramme packets. There is, however, only one

Figure 210

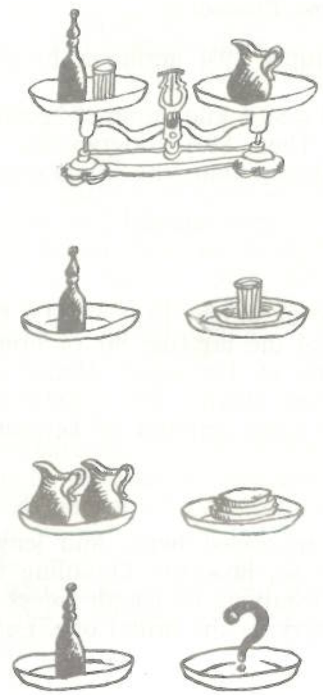


Figure 211

500-gramme weight and hammer that weighs 900 grammes.
How should one go about it using the weight and the hammer?

Archimedes's Problem

The most ancient of brain-twisters pertaining to weighing is undoubtedly the one the tyrant of Syracuse Hieron gave to the famous mathematician Archimedes.
The legend has it that Hieron entrusted a craftsman to manufacture a crown for a statue and ordered to give him the required amount of gold and silver. When it was ready, the crown weighed as much as the initial amounts of gold and silver had originally weighed together, but the craftsman was alleged to have stolen some of the gold having replaced it by silver.

Hieron called in Archimedes and asked him to determine how much gold and silver respectively the crown contained.
Archimedes solved the problem proceeding from the fact that in water pure gold loses one twentieth of its weight, and silver one tenth.
If you want to try your hand at the problem suppose that the craftsman was given 8 kilogrammes of gold and 2 kilogrammes of silver and when Archimedes weighed the crown under water the result was 9 1/4 kilogrammes, not 10 kilogrammes. Given that the crown was made of solid metal, without any voids, how much gold had the craftsman stolen?

77

One Million Times the Same Product

The mental arithmetic here is as follows. We must multiply 89.4 grammes by one million, i. e. by one thousand thousands.

We can do the multiplication in two steps: $89.4 \times 1{,}000 = 89.4$ kilogrammes because the kilogramme is 1,000 times larger than the gramme. Then, 89.4 kilogrammes $\times$ $\times$ $1{,}000 = 89.4$ tonnes, because the tonne is 1,000 times larger than the kilogramme.

The weight we seek is thus 89.4 tonnes.

Honey and Kerosene

Since honey is twice as heavy as kerosene, the difference in weight ($500 - 350 = 150$ grammes) is the weight of the kerosene in the volume of the jar (the jar of honey weighs as much as a jar containing a double amount of kerosene). Hence we determine the weight of the jar: $350 - 150 = 200$ grammes. Really: $500 - 200 = 300$ grammes, i. e. the honey is two times heavier than the same amount of kerosene.

A Log

A common answer is that a log, whose thickness has increased twice, and length decreased twice, should be the same weight. This is not so, however. Doubling the diameter increases the volume of a round log *fourfold*, but halving its length *halves* its volume. The net result is that the final log is twice as heavy as the initial one, i. e. it weighs 60 kilogrammes.

Under Water

Each immersed body becomes lighter by the weight of the water displaced by it. This law, discovered by Archimedes, will help us to answer the problem.

The 2-kg boulder has a larger volume than the 2-kg iron weight because the material of the boulder (granite) is lighter than iron. Accordingly, the boulder will displace a larger volume of water than the weight and, according to Archimedes's principle, loses more than the weight. The weight will thus outweigh the boulder under water.

Decimal Balance

When immersed in water, an iron object loses one eighth of its weight.* Thus both the nails and the weights will when immersed have only 7/8 of their former weight. Since the weights were 10 times lighter than the nails before immersion and they continue to be 10 times lighter after immersion, the equilibrium will not be disturbed.

* The figure wasn't given in the statement of the problem as the exact share of the weight lost is immaterial here.

A Piece of Soap

Three quarters of a piece of soap plus 3/4 kilogrammes weigh as much as the whole piece. But a whole piece is 3/4 plus 1/4, hence 1/4 of a piece weighs 3/4 kilogrammes and the whole piece weighs four times as much as 3/4 kilogrammes, i.e. 3 kilogrammes.

Cats and Kittens

A comparison of both weighings shows that replacing a cat by a kitten reduces the weight by 2 kilogrammes. It follows that a cat is 2 kilogrammes heavier than a kitten. With this in mind we in the first weighing replace all the four cats by kittens to obtain $4 + 3 = 7$ kittens that will together weigh not 15 kilogrammes but $2 \times 4 = 8$ kilogrammes less. Consequently, the seven kittens weigh $15 - 8 = 7$ kilogrammes. Hence a kitten weighs 1 kilogramme, and a cat weighs $1 + 2 = 3$ kilogrammes.

Shell and Beads

Compare the first and second weighings. You'll see that in the first weighing the shell can be replaced by one cube and eight beads. We'll then have four cubes and eight beads on the left pan balanced by 12 beads. If we now remove eight beads from each pan, we won't upset the balance. There'll four cubes now remain on the left pan, and four beads on the right. One cube thus weighs the same as one bead.

We can now work out the weight of the shell: replacing (second weighing) the cube on the right pan by a bead gives that the weight of the shell is equal to that of nine beads.

The result can be checked easily.

In the first weighing, replace the cubes and shell on the left pan by an appropriate number of beads. You'll thus obtain $3 + 9 = 12$, as required.

Fruit

In the first weighing we replace one pear by six peaches and an apple. We may do so because the pear weighs as much as the six peaches and apple. We then obtain four apples and six peaches on the left pan and 10 peaches on the right. Removing the six peaches from each pan gives that the four apples weigh as much as four peaches. Accordingly one peach weighs the same as one apple.

Now it's easy to figure out that a pear weighs the same as seven peaches.

How Many Glasses?

The problem has several different solutions. The following is just one of them.

In the third weighing we replace each jug by a bottle and a glass (we know from the first weighing that the balance should remain in equilibrium). We thus find that two bottles and two glasses are balanced by three saucers. It thus appears that four glasses and two saucers are balanced by three saucers.

Removing two saucers from each pan shows that four glasses are balanced by one saucer.

Hence one bottle is balanced (compare with the second weighing) by five glasses.

With a Weight and a Hammer

The procedure to be followed is like this. First put the hammer on one pan and the weight on the other. Then add just enough sugar for the pans to be in equilibrium. It's clear that the sugar weighs $900 - 500 = 400$ grammes. The operation is performed three more times. The remaining sugar weighs $2{,}000 - (4 \times 400) = 400$ grammes.

It is only remains now to halve each of the five 400-gramme packets obtained. It's a straightforward exercise: the contents of a 400-gramme packet are divided between two packets put on different pans until the balance balances.

Archimedes's Problem

If the crown ordered had been made purely of gold, it would have weighed 10 kilogrammes in air losing when immersed 1/20 part of its weight, i.e. 1/2 kilogramme. But we know that in fact the crown lost in water $10 - 9\ 1/4 = 3/4$ kilogramme, not 1/2 kilogramme. This was because it contained silver – a metal that in water loses 1/10 part of its weight, not 1/20 part. The crown thus contained an amount of silver sufficient for it to lose in water 3/4 kilogramme, rather than 1/2 kilogramme, i.e. 1/4 kilogramme more. Suppose in the purely golden crown one kilogramme of gold were replaced by silver, the crown would when immersed lose another $1/10 - 1/20 = 1/20$ kilogramme. Consequently, in order to decrease the crown's weight by 1/4 kilogramme it was necessary to replace with silver as many kilogrammes of gold as there were 1/20ths in 1/4: $1/4 \div 1/20 = 5$. So, the crown contained 5 kilogrammes of silver and 5 kilogrammes of gold instead of the 2 kilogrammes of silver and the 8 kilogrammes of gold the craftsman was given. Thus, 3 kilogrammes of gold had been stolen and replaced by silver.

Problems on Clocks and Watches

Three Clocks

In my home there are three clocks. On the 1st of January they all showed true time. But only the first clock kept perfect time, the second was a minute slow a day, and the third was gaining a minute a day. Should the clocks continue like this, how long would it take for them all to show true time again?

Two Clocks

Yesterday I checked my wall clock and alarm clock and set them correctly. The wall clock is 2 minutes slow an hour, the alarm clock gains 1 minute an hour.

Today both clocks stopped simultaneously since they had run down. The wall clock shows 7 o'clock and the alarm clock 8 o'clock.

At what time yesterday did I set the clocks?

What Time Is It?

"Where are you hurrying to?"

"To catch the 6 o'clock train. How long have I got left?"

"50 minutes ago there were four times more minutes after three."

What does this strange answer mean? What time was it?

Figure 212

When Do the Hands Meet?

At 12 o'clock one hand is above the other. But you may have noticed that it is not the only moment when the hands meet: they do so several times a day.

Can you say when all those moments are?

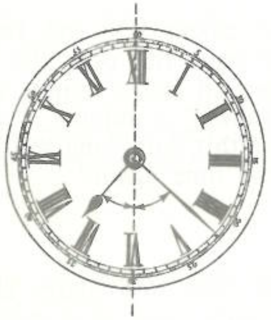
Figure 213

When are the Hands Pointing in Opposite Directions?

By contrast, at 6 o'clock both hands point in opposite directions. But is it only at 6 o'clock that this is the case or there are some other such moments during the next 12 hours?

12

Figure 214



On Either Side of Six O'Clock

I glanced at a clock and noticed that both hands were equally separated from 6. What time was it?

The Minute Hand Ahead of the Hour Hand

When is the minute hand as far ahead of the hour hand as the hour hand in turn is ahead of the figure 12 on the face? And maybe there are several such moments during the day or none at all?

Vice Versa

If you observe a clock attentively, you may have noticed the reverse arrangement of the hands as compared with that just described: viz. the hour hand is as far ahead of the minute one as the minute hand is ahead of the figure 12.

When does this happen?

Three and Seven

A clock strikes three. And while it does so 3 seconds elapse. How long does it take the clock to strike seven?

I warn you, just in case, that this isn't a joke, i.e. it's not a trick question.

Ticking

Lastly, make a small experiment. Put your watch on a table, move a few steps aside and listen to the ticking. If it's sufficiently quiet in the room, you'll hear that your watch sounds, as it were, in intervals: ticks for a while, then is silent several seconds, and then starts ticking again, and so on.

Explain!

12

Three Clocks

720 days. During this time the second clock will lose 720 minutes, i.e. exactly 12 hours, and the third clock will have gained exactly the same time. Then all the three clocks will show as they did on the 1st of January, i.e. true time.

Two Clocks

The alarm clock is gaining 3 minutes an hour compared with the wall clock. Thus it gains an hour, i.e. 60 minutes, every 20 hours. But during these 20 hours the alarm clock gains 20 minutes compared with true time. This implies that both clocks were set correctly 19 hours 20 minutes before, i.e. at 11.40.

What Time Is It?

Between 3 and 6 o'clock there are 180 minutes. The number of minutes to go to 6 o'clock is easily found by dividing $180 - 50 = 130$ minutes into two parts, one of which being four times larger than the other. Hence, we'll have to find 1/5 part of 130. It was thus 26 minutes to 6 o'clock.

In fact, 50 minutes before it was $26 + 50 = 76$ minutes to go to 6 o'clock. Accordingly, $180 - 76 = 104$ minutes had passed since 3 o'clock, which is four times longer than the time to go to 6 o'clock.

When Do the Hands Meet?

We start our observation at 12 o'clock, when both hands meet. Since the hour hand moves 12 times slower than the minute one (it takes 12 hours to make a complete circle, and the minute one 1 hour), the hands cannot, of course, meet during the next hour. But after the hour has passed and the hour hand come to the 1 o'clock mark (having completed 1/12th of the full circle), the minute hand has made a complete turn and is again at 12, i.e. 1/12 part of the circle behind the hour hand. The condition of the race is now different since the hour hand moves slower than the minute one, but is ahead of the minute hand which has to overtake it. If the race lasted an hour, the minute hand would have gone round a complete circle, and the hour hand 1/12 part of the circle, i.e. the minute hand would have travelled 11/12 part of the circle more. But to overtake the hour hand, the minute hand must only cover 1/12 of the circle which is the distance separating them. This requires a period of time that is the same fraction of an hour as 1/12th is a fraction of 11/12 i.e. one eleventh. Thus, the hands will meet in 1/11 hour, i.e. in $60/11 = 5\ 5/11$ minutes.

The hands will thus meet 5 5/11 minutes after the first hour has elapsed, i.e. at 5 5/11 minutes past one.

What about the next meeting?

You should be able to see that it'll occur 1 hour 5 5/11 minutes later, i.e. at 10 10/11 minutes past 2 o'clock. The next meeting occurs another 1 hour 5 5/11 minutes later, i.e. at 16 4/11 minutes past 3 o'clock, and so forth. You may have already guessed that, all in all, there'll be 11 such meetings. The 11th comes $1\ 1/11 \times 11 = 12$

hours after the first one, i.e. at 12 o'clock. In other words, it coincides with the first meeting, with future meetings occurring at the previous times.

Let's list the times of all the meetings:

1st–5 5/11 minutes past	1 o'clock
2nd–10 10/11 minutes past	2 o'clock
3rd–16 4/11 minutes past	3 o'clock
4th–21 9/11 minutes past	4 o'clock
5th–27 3/11 minutes past	5 o'clock
6th–32 8/11 minutes past	6 o'clock
7th–38 2/11 minutes past	7 o'clock
8th–43 7/11 minutes past	8 o'clock
9th–49 1/11 minutes past	9 o'clock
10th–54 6/11 minutes past	10 o'clock
11th–	12 o'clock

When Are the Hands Pointing in Opposite Directions?

The approach here is very much like that in the previous problem. We'll again begin at 12 o'clock when both hands meet. We want to find the time required for the minute hand to get ahead of the hour hand by exactly half a circle, it is then that the hands are pointing in opposite directions. We already know (see the previous problem) that during an hour the minute hand gets ahead of the hour hand by 11/12 part of the circle. For it to get ahead by only 1/2 a circle takes less than an hour by so many times as 1/2 is less than 11/12, i.e. 6/11 part of an hour. Accordingly, after 12 o'clock the first time the hands point in opposite directions is in 6/11 hours, or 32 8/11 minutes. Look at a watch at this time and you'll see that the hands are really pointing in opposite directions.

Is this the only moment when we have such an arrangement? Of course, not. The hands are so arranged *32 8/11 minutes after each meeting.* We already know that during a 12 hour's time there are 11 such meetings. Hence the hands point opposite ways 11 times every 12 hours. These moments are easily found:

12 o'clock + 32 8/11 minutes = 32 8/11 minutes past 12 o'clock
1 o'clock 5 5/11 minutes + 32 8/11 minutes = 38 2/11 minutes past 1 o'clock
2 o'clock 10 10/11 minutes + 32 8/11 minutes = 43 7/11 minutes past 2 o'clock
3 o'clock 16 4/11 minutes + 32 8/11 minutes = 49 1/11 minutes past 3 o'clock, and so on.

I leave it for you to find the remaining moments.

On Either Side of Six O'Clock

The problem is solved like the previous one. Imagine that both hands are at 12 and that the hour hand has shifted by a certain part of a circle to be denoted by x. Meanwhile the minute hand has turned by $12x$. If the time that has passed is less than one hour, then to meet the conditions of our problem the minute hand must travel a full circle less the angle covered by the hour hand since 12. In other words,

$$1 - 12x = x.$$

Hence $1 = 13x$ and $x = 1/13$ part of a circle. The hour hand covers this fraction of

a circle in 12/13 part of an hour, i.e. when it is $(12/13) \times 60$ minutes or 55 5/13 minutes past 12 o'clock. During the same period of time, the minute hand covers 12 times more, i.e. 12/13 part of a circle. You see that both hands are equally separated from 12, and hence equally separated from 6, too.

We've found one location of the hands, namely one that comes about in the *first* hour. During the *second* hour this occurs once more and you can find it arguing along the same lines as before, from the relation

$$1 - (12x - 1) = x \text{ or } 2 - 12x = x.$$

Hence $2 = 13x$ and $x = 2/13$ of a circle. So the hands will be in the right position at $(1\ 11/13) \times 60$ minutes or at 50 10/13 minutes past 1 o'clock.

The hands will meet our requirement next time when the hour hand has shifted 3/13 of a circle away from 12, i.e. at 2 10/13 o'clock, and so on. All told, there are 11 such positions, the hands changing sides after 6 o'clock.

The Minute Hand Ahead of the Hour Hand

If we start looking at a clock at 12 o'clock exactly, then during the first hour we won't see the position desired. Why? Because the hour hand covers 1/12 part of what the minute hand does, and hence lags behind the minute hand far more than is required for the arrangement we seek. Whichever the angle through which the minute hand turns about 12, the hour hand will only be at 1/12 part of that angle, not a half as is desired. But suppose an hour has elapsed and the minute hand is at 12 and the hour hand at 1, i.e. 1/12 part of a complete turn ahead of the minute hand. Let's see if such an arrangement of the hands may come about during the second hour. Suppose that the moment has come when the hour hand has turned by a fraction of a circle that we'll denote by x. Meanwhile the minute hand has covered 12 times more, i.e. $12x$. If now we subtract from this a complete turn, the difference $12x - 1$ must be twice as large as x, i.e. $2x$. Thus $12x - 1 = 2x$, whence it follows that a complete turn equals $10x$ (because $12x - 10x = 2x$). But if $10x$ equals a complete turn, then $1x = 1/10$ part of a turn. We've thus arrived at the solution: the hour hand must have moved by 1/10 part of a turn past 12 o'clock. This takes 12/10 hours or 1 hour 12 minutes. The minute hand will then be two times farther away from 12, i.e. at 1/5 of a turn away, which corresponds to $60/5 = 12$ minutes, as required.

We've found one solution to the problem. But there are other ones and during a period of 12 hours the hands come to be arranged in the right way several times. We'll try to find the other solutions.

To find the next time we'd have to wait till 2 o'clock and now the minute hand is at 12 and the hour hand at 2. Reasoning along the same lines as before we arrive at

$$12x - 2 = 2x,$$

whence two complete turns are equal to $10x$, and hence $x = 1/5$ part of a complete turn. This corresponds to the moment $12/5 = 2$ hours 24 minutes.

I leave it to you to work out further moments. You'll find that the hands arrange themselves in the right way at the following 10 instants in time:

1.12	7.12
2.24	8.24

3.36	9.36
4.48	10.48
6.00	12.00

The answers "6.00" and "12.00" might appear wrong, but only at first sight. In fact, at 6 o'clock the hour hand is at 6 and the minute one, at 12, i.e. exactly twice as far. At 12 o'clock the hour hand is separated from 12 by "zero" and the minute one, if you wish, by "double zero" (because double zero is just zero). So, this case, too, meets the restrictions of our problem.

Vice Versa

After the treatment we have just given, this problem is an easy exercise. Using the same arguments as above we can determine that for the first time the required arrangement will occur at the time given by

$$12x - 1 = x/2.$$

Therefore $1 = 11\ 1/2\ x$, or $x = 2/23$ of a turn, i.e. $(2/23) \times 12$ hours or 1 1/23 hours after 12 o'clock. Hence at 2 14/23 minutes past 1 o'clock the hands will be arranged correctly. The minute hand will then be midway between 12 o'clock mark and 1 1/23 hours mark, i.e. at 12/23 hours mark, which is exactly 1/23 part of a turn (the hour hand will be at 2/23 part of a turn).

The hands will be arranged in the required manner for the second time at a time which can be found from the relation

$$12x - 2 = x/2.$$

It follows that $2 = 11\ 1/2\ x$ and $x = 4/23$, the time we seek is 5 5/23 minutes past 2 o'clock.

The third moment is 7 19/23 minutes past 3 o'clock, and so forth.

Three and Seven

The commonest answer is "7 seconds". But, as we'll now see, that is wrong.

When the clock strikes three we have two gaps: (1) between the first and second strokes, (2) between the second and third strokes.

Each gap thus lasts 1 1/2 seconds.

But when the clock strikes seven, there are six such gaps, which gives 9 seconds.

Ticking

The enigmatic interruptions in the ticking are only due to fatigue in your ears. From time to time your perception of sound becomes blunted for a second or two so that in these intervals you won't hear the ticking. This aural fatigue passes off after a short while and previous ability to perceive the sound returns with the result that you again hear the ticking. Then another fatigue period comes on, and so forth.

13 Problems on Transport

A Plane's Flight

An aircraft covers the distance from town A to town B in 1 hour 20 minutes. However, it takes it 80 minutes to get back.

How could you explain it?

Two Locomotives

You may have seen a train driven by two locomotives, one at the front and the other at the back. But have you ever given any thought as to what happens to the couplings between the carriages and to their buffers? The front locomotive only pulls the carriages when the coupling is taut, in which case the buffers do not press against each other and so the rear locomotive cannot be pushing. On the other hand, when the rear locomotive pushes the train, the buffers press hard against each other, which makes the coupling become slack, thus rendering the front locomotive useless.

It turns out that the locomotives cannot be moving the train at the same time since only one of them is working at a time.

Why then do they employ two locomotives?

The Speed of a Train

You are travelling in a train and want to find its speed, could you work it out from the clatter of the wheels?

Two Trains

Two trains once left their respective stations for the other's station simultaneously. The first arrived at its destination an hour after the two trains had met each other. The second reached its destination 2 hours 15 minutes after the same event.

How many times faster was the first train?

The problem can be done using mental arithmetic.

How Does a Train Start From Rest?

You may have noticed that before making a train move forward the engine-driver sometimes makes it push back. Explain why.

A Race

Two sailing boats are competing against each other. They must sail 24 kilometres there and back in the shortest time possible. The first boat covered the whole route with a uniform speed of 20 kilometres an hour whilst the second boat sailed the outward leg at 16 kilometres an hour and sailed back at 24 kilometres an hour.

The first boat won, though it would seem that the second one should have gained during the return trip exactly what it lost out during the first section of the route. It should thus have come in at the same time as the first boat.

Why did it lag behind?

Steaming Up and Down the River

A steamer makes 20 kilometres an hour downstream and 15 kilometres an hour upstream. A trip between two towns takes it 5 hours less than the return trip.

What is the distance between the towns?

A Plane's Flight

There is actually nothing to explain here, the aircraft takes the same time to travel in both directions.

The problem has a catch for the inattentive reader who might think that 1 hour 20 minutes and 80 minutes are different times. Strange as it might seem, many people swallow this bait, and people used to adding up are more likely to make it than those who aren't. The explanation lies in the habit of dealing with metric system of measures and money. We are apt to treat "1 hour 20 minutes" and "80 minutes" just like "1 pound 20 pence" and "80 pence", say, or "1 dollar 20 cents" and "80 cents". So it's really a psychological problem.

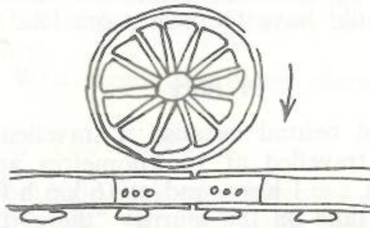
Two Locomotives

The way it works out is as follows. The front locomotive does not take care of the whole of the train, but only part of it, about half the carriages. The rest of them are pushed by the rear locomotive. The couplings between the first group of carriages are taut whilst they are slack between the rear ones which are being pushed buffer to buffer.

The Speed of a Train

You must have noticed that when travelling in a train you feel regular jerks all the time which the springs, however good, cannot suppress. These jerks come from the wheels being slightly jarred at rail junctions (Fig. 215) and are transferred throughout the carriage.

Figure 215



This nuisance, which is also bad both for the carriages and the tracks, lends itself for measuring the speed of the train. You only need to count the number of jerks you feel in one minute to find how many rails you've passed. Now you only have to multiply this number by the length of a rail to arrive at the distance covered by the train during that minute.

The regulation length of a rail is about 15 metres*. So, multiply the number of jerks a minute by 15 and then by 60, divide the result by 1,000 and you'll obtain the

* You may work out the length of a rail by pacing it out, seven paces amounting to about 5 metres.

number of kilometres covered by the train per hour. So,

$$\frac{(\text{number of jerks}) \times 15 \times 60}{1{,}000} = \text{kilometres per hour.}$$

Two Trains

The faster train arrives at the meeting point having covered a distance that is larger than the distance covered by the slower train by as many times as the speed of the faster train is higher than that of the slower train. After the meeting each train has to pass the distance that had been covered by the other one. In other words, the faster train covered a distance after the meeting that was as many times shorter than the distance covered by the slower train as its speed was higher. If we denote the ratio of the two speeds by x, then the faster train took x^2 times less time than the other to cover the distance from the meeting point to the respective station. Hence $x^2 = 2\ 1/4$ and $x = 1\ 1/2$, i.e. the first train is 1.5 faster than the second train.

How Does a Train Start From Rest?

When a train arrives at a station and comes to rest, the couplings between the carriages are taut. If the locomotive is to begin to pull the train like this, it would have to start the whole of the train from rest *at once*, which might be too difficult a task for it. On the other hand, if the locomotive first pushes the train backwards the couplings are no longer taut and the train is started from rest carriage by carriage *in succession* and that is much easier.

In other words, the engine-driver does what a coachman does sometimes when the coach is heavily loaded, i.e. he starts the coach and only then jumps on it, otherwise the horse would have to push more load from rest.

A Race

The second boat lagged behind because it travelled at 24 kilometres an hour *for a shorter time* than it travelled at 16 kilometres an hour. In fact, it travelled at 24 km/h for 24/24 hours, i.e. 1 hour, and at 16 km/h for 24/16 hours, i.e. 1 1/2 hours. Therefore, it lost more time on the journey "there" than gained on the way "back".

Steaming Up and Down the River

Travelling downstream the steamer covers 1 kilometre in 3 minutes whilst travelling upstream it covers 1 kilometre in 4 minutes. In the first case, the steamer gains 1 minute every kilometre, and as the total gain is 5 hours, or 300 minutes, the distance between the towns is 300 kilometres.

Really,

$$\frac{300}{15} - \frac{300}{20} = 20 - 15 = 5.$$

214–215 ?

14

A Glass of Peas

Of course, you've seen peas many times and held a glass in your hand, so that you must know the sizes of these things. Imagine a glass filled to the brim with dry peas. Thread all the peas on a piece of string like beads.

If the string is stretched, how long would it be approximately?

Water and Wine

One bottle contains a litre of wine, another a litre of water. A spoonful of wine is transferred from the first bottle into the second, and then a spoonful of the mixture thus obtained is transferred from the second bottle into the first one.

What do we now have, more water in the first bottle or more wine in the second?

A Die

Figure 216 shows a die, i.e. a cube with from 1 to 6 points on its six faces.

Peter bets that if the cube is thrown four times in succession, then it is bound to show 1 at least once.

But Vladimir argues that the 1 will either not appear at all with the four throws or it will show more than once.

Who stands the better chance of winning?

Figure 216

The Yale Lock

The Yale lock was invented by an American, Linus Yale, Jr., in 1865, and has come to be almost universally used ever since. Despite its long history, some people question the possibility of having a large number of versions of the lock. But we need only to look at the construction of the lock to see that it provides for an almost unlimited number of variations.

Figure 217 depicts the front view of the Yale lock. You see a small circle around the key hole which is the end face of the cylinder passing through the depth of the lock. The lock opens when the cylinder turns, but this is the crunch. The cylinder is secured by five short

steel pins (Fig. 217, right). Each pin actually consists of two pins, and the cylinder can only be turned when the double pins are so arranged that the cut lie at the boundary of the cylinder.

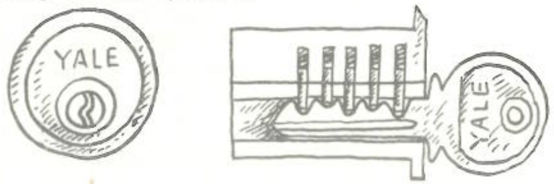


Figure 217

The pins are arranged this way using a key with serrated edge. You just insert the key into the keyhole and the pins are lifted to the height required for the lock to open.

You can easily see now that the number of the various combinations of heights in the lock can be exceedingly large. It depends on the number of ways in which each pin may be severed.

Suppose that each pin may be divided into two parts in 10 ways only. Try and work out the number of combinations possible for the Yale lock.

How Many Portraits?

Draw a portrait on a sheet of cardboard and cut it into several–say nine–stripes. Next draw other stripes showing various parts of the face so that any two neighbouring stripes belonging to different portraits might be fitted into another portrait without interrupting the lines. If you prepare, say, four stripes* for each part of the face, you'll have 36 stripes all in all.

* These could be conveniently glued onto four faces of a square block.

Figure 218

You'll now be able to make up a variety of faces by taking nine stripes each time.

Shops once used to sell ready-made sets of these stripes (or blocks) to make up portraits (Fig. 218). It was claimed that of 36 stripes one could produce a *thousand* various faces.

Is it so?

Abacus

Perhaps you can use the abacus and can set, say, 25 pounds, on it. But the problem becomes more difficult if you must shift not seven beads, as usual, but 25 beads. Just try.

To be sure, nobody is going to do so in practice, but the problem is not intractable and the answer is rather curious.

Leaves of a Tree

If we were to take all the leaves from an old tree, say a lime-tree, and place them side by side without any breaks, how long approximately would the line be? Would it be possible, for example, to encircle a large house with it?

A Million Steps

You must know what a million is and can estimate the length of your step, so that you should easily be able to say how far would a million steps take you? More than 10 kilometres away? Or less?

Cubic Metre

A teacher asked his class if they were to put all the millimetre cubes contained in cubic metre, on top of each other, how high would the column be?

"It'd be higher than the Eiffel Tower (300 metres)!" one student exclaimed.

"Even higher than Mont Blanc (5 kilometres)!" another answered. Which was closer to the truth?

Whose Count Was Higher?

Two people kept count of the passers-by on a pavement over a period of an hour. One of them stood near the gate of a house whilst the other strolled to and fro along the pavement.

Whose count was higher?

A Glass of Peas

Any guess here will lead you to an error. A calculation, however crude, is in order.
A dry pea is about 1/2 centimetre across. A centimetre cube contains no less than $2 \times 2 \times 2 = 8$ peas (if tightly packed, even more). In a glass of capacity 250 cubic centimetres there are no less than $8 \times 250 = 2{,}000$ peas. When strung these would give a line $1/2 \times 2{,}000 = 1{,}000$ centimetres long, i.e. 10 metres.

Water and Wine

In solving the problem we mustn't overlook the fact that the final volume of liquid in the bottles was equal to the initial one, 1 litre. We then argue as follows. Let, after both transferrals, the second bottle contain n cubic centimetres of wine, and hence $(1{,}000 - n)$ cubic centimetres of water. Where have the missing n cubic centimetres of water gone? Clearly, these are to be found in the first bottle. Accordingly, the wine in the end contains as much water as there is wine in the water.

A Die

The number of all the possible events after four throws of the die is $6 \times 6 \times 6 \times 6 = 1{,}296$. Suppose that the die has already been thrown once and a 1 appeared. Then for the three remaining throws the number of all the possible events, favourable for Peter, i.e. the occurrence of any face save for the 1, will be $5 \times 5 \times 5 = 125$. In exactly the same way, 125 outcomes favourable for Peter are possible if the 1 appears only in the second, only in the third or only in the fourth throw. So, there are $125 + 25 + 125 + 125 = 500$ various possibilities for the 1 to appear once, and only once, in the four throws. As to the unfavourable outcomes, there are $1{,}296 - 500 = 796$ since all the remaining events are unfavourable.
Thus, we see that Vladimir stands better chance to win than Peter: 797 against 500.

The Yale Lock

It's easily seen that the number of different locks possible is $10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$. Each of these locks can only be opened by their own key. It is very comforting for the owner of the lock that there are 100,000 versions of the lock and key as the lock picker has only one chance in 100,000 to hit upon the right key.
Our calculation is very rough since it assumes that each pin can only be divided in 10 different ways. Clearly it could actually be done in a larger number of ways, thus notably increasing the number of different locks possible. This shows the advantage of the Yale lock.

How Many Portraits?

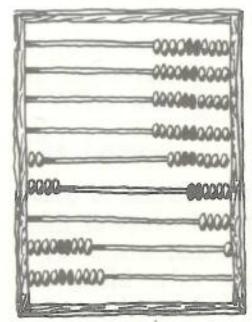
Far more than a thousand. To show this is true make each of the nine sections of a portrait with one of the Roman numerals I, II, III, IV, V, VI, VII, VIII and IX. For

each section there are four stripes, so we'll mark these by 1, 2, 3 and 4.
Take stripe I.1. It may go to II.1, II.2, II.3, or II.4, i.e. we may have four combinations. But since section I may be represented by four stripes (I.1, I.2, I.3, or I.4) each of which may be connected to II in four ways, then the two upper sections I and II may be joined in $4 \times 4 = 16$ various ways.
To each of these 16 arrangements we may attach section III in four ways (III.1, III.2, III.3, or III.4). Consequently, the first three sections may be combined in $16 \times 4 = 64$ various ways.
Reasoning along the same lines, we find that I, II, III, and IV may be arranged in $64 \times 4 = 256$ various ways: I, II, III, IV, and V, in 1,024 ways: I, II, III, IV, V, and VI, in 4,096 ways, and so forth. Lastly, all the nine sections may be fitted together in $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 262{,}144$ ways. Not one thousand but more than *a quarter of a million* different portraits!
The problem is a very instructive one and it goes to explain why it's only exceptionally rarely that we may come across two similar faces. We've just seen that if the human face were characterized by as few as nine features with only four versions possible, then it would be more than 260,000 various faces in existence. In actuality, there are more than nine features and they may vary in more than four ways. In fact, if there were 20 features varying in 10 ways each, we would have 10^{20}, or 100,000,000,000,000,000,000, ways.
Incidentally, this is many times greater than the world's population.

Abacus

You can set 25 pounds using 25 beads in the following way:

Figure 219



Actually, this gives 20 pounds + 4 pounds + 90 pence + 10 pence = 25 pounds.
The number of beads is $2 + 4 + 9 + 10 = 25$.

Leaves of a Tree

A small town, let alone a house, could be encircled with the leaves from a tree if we arranged them in a line because the line would be about 12 kilometres long! Really, the foilage of mature tree includes no less than 200-300 thousand leaves. If for definiteness we stick to 250 thousand and take a leaf to be 5 centimetres wide, we'll have a line 1,250,000 centimetres long, which is 12,500 metres, or 12.5 kilometres.

A Million Paces

A million paces is much more than 10 or even 100 kilometres. If an average pace is about 3/4 metre long, then 1,000,000 paces = 750 kilometres. Since the distance from Moscow to Leningrad is about 640 kilometres, then a million paces would take farther than Leningrad.

A Cubic Metre

Both answers are far from the true figure because the column would be 100 times higher than the highest mountain on Earth. Indeed, in a cubic metre there are 1,000 × × 1,000 × 1,000 = 1 milliard cubic millimetres. If you put one on top of another they would form a column 1,000,000,000 millimetres high or 1,000,000 metres, or 1,000 kilometres.

Whose Count Was Higher?

The counts were equal.

Instructor and Student

The story related below is said to have occurred in Ancient Greece. The teacher and thinker Protagoras (485-410 B.C.) undertook to teach a young man the art of being a barrister. The two sides made a deal that the student pay the fee just after he had made some achievement, i.e. after he had won his first trial.

The student passed the course and Protagoras was waiting for his reward, but the student wouldn't appear in a court of justice. What was to be done? To get his fee the teacher sued his student. He argued thus: if he won the case, the money would be recovered by the court, whereas if he lost the case, and hence his student won it, the money would again be paid according to their deal.

The student, however, regarded Protagoras's case as absolutely hopeless (he seems to have learned something from his teacher) and reasoned as follows: if the judge decided against him, he wouldn't pay according to the terms of the deal since he would have lost his first case, whereas if the judge decided in his favour, again he wouldn't have to pay since that would be the decision of the court.

The judge was embarrased but after a great deal of thought he hit upon an idea and passed a decision that, without violating the terms of the deal, gave the teacher an opportunity to recover his fee.

What was the decision?

The Legacy

Here is another ancient problem that was a favourite with lawyers in Ancient Rome.

A widow has to share a legacy of 3,500 sestertii with her child who was about to be born. According to Roman law, if the child were a boy, his mother got a half of the son's share but if it were a girl, the mother got double the share of the daughter. But it so happened that twins were born, a boy an a girl.

How was the legacy to be shared so that the law was completely satisfied?

Pouring

Consider of jug containing 4 litres of milk. The milk must be divided equally between two friends, but the

only containers available are two empty jugs, one of which holds 2 1/2 litres and the other holds 1 1/2 litres.

How can the milk be divided using the three jugs? Of course, it'll be necessary to pour the milk from one jug into another. But how?

Two Candles

The electricity failed in my flat because the fuse had blown. I lighted two candles that had been specially prepared on my desk, and worked on in their light until the failure was set right.

The next day they wanted to know how long the electricity was off. I had not noticed the time when the electricity failed and was restored, and I didn't know the initial length of the candles. I only knew that the candles were the same length but different thicknesses, and that the thicker one took 5 hours to burn down completely whilst the thinner one took 4 hours. Both were new before I had lighted them up. But I didn't find the ends of the candles: somebody had thrown them away. I was told that the stubs were so small that it wouldn't have paid to keep them.

"But couldn't you remember their lengths?" I asked.

"They weren't the same. One was four times longer than the other".

All my attempts to squeeze out something more failed. I had to be content with the above information and try to work out how long the candles had been burning.

How would you handle the problem?

Three Soldiers

Three soldiers were having a problem, too. They had to cross a river without a bridge. Two boys with a boat agreed to help the soldiers but the boat was so small it could only support one soldier and even then a soldier and a boy couldn't be in the boat for fear of sinking it. None of the soldiers could swim.

It would seem that under these conditions only one soldier could cross the river. However, all three soldiers were soon on the other bank and returned the boat to the boys.

How did they do it?

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A Herd of Cows

Here is one of the versions of a curious ancient problem.

A father distributed his herd amongst his sons. To his eldest he gave one cow plus 1/7 of the remaining cows; to his second eldest, two cows plus 1/7 of the remaining cows; to the third eldest, three cows plus 1/7 of the remaining cows; to the fourth eldest, four cows and 1/7 of the remaining cows, and so forth. The herd was distributed among his sons without remainder.

How many sons and how many cows were there?

Square Metre

When a boy was told for the first time that a square metre contains a million square millimetres, he wouldn't believe it.

"Why so many?" he was surprised. "Here I've got a sheet of graph paper that is exactly one metre long and one metre wide. And are there million millimetre squares here? I don't believe it!"

"Count them then," somebody advised.

The boy decided to do so and count all the squares. He got up early in the morning and set about counting them neatly marking each square he had counted with a point.

Each mark took him a second so the going was rather fast. He worked like blazes, still do you think he managed to make sure that a square metre has a million square millimetres on the same day?

A Hundred Nuts

A hundred nuts are to be divided between 25 people so that nobody gets an even number of nuts.

Could you do it?

Dividing Money

Two people were making porridge on a camp-fire. One contributed 200 grammes of cereals, the other 300 grammes. When the porridge was ready, they were joined by a passer-by who partook of their meal and paid them 50 pence.

How should they divide the money?

Sharing Apples

Nine apples must be shared out amongst 12 children so that no apple is divided into more than four parts. On the face of it the problem is insolvable, but those who knows about fractions can solve it easily.

Once you have solved that one it should be easy to handle another problem in the same vein: to divide seven apples among 12 boys so that none of the apples is divided into more than four parts.

A Further Apple Problem

Five friends came to see Peter. Peter's father wanted to treat all six boys to apples but there were only five apples. What was to be done? Everyone had to have his fair share. The apples, of course, had to be cut but not into small pieces since Peter's father wouldn't cut them into more than three. So, the problem was to divide the five apples equally among the six boys so that none of the apples was cut into more than three pieces. How was Peter's father to get out of his predicament?

One Boat for Three

Three sports enthusiasts possess one boat. They keep it on a chain with three locks so that each of them could use it but a stranger couldn't. Each of them has his own key but he can still unlock the boat without waiting for his friends and their keys.

How did they arrange it?

Waiting for a Tram

Three brothers came to a tram stop. There was no tram in sight and the eldest brother suggested they wait.

"Why wait?" the second brother asked, "we'd better go on. When the tram catches up with us, we can jump onto it, but by then we'd have got part of the way home and thus we'll get there sooner."

"If we decide to go," the youngest brother objected, "then we'd better go backwards not forwards: since then we'll meet an oncoming tram sooner and so get home sooner."

Since the brothers couldn't persuade each other, each went his own way. The eldest stayed to wait, the second went on, and the youngest walked back down the route. Which of the three got home sooner? Who was the most reasonable?

Instructor and Student

The decision was to decide against Protagoras but give him the right to bring the case before the court a second time. After the student had won his first trial, the second one should undoubtedly be decided in favour of the instructor.

The Legacy

The widow gets 1,000 sestertii, the son 2,000 sestertii, and the daughter 500 sestertii. This fulfils Roman law since the widow gets a half of the son's share and double the daughter's.

Pouring

Seven pourings will be required as is shown in the table:

Pouring	4 l	1 1/2 l	2 1/2 l
1	1 1/2	—	2 1/2
2	1 1/2	1 1/2	1
3	3	—	1
4	3	1	—
5	1/2	1	2 1/2
6	1/2	1 1/2	2
7	2	—	2

Two Candles

We'll construct a simple equation. We'll denote the time (in hours) that the candles burned by x. Each hour 1/5 part of the original length of the thick candle and 1/4 part of the original length of the thin candle burns away. Accordingly, the thick candle's stub will be $1 - x/5$ of its original length and the thin candle's stub $1 - x/4$ of the original length. We know that the candles were originally equally long and that the four times the length of the thick stub, i.e. $4(1 - x/5)$, was equal to the length of the thin stub $(1 - x/4)$. Thus,

$$4\left(1 - \frac{x}{5}\right) = 1 - \frac{x}{4}.$$

Solving the equation gives that $x = 3\ 3/4$ hours, i.e. the candles had burned for 3 hours 45 minutes.

Three Soldiers

The following six crossings were made:

1st crossing. Both boys go to the opposite bank and one of them brings the boat back to the soldiers (the other stays on the opposite bank).

2nd crossing. The boy that brought the boat back stays on the bank with the soldiers and a soldier crosses the river in the boat. The boat returns with the other boy.
3d crossing. Both boys cross the river and one of them returns with the boat.
4th crossing. The second soldier crosses and the boat returns with the boy.
5th crossing. Like the third one.
6th crossing. The third soldier crosses and the boat returns with the boy. The boys continue on their journey and the three soldiers are on the opposite bank.

A Herd of Cows

Arithmetically (i. e. without resorting to equations), the problem should be approached from the end.

The youngest son got as many cows as there were sons for he could not get an additional 1/7 of the remaining herd as there were no cows left.

Further, the next son got one cow less than there were sons, plus 1/7 of the remaining cows. Accordingly, the share of the youngest son amounts to 6/7 of the share of the remainder.

It thus follows that the number of cows the youngest son got must be divisible by six.

Let's assume that the youngest son received six cows and see if this assumption is good. It follows from the assumption that there were six sons. The fifth son got five cows plus 1/7 of seven, i. e. six cows all in all. Thus, the two youngest sons got $6 + 6 = 12$ cows, which accounts for 6/7 part of the herd left after the fourth son has received his share. The total residue was $12 \div 6/7 = 14$ cows, hence the fourth son got $4 + 14/7 = 6$ cows.

We'll now work out the residue after the third son got his share: $6 + 6 + 6 = 18$ cows is 6/7 part of the residue. Therefore, the total residue was $18 \div 6/7 = 21$ cows. The third son got $3 + 21/7 = 6$ cows.

In exactly the same way we'll find that the second and first sons also got six cows each.

Our assumption that there were six sons and 36 cows appears to be plausible.

But are there other solutions? Assume that there were 12 sons, not six. It turns out that this assumption is unsuitable. The number 18 won't do either. Other multiples of six would be unreasonable since there couldn't be 24 or more sons.

Square Metre

No, the boy would not be able to verify the fact in one day. Even if he counted for 24 hours without interruption, he would have counted only 86,400 squares since there are only 86,400 seconds in 24 hours. To count to one million he would have to work for almost 12 days without stopping, and for a month if he worked 8 hours a day.

A Hundred Nuts

Many people would immediately set about trying a variety of combinations, but their efforts would all be to no avail. If you give some thought to the problem, you'll understand the futility of all their efforts since the problem is insolvable.

If you could break 100 into 25 odd summands, you would have been able to make an odd number of odd numbers add up to 100 which is an even number, and that is clearly impossible.

In fact, we would have to obtain 12 pairs of odd numbers and one more odd number. Each pair of odd numbers yields an even number, so 12 pairs of even numbers must add up to an even number. If then we add an odd number to the total, we'll end up with an odd result. Thus 100 can never be composed of such summands.

Dividing Money

Most people answer that the one who contributed the 200 grammes should get 20 pence and the other 30 pence. This division is not fair.

We'll argue as follows: 50 pence was paid for one portion of food.

Since there were three eaters, the cost of the porridge (500 grammes) should be 1 pound 50 pence. The person who contributed the 200 grammes gave 60 pence worth of food in terms of money (since a hundred grammes costs $150 \div 5 = 30$ pence). However, he also consumed 50 pence worth of porridge, hence he must get back $60 - 50 = 10$ pence.

The contributor of the 300 grammes (i. e. 90 pence in terms of money) must get $90 - 50 = 40$ pence.

Thus, out of the 50 pence one person should have 10 pence and the other person 40 pence.

Sharing Apples

It's possible to share nine apples equally between 12 children without cutting any apple into more than four parts.

Six apples should be divided in two each to yield 12 halves. The remaining three apples should each be divided into four equal parts to yield 12 quarters. Now each child receives a half and a quarter. So each will get 3/4 of an apple as required, because $9 \div 12 = 3/4$.

Reasoning along the same lines· it's possible to divide seven apples among 12 children so that each child gets an equal share and no apple needs to be cut into more than four parts. In this case each child should get 7/12 of an apple, but notice that $7/12 = 3/12 + 4/12 = 1/4 + 1/3$.

Therefore three apples are divided into four parts and the four remaining apples into three parts each. We thus obtain 12 quarters and 12 thirds.

In consequence, each child can be given a quarter and a third, or 7/12.

A Further Apple Problem

The apples were divided thus: three apples were each cut in half to yield six halves that were distributed among the children and the remaining two apples were each cut into three to obtain six thirds that were also given to the children.

Consequently, each boy got a half and a third of an apple, i. e. all the boys got their equal share, and none of the apples was cut into more than three equal parts.

One Boat for Three

The locks should be connected as shown in Fig. 220. You can see quite easily that each of the boat's owners can open the chain of the three locks using his key.

Figure 220

Waiting for a Tram

The youngest brother, who went backwards, saw an oncoming tram and jumped into it. When the tram came to the stop where the eldest brother was waiting, he got in too. A short while later the tram caught up the third brother who was walking homewards and collected him. All the three brothers found themselves in the same tram and, of course, arrived home at the same time.

The most reasonable brother was the eldest one since he waited quietly at the stop.

228–229 Problems from *Gulliver's Travels*

Beyond doubt the most fascinating pages in *Gulliver's Travels* are those describing his unusual adventures in the country of tiny Lilliputians and in the country of giant Brobdingnagians. In Lilliput the dimensions–height, width, thickness–of people, animals, plants and other things were 1/12 of those here. By contrast, in Brobdingnag they were 12 times larger. We can easily understand why the author of the *Travels* choose the number 12, if we remember that in the British system of units there are 12 inches in a foot. A 12-fold increase or decrease doesn't seem to be very much of a change but the nature and way of life in this fantastic countries was strikingly different from those we are used to. Every now and then the differences are so amazing that can serve as a material for interesting problems.

Animals of Lilliput

Gulliver relates: "Fifteen hundred of the Emperor's largest horses... were employed to draw me towards the metropolis."

Doesn't it seem to you that 1,500 horses are a bit too many taking into account the relative dimensions of Gulliver and Lilliputian horses?

Also, Gulliver tells us a no less amazing thing about the cows, bulls, and sheep, for when he left he just "put them into his pocket".

Is it all possible?

Hard Bed

Lilliputians made the following bed for their giant guest: "Six hundred beds of the common measure were brought in carriages, and worked up in my house; a hundred and fifty of their beds sewn together made up the breadth and length, and these were four double, which however kept me but very indifferently from the hardness of the floor, that was of smooth stone".

Why was Gulliver so incomfortable on the bed? And is this computation correct?

Gulliver's Boat

Gulliver left Lilliput in a boat washed up on the shore by chance. The boat seemed monstrous to the

Lilliputians, it surpassed by far the largest ships of their fleet.

Could you work out the displacement* of the boat in Lilliputian tonnes if its weight-carrying capacity was 300 kilogrammes?

Hogsheads and Buckets of Lilliputians

Gulliver is drinking:

"I made another sign that I wanted drink... They slung up with great dexterity one of their largest hogsheads; then rolled it towards my hand, and beat out the top; I drank it off at a draught, which I might well do, for it hardly held half a pint... They brought me a second hogshead, which I drank in the same manner, and made signs for more, but they had none to give to me".

Elsewhere in the book Gulliver describes the Lilliputian buckets as being no larger than a thimble.

Why should such tiny hogsheads and buckets exist in a country where everything is only 1/12th normal size?

Figure 221

Food Allowance and Dinner

Lilliputians set the following daily allowance of food for Gulliver:

"... the said Man Mountain shall have a daily allowance of meat and drink, sufficient for the support of 1,728 of our subjects."

Elsewhere Gulliver relates:

"I had three hundred cooks to dress my victuals, in little convenient huts built about my house, where they and their families lived, and prepared me two dishes apiece. I took up twenty waiters in my hand, and placed them on the table; a hundred more attended below on the ground, some with dishes of meat, and some with barrels of wine and other liquors slung on their shoulders; all which the waiters above drew up as I wanted, in a very ingenious manner, by certain cords, as we draw the bucket up a well in Europe".

How did they come to fix on that number? And what is the use of all that army of servants to feed just one man? After all, he's only a dozen times taller than a Lilliputian. Are the allowance and appetites compatible with the relative sizes of Gulliver and the Lilliputians?

Figure 222

* The displacement of a ship is the largest load (including the weight of the ship itself) that the ship can support.

Figure 223

Three Hundred Tailors

"Three hundred tailors were employed... to make me clothes."

Was this army of tailors really necessary to have clothes made for a man who is only a dozen times larger than a Lilliputian?

Gigantic Apples and Nuts

In the part "A Voyage to Brobdingnag" devoted to Gulliver's stay in the country of giants we read about some of the hero's trouble-filled adventures. So once he was in the gardens of the court under some apple-trees and the Queen dwarf "when I was walking under one of them, shook it directly over my head, by which a dozen apples, each of them near as large as a Bristol barrel, came tumbling about my ears; one of them hit me on the back as I chanced to stoop, and knocked me down flat on my face."

On another occasion "an unlucky schoolboy aimed a hazelnut directly at my head, which very narrowly missed me; otherwise, it came with so much violence, that it would have infallibly knocked out my brains; for it was almost as large as a small pumpkin".

What do you think was the weight of the apples and nuts in Brobdingnag?

A Ring of the Giants

The collection of rarities brought by Gulliver from Brobdingnag includes "a gold ring which one day she (the Queen) made me a present of in a most obliging manner, taking it from her little finger, and throwing it over my head like a collar."

Is it possible that a ring from a little finger would fit on Gulliver like a collar and how much, approximately, would the ring weigh?

Books of the Giants

About books of Brobdingnagians Gulliver tells us the following: "I had liberty to borrow what books I pleased. The Queen's joiner had contrived... a kind of wooden machine five and twenty foot high, formed like a standing ladder; the steps were each fifty foot long. It was indeed a movable pair of stairs, the lowest end placed at ten foot distance from the wall of the

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chamber. The book I had a mind to read was put up leaning against the wall. I first mounted to the upper step of the ladder, and turning my face towards the book, began at the top of the page, and so walking to the right and left about eight or ten paces according to the length of the lines, till I had gotten a little below the level of my eye; and then descending gradually till I came to the bottom; after which I mounted again, and began the other page on the same manner, and so turned over the leaf, which I could easily do with both my hands, for it was as thick and stiff as a pasteboard, and in the largest folios not above eighteen or twenty foot long."

Does this make sense?

Collars for the Giants

Finally, consider a problem of this kind that is not directly taken from *Gulliver's Travels.*

You may know that the size of a collar is nothing but the number of centimetres of its length. If your neck is 38 centimetres round, your collar size is 38. On average an adult's neck is 40 centimetres round. If Gulliver wished to order some collars in London for a Brobdingnagian, what number would he require?

16

Animals of Lilliput

It's calculated in the answer to "Food Allowance and Dinner" that Gulliver's volume was 1,728 times larger than that of a Lilliputian. Crearly, he was that many times heavier. For Lilliputians it was as difficult to transport his body as it would have been to transport 1,728 grown-up Lilliputians. That is why the cart with Gulliver had to be pulled by so many Lilliputian horses.

Figure 224

Animals in Lilliput were also 1,728 times smaller in volume, and hence as much ighter than ours.

Our cow is about 1.5 metres high and weighs 400 kilogrammes. A cow in Lilliput would be 12 centimetres high and weigh 400/1,728 kilogrammes, i.e. less than 1/4 kilogrammes. A toy cow like this really could be carried about in a pocket.

Gulliver gives a true account of relative sizes:

"The tallest horses and oxen are between four and five inches in height, the sheep an inch and a half, more or less; their geese about the bigness of a sparrow, and so the several gradations downwards, till you come to the smallest, which to my sight were almost invisible... I have been much pleased with observing a cook pulling a lark, which was not so large as a common fly; and a young girl threading an invisible needle with invisible silk."

Hard Bed

The calculation is quite correct. If a Lilliputian bed is 12 times shorter, and of course 12 times narrower than a conventional bed, then its surface area would be 12 × 12 times smaller than the surface of our bed. Accordingly, for his bed Gulliver required 144 (i.e. to make a round number, about 150) Lilliputian beds. The bed would however have been exceedingly thin – 12 times thinner than ours. Thus even four layers of such beds would not have been soft enough for Gulliver since the resultant mattress was three times thinner than ours.

Gulliver's Boat

We know from the question that the boat could carry 300 kilogrammes, i.e. its displacement was about 1/3 tonne. A tonne is the weight of 1 cubic metre of water, hence the boat displaced 1/3 of our cubic metre. But all the linear dimensions in Lilliput are 1/12 of ours, and volumes are 1/1,728 of ours. So 1/3 of our cubic metre contains about 575 Lilliputian cubic metres and thus Gulliver's boat had a displacement of 575 tonnes or thereabout since we arbitrarily took the figure 300 kilogrammes.

Today we have ships with displacements of tens of thousands of tonnes ploughing the seas, so a ship with a 575-tonne displacement should not be a wonder. We should remember though that at the time of writing (early in the 18th century) 500–600-tonne ships were still rare.

Hogsheads and Buckets of Lilliputians

Lilliputian vessels were 12 times smaller than ours in every dimensions – height, width, and length – and 1,728 times smaller in volume. If we assume that our bucket contains about 60 glasses, we can work out that a Lilliputian bucket contains only 60/1,728, i.e. about 1/30 of a glass. This is just larger than a tea-spoonful but not really much larger than the volume of a large thimble.

If the capacity of a Lilliputian bucket is thus a tea-spoonful, the capacity of a 10-bucket hogshead would not be much larger than half a glass. No wonder Gulliver couldn't quench his thirst with two such hogsheads.

Food Allowance and Dinner

The computation is perfectly correct. We shouldn't forget that Lilliputians were an exact, though smaller, replica of conventional people with normally proportioned members. Consequently, they were not only 12 times shorter, but also 12 times narrower and 12 times thinner than Gulliver, and their volume was 1/1,728 of that of Gulliver. And to support the life of such a body requires respectively more food. That's why Lilliputians calculated that Gulliver needed an allowance sufficient to support 1,728 Lilliputians.

We now see the purpose of so many cooks. To make 1,728 dinners requires no less than 300 cooks taking that one Lilliputian cook can make half a dozen Lilliputian dinners. An accordingly larger number of people is required to haul the load up to Gulliver's table, which can be estimated to be the height of a three-storey building in Lilliput.

Three Hundred Tailors

The surface of Gulliver's body was 12×12, i.e. 144 times larger than that of a Lilliputian. This is clearer if we imagine that each square inch of the surface of a Lilliputian's body corresponds to a square foot on the surface of Gulliver's body. We know, however, that there are 144 square inches in a square foot. Thus Gulliver's suit would take 144 times more fabric than that of a Lilliputian, and hence more

working time. If, say, one tailor can make one suit in two days, then to make 144 suits in a day (or one of Gulliver's suits) may require 300 tailors.

Gigantic Apples and Nuts

An apple that weighs about 100 grammes here should correspond to an apple in Brobdingnag that is as many times heavier as it is bigger in volume, i.e. 1,728 times heavier than here. Thus Brobdingnagian apples are about 173 kilogrammes. If such an apple falls from a tree and hits a man on the back, he would only just survive the blow. Gulliver thus got off lightly.

Figure 225

A Brobdingnagian nut must have weighed 3–4 kilogrammes, if we take that our nut weighs about 2 grammes. Such a gigantic nut might be about a dozen centimetres across. A 3-kilogramme, hard object thrown with the speed of the nut clearly could smash the skull of a normal-size man. Elsewhere in the book Gulliver recalls: "There suddenly fell such a violent shower of hail, that I was immediately by the force of it struck to the ground: and when I was down, the hailstones gave me such cruel bangs all over the body, as if I had been pelted with tennis balls." Quite plausible, because each piece of hail in this country of giants must weigh no less than a kilogramme.

A Ring of the Giants

A normal little finger is about 1 1/2 centimetres across. Multiplying this by 12 gives 18 centimetres and a ring of such a diameter has a circumference of 56 centimetres, i.e.

Figure 226

it's sufficiently large for a normal head to go through it.

As to the weight of such a ring, if a normal ring weighs 5 grammes its counterpart in Brobdingnag must have weighed 8 1/2 kilogrammes!

Books of the Giants

If we start from the size of books current in our times (about 25 centimetres long and 12 centimetres wide), then Gulliver's account might appear to be a slight exaggeration.

Figure 227

You could handle a book 3 metres high and 1 1/2 metres wide without a ladder and without having to move to the left or right by 8–10 steps. In the days of Swift, early in the 18th century, the conventional format of books (tomes) was far larger than now. 20 × 30 cm formats were not uncommon, which when multiplied by 12 gives 360 × × 240 centimetres. It is impossible to read a 4-metre book without a ladder. But a real tome of the time might be as large as a newspaper.

However, the modest tome we mentioned would in the country of giants weigh 1,728 times more than here, i. e. about 3 tonnes. Assuming that it has 500 sheets, each of its sheets would weigh about 6 kilogrammes, perhaps a bit too much for fingers.

Collars for the Giants

The neck of a giant will be 12 times larger than that of a normal man. And if a normal man needs a collar of size 40, the giant would need a 40 × 12 = 480 size collar.

* * *

We thus see that all the whimsical things in Swift seem to have been carefully calculated. Responding to certain critisisms of his poem *Eugine Onegin* Alexander Pushkin once noted that in his book "time is calculated with a calendar". In exactly the same way Swift could say that all his objects had conscientiously been computed using the laws of geometry.

236–237 Stories about Giant Numbers

Reward

According to legend, the following happened in ancient Rome.

I. General Terentius had returned to Rome with booty after a victorious campaign. Back in the capital he was received by the Emperor.

The reception was very warm and the Emperor thanked him cordially for his services to the Empire promising to confer on him a high office in the Senate.

But Terentius didn't want this. He said:

"I have won many victories to exalt your grandeur, Sire, and to cover your name with glory. I have been unafraid of death and if I had many lives, I'd sacrifice all of them to you. But I'm tired of fighting, my youth had passed and my blood flows slower in my veins. The time has come for me to retire to my father's home and revel in the joys of domestic life."

"What would you like to receive from me, Terentius?" the Emperor asked.

"Hear me out with indulgence, Sire! In all these long years of battle, imbruing my sword with blood, I have had no time to take care of my well-being. I'm poor, Sire..."

"Proceed, brave Terentius."

The encouraged general went on to say: "If it is your desire to reward your humble servant, then may your generosity help me live out the remainder of my days in peace and comfort at home. I do not seek honour or high office as I would like to retire from power and public life to live peacefully. Sire, please award me with money to provide for the rest of my life."

The Emperor – so the legend goes – wasn't distinguished for his lavishness. He liked to save money for himself but was miserly with it to others. The general's request plunged him in a deep reverie.

He asked, "What sum, Terentius, would you consider sufficient?"

"A million denarii, Sire."

The Emperor grew pensive again. The general waited, his head down.

Finally, the Emperor spoke:

"Valiant Terentius, you are a great warrior and your prodigies of valour have earned you a lavish reward! I will give you wealth. Tomorrow at noon you will hear

my decision."

Terentius bowed and walked out.

II. On next day at the hour appointed the general came to the Emperor's palace.

"Greetings, brave Terentius!" the Emperor said. Terentius bowed his head humbly.

"Sire, I came to hear your decision. You kindly promised to reward me."

The Emperor answered: "It's not my intention that such a noble warrior like you should have some miserable reward for his heroic deeds. Listen to me. There are in my treasury 5 million copper brasses*. You shall go to the treasury and take one coin, then you shall return here and place it at my feet. On the following day you shall again go to the treasury, take a coin worth 2 brasses and place it here near the first one. On the third day you are to bring a coin worth 4 brasses and on the fourth day bring a coin worth 8 brasses, on the fifth, 16, etc., double the value of the previous coin. I will order appropriate coins be produced for you and while you have the strength, you will take them from my treasury. Nobody may help you, you must rely on your own power only. You will stop when you notice that cannot move a coin any more and then our deal will come to an end. All the coins that you will have managed to bring here will belong to you and you shall keep them as your reward."

Figure 228

Terentius listened eagerly to the Emperor's words. He visualized the multitude of coins, each one more than another, that he would bring out of the treasury.

"I'm happy with your favour," he beamed. "Really generous is your reward!"

III. Terentius started his daily visits to the treasury. It was located close to the Emperor's hall so the first trips with the coins cost Terentius very little effort.

On the first day he only brought 1 brass. This was a small coin 21 millimetres across and weighing 5 grammes.

His trips upto the sixth day were also very easy and he brought the coins double, fourfold, sixteen-fold, and thirty-two-fold the weight of the first.

* A brass is a fifth of a denarius.

The seventh coin weighed 320 grammes and was 8 1/2 centimetres across.*

On the eighth day Terentius had to carry out a coin that was worth 128 units. It weighed 640 grammes and was about 10 1/2 centimetres wide.

On the ninth day he carried into the Emperor's hall a coin corresponding to 256 unit coins. It was 13 centimetres wide and weighed more than 1 1/4 kilogrammes.

On the twelfth day the coin was almost 27 centimetres across and 10 1/4 kilogrammes in weight.

The Emperor who up until that day was very kind to the general now couldn't conceal his triumph. He saw that after 12 days only slightly more than 2,000 brass units had been brought.

Further, on the thirteenth day the brave Terentius brought out a coin that was worth 4,096 units. It was 34 centimetres wide and weighed 20 1/2 kilogrammes.

On the fourteenth day Terentius had a heavy coin that was 42 centimetres across and weighed 41 kilogrammes.

"Are you tired, my brave Terentius?" the Emperor could hardly help smiling.

"No, Sire," the general responded grimly wiping his brow.

The fifteenth day came. This time Terentius's burden was really heavy. He trudged slowly to the Emperor carrying a huge coin corresponding to 16,384 unit coins. It was 53 centimetres wide and weighed 80 kilogrammes, the weight of a tall warrior.

On the sixteenth day the general staggered with the burden on his back. It was a coin equal to 32,768 units, its diameter being 67 centimetres and weighing 164 kilogrammes.

The general was exhausted and gasping. The Emperor smiled...

When Terentius came to the Emperor the next day, there was a roar of laughter. He could no longer carry his coin in his hands and rolled it in front of him. The coin was 84 centimetres and 328 kilogrammes, and corresponded to 65,536 unit coins.

The eighteenth day was the last day of Terentius's enrichment, for his visits to the treasury and trips to

* If a coin's volume is 64 times that of a normal one, then it is only four times wider and thicker, because $4 \times 4 \times 4 = 64$. We should have this in mind when working out the sizes of further coins.

the Emperor's hall ended on that day. This time he had to fetch a coin worth 131,072 unit coins. It was more than a metre across and weighed 655 kilogrammes. Using his spear as a lever Terentius rolled it into the hall with a huge effort. The mammoth coin fell thundering at the Emperor's feet.

Terentius was completely worn out.

"Can't do any more... Enough for me," he wispered.

The Emperor could hardly conceal his pleasure at the total triumph of his ruse. He ordered the treasurer to compute the total of all the brasses brought into the hall by Terentius.

The treasurer reckoned quickly and said: "Sire, thanks to your generosity the victorious warrior Terentius has got a reward of 262,143 brasses."

So the close-fisted Emperor gave the general about 1/20 part of the million of denarii Terentius had requested.

* * *

Let's check the treasurer's calculation and the weight of the coins. Terentius brought out:

Day	Coin in brasses	Weight in grammes
1	1	5
2	2	10
3	4	20
4	8	40
5	16	80
6	32	160
7	64	320
8	128	640
9	256	1,280
10	512	2,560
11	1,024	5,120
12	2,048	10,240
13	4,096	20,480
14	8,192	40,960
15	16,384	81,920
16	32,768	163,840
17	65,536	327,680
18	131,072	655,360

The totals for these columns can be calculated easily using the proper rule*, thus the second column totals

* Each number in this column equals the sum of the previous ones plus one. Therefore, when it's necessary to sum up all the numbers in the column, e. g. from 1 to 32,768, we need only find the next number and subtract one, i.e. $32{,}768 \times 2 - 1$. The result is 65,535.

262,143. Terentius requested a million of denarii, i.e. 5 million brasses. Accordingly, he got

$5{,}000{,}000 \div 262{,}143 = 19$ times

less than he requested.

Legend about Chess-Board

I. Chess is one of the world's most ancient games. It has been in existence for centuries so it is no wonder that it has given rise to many legends whose truthfulness cannot be checked because of the remotedness of the events. One of these legends I want to relate. You do not need to be able to play chess to understand it, it is sufficient for you to know that it involves a board divided into 64 cells (black and white alternately).

The play of chess was invented in India. When the Indian king Sheram got to know about it he was amazed at its ingeniousness and the infinite variety of positions it afforded. Having learned that the play was invented by one of his subjects, the king summoned him in order to reward him personally for such a stroke of brilliant insight.

The inventor, named Seta, came before the sovereign's throne. He was a simply dressed scribe who earned his living giving lessons to pupils.

Figure 229

"I want to reward you properly, Seta, for the beautiful play you invented," the king said.
The sage bowed.
"I'm rich enough to fulfil any of your desires," the king went on to say. "Name a reward that would satisfy you and you'll get it".
There was a silence.
"Don't be shy! What's your desire? I'll spare nothing to meet your wish!"
"Great is your kindness, oh sovereign. Give me some time to sleep on it. Tomorrow, upon consideration, I'll name you my wish."
When the next day Seta came to the throne he amazed the king by the unprecedented modesty of his desire.
Seta said: "Sovereign, order that one grain of wheat be given to me for the first cell of the chess-board."
"A simple wheat grain?" the king was shocked.
"Yes, sovereign. For the second cell let there be two grains, for the third four, for the fourth eight, for the fifth 16, for the sixth 32..."
"Enough!" the king was exasperated. "You'll get your grains for all the 64 cells of the board according to your wish: for each twice as much as for the previous one. But let me tell you that your wish is unworthy of my generosity. By asking for such a miserable reward you show disrespect for my favour. Truly, as a teacher you might give a better example of gratitude for the kindness of your king. Go away! My servants will bring you the bag of wheat."
Seta smiled, left the hall and began to wait at the palace gates.

II. At dinner the king remembered about the inventor of chess and asked if the foolish Seta has collected his miserable reward.
The answer was: "Sovereign, your order is being fulfilled. The court mathematicians are computing the number of grains required."
The king frowned–he wasn't used to having his orders fulfilled so slowly.
At night, before going to bed the king Sheram again inquired how long before had Seta left the palace with his bag of wheat.
"Sovereign, your mathematicians are working hard and hope to finish their calculations before dawn."
"Why so long?" the king was furious. "Tomorrow, before I wake up everything, down to the last grain, must be given to Seta. I never give my order twice!"

First thing in the morning the king was told that the chief mathematician humbly asked to make an important report.
The king ordered him in.
Sheram said: "Before you bring out your business I'd like to know if Seta has at last received the miserable reward that he asked for."
The old man responded: "It's exactly because of this that I dared to bother you at such an early hour. We've painstakingly worked out the number of grains that Seta wants to have. The number is so enormous..."
"No matter how enormous it is", the king interrupted him arrogantly, "my granaries won't be depleted! The reward is promised and must be given out..."
"It's beyond your power, oh sovereign, to fulfil his wish. There is not sufficient grain in all your barns to give Seta what he wants. And there is not enough in all the barns throughout the kingdom. You would not find that many grains in the entire space of the earth. And if you wish to give out the promised reward by all means, then order all the kingdoms on earth to be turned into arable fields, order all the seas and oceans dried up, and order the ice and snowy wastes that cover the far northern lands melted. Should all the land be sown with wheat and should the entire yield of these fields be given to Seta, then he'd receive his reward."
The king attended to the words of the elder with amazement.
"What is this prodigious number?"
"18,446,744,073,709,551,615, oh sovereign!"

III. Such was the legend. There is now no way of knowing if it's true, but that the reward is expressed by this number you could verify by some patient calculations. Starting with unity you'll have to add up number 1,2,4,8, etc. The result of the 63th doubling will be what the inventor should receive for the 64th cell of the board.
If you use the rule explained at the end of the previous problem, you can easily obtain the number of grains to be received by the inventor (we double the last number and subtract one). Hence the calculation comes down to multiplying together 64 twos:

$2 \times 2 \times 2 \times 2 \times 2$, etc.,–64 times.

To facilitate computation divide the 64 multipliers into six groups with 10 twos in each and one last group with four twos. It's easy to see that the product of 10 twos is 1,024, and of four twos, 16. The desired result is thus $1{,}024 \times 1{,}024 \times 1{,}024 \times 1{,}024 \times 1{,}024 \times 1{,}024 \times 16$.

Multiplying $1{,}024 \times 1{,}024$ gives 1,048,576.

It now remains to find $1{,}048{,}576 \times 1{,}048{,}576 \times \times 1{,}048{,}576 \times 16$, subtract one from the result to arrive at the sought-for number of grains: 18,446,744,073,709,551,615.

If you want to imagine the enormousness of this numerical giant just estimate the size of a barn that would be required to house this amount of grain. It's known that a cubic metre of wheat contains about 15,000,000 grains. Consequently, the reward of the inventor of chess would occupy about 12,000,000,000,000 cubic metres, or 12,000 cubic kilometres. If the barn were 4 metres high and 10 metres wide its length would be 300,000,000 kilometres, twice the distance to the Sun!

The Indian king could never grant such a reward. Had he been good at maths, he could have freed himself of the debt. He should have suggested to Seta to count off the grains he wanted himself.

In fact, if Seta kept on counting day in day out he would have counted only 86,400 grains in the first 24 hours. A million would have required no less than 10 days of continual reckoning and thus to process 1 cubic metre of wheat would have required about half a year. In a ten year's time he would have handled about 20 cubic metres. You see that even if Seta had devoted a lifetime to his counting, he would still have only obtained a miserable fraction of the reward he desired.

Prolific Multiplication

A ripe poppy head is full of tiny seeds, each of which can give rise to a new plant. How many poppy plants shall we have, if all the seeds germinate? To begin with we should know how many seeds there are in a head. A boring business, but if you summon up all your patience you'll find that one head contains about 3,000 seeds.

What follows from this? If there is enough space around our poppy plant with adequate soil, each seed will produce a shoot with the result that the following

summer 3,000 poppies will grow. A whole poppy field from just one head.

Let's see what will happen next. Each of the 3,000 plants will produce no less than one head (more often several heads), with 3,000 seeds each. Having germinated, the seeds of each head will give 3,000 new plants, and hence during the following year we are going to have

$3{,}000 \times 3{,}000 = 9{,}000{,}000$ plants.

Calculation gives that in the third year the offspring of our initial head will already reach

$9{,}000{,}000 \times 3{,}000 = 27{,}000{,}000{,}000$.

In the fourth year there will be

$27{,}000{,}000{,}000 \times 3{,}000 = 81{,}000{,}000{,}000{,}000$ offspring.

In the fifth year our poppies will engulf the earth, because they'll reach the number

$81{,}000{,}000{,}000{,}000 \times 3{,}000 = 243{,}000{,}000{,}000{,}000{,}000$.

But the surface area of all the land, i.e. all the continents and islands of the earth, amount to 135,000,000 square kilometres, or 135,000,000,000,000 square metres – about 2,000 times less than the number of the poppy plants grown.

You see thus that if all the poppy-seeds from one head germinated, the offspring of one plant could engulf the earth in five years so that there were about 2,000 plants of each square metre of land. Such a numerical giant lives in a tiny poppy seed!

A similar calculation made for a plant other than the poppy, one which yields less seeds, would lead to the same result with the only distinction that its offspring would cover the lands of the earth in a longer period than five years. Take a dandelion, say, which gives about 100 seeds annually. Should all of them germinate, we would have:

Year	Number of plants
1	1
2	100
3	10,000
4	1,000,000
5	100,000,000
6	10,000,000,000
7	1,000,000,000,000
8	100,000,000,000,000
9	10,000,000,000,000,000

This is 70 times more than the square metres of land available on the globe.

In consequence, the whole Earth would be covered by dandelions in the ninth year with about 70 plants on each square metre.

Why then don't we observe in reality these tremendous multiplications? Because the overwhelming majority of seeds die without producing any new plants, they either fail to hit a suitable patch of soil and don't germinate at all, or having begun to germinate are suppressed by other plants, or are eaten by animals. If there were no massive destruction of seeds and shoots, any plant would engulf our planet in a short period.

This is true not only of plants but of animals, too. If it were not for death, the offspring of just one couple of any animal would sooner or later populate all the land available. Swarms of locust covering huge stretches of land may give some idea of what might happen on earth if death didn't hinder the multiplication of living things. In two decades or so the continents would be covered with impenetrable forests and steppes inhabited by incountable animals struggling for their place under the sun. The oceans would be filled to the brim with fish so that any shipping would be impossible. And the air would not be transparent because of the mists of birds and insects...

Before we leave the subject, we'll consider several real-life examples of uncannily prolific animals placed in favourable conditions.

At one time America was free of *sparrows*. The bird that is so common in Europe was deliberately brought to the United States to have it exterminate the destructive insects. The sparrow is known to eat in quantity voracious caterpillars and other garden and forest pests. The sparrows liked their new environment, since there were no birds of prey eating them, and so they began to multiply rapidly. The number of insects began to drop markedly and before long the sparrows, for want of animal food, switched to vegetable food and went about destroying crops*. The Americans were even forced to initiate a sparrow control effort which appeared to be so expensive that a law was passed forbidding the import to America of any animals.

* In the Hawaii they even completely superseded small endemic birds.

A further example. There were no *rabbits* in Australia when the continent was colonized by the Europeans. The rabbits were brought to Australia in the late 18th century and as there were no carnivores that might be their enemies they began to multiply at a terrifically fast rate. Hordes of rabbits soon inundated Australia inflicting enormous damage to agriculture. They became a plague of the country and their eradication required great expense and effort. Later the same situation with rabbits occurred in California.

A third instructive story comes from Jamaica. The island was suffering from an abundance of poisonous snakes. To get rid of them it was decided to introduce the *secretary-bird*, an inveterate killer of poisonous snakes. The number of snakes soon dropped all right, but instead the island got to be infested with the rats that earlier were controlled by the snakes. The rats wrought dreadful havoc amongst the sugar cane fields and posed an urgent problem. It's known that an enemy of the rats in the Indian *mongoose*, and so it was decided to bring four pairs of these animals to the island and allow them multiply freely. The mongooses adapted perfectly to their new land and in a short period of time inhabited the island. In less than a decade they had almost wiped out the rats. But alas, having destroyed the rats, the mongooses began to consume whatever came their way and turned into omnivores. They started killing puppies, goat-kids, piglets, poultry. And when they had multiplied still further they set about devastating orchards, fields and plantations. So the inhabitants of the island were compelled to start combating their previous allies, but with limited success.

Free Dinner

Ten young people decided to celebrate leaving school by a dinner at a restaurant. When all had gathered they started arguing as to how they were to sit at the table. Some suggested that they sit in alphabetic order, others, by age, yet others, by their academic record, or even by their height.

The argument dragged on, but nobody sat down at the table.

It was the waiter who made it up between them. He said:

"My young friends, you'd better stop arguing, sit at

the table arbitrarily and listen to me."

The ten set anyhow and the waiter continued: "Let somebody record the order in which you are sitting now. Tomorrow come here again to dine and sit in another order. The day after tomorrow you sit in a new order, and so on, until you have tried out all the arrangements possible. When you come to sit in exactly the same order as you are sitting now, then upon my word, I'll start to treat you to the finest dinners without charge."

The party liked the suggestion. It was decided to come every night and try all the ways of sitting at the table in order to enjoy the free dinner as soon as possible.

They didn't live to see it, however. And not because the waiter didn't keep his word, but because the number of arrangements was too great. Specifically, it is 3,628,800. You can see this number of days equals to almost 10,000 years!

It might seem unlikely to you that as few as 10 people might be arranged in such an enormous number of various ways but we can check it.

To begin with, we must learn how to find the number of permutations possible. To make our life easier we'll begin with a small number of objects, say three. Let's label them A, B, and C.

We would like to know in how many ways it's possible to permute them. We argue as follows: if for the moment we put B aside, the two remaining objects may be arranged in two ways. We will now attach B to each of the two pairs. We may place it in each of three ways:

(1) B *behind* the pair;
(2) B *in front* of the pair;
(3) B *between* the members of the pair.

Clearly, there are no other positions for B besides these three. But as we have two pairs, AB and BA, then there are $2 \times 3 = 6$ ways of arranging the objects.

Further, we'll repeat the argument for four objects. Let there be four objects A, B, C, and D. We'll again put aside one object and make all the possible permutations with the remaining three. We know already that there are six of these. In how many ways may we attach D to each of the six arrangements of three? Obviously, we may place it as follows;

(1) D *behind* the triple;
(2) D *in front* of the triple;

(3) D *between* the first and second objects;
(4) D *between* the second and third objects.

We thus get $6 \times 4 = 24$ permutations, and since $6 = 2 \times 3$, and $2 = 1 \times 2$, then the number of permutations may be represented as the product $1 \times 2 \times 3 \times 4 = 24$.

Reasoning along the same lines we'll find that for five objects, too, the number of permutations is $1 \times 2 \times 3 \times 4 \times 5 = 120$.

For six objects: $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$, and so on.

Return now to the case of the 10 diners. The number of permutations possible here is obtainable if we take the trouble of multiplying together $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$. This will in fact give the above-mentioned 3,628,800.

The calculation would be more complex if among the 10 diners there were five girls who wanted to alternate with the boys. Although the number of the possible permutations is far less in this case, it's somewhat more difficult to work it out.

Let one boy seat at the table somewhere. The remaining four may only be seated in alternate chairs (leaving the vacant places for the girls) in $1 \times 2 \times 3 \times 4 = 24$ various ways. Since the total number of chairs is 10, the first boy may be seated in 10 ways, hence the number of all possible arrangements for the boys is $10 \times 24 = 240$.

What is the number of ways in which the five girls may occupy the vacant chairs between the boys? Clearly, $1 \times 2 \times 3 \times 4 \times 5 = 120$. Combining each of the 240 positions for the boys with each of the 120 positions of the girls we obtain the number of all the possible arrangements: $240 \times 120 = 28{,}800$.

The above number is smaller by far than the previous one though it would require almost 79 years to work through them all. Should the young guests of the restaurant live to be 100, they could get the free dinner, if not from the waiter himself, then from his successor.

Out of Seven Digits

Write the seven digits from 1 to 7 one after the other: 1 2 3 4 5 6 7.

It's easy to connect them by the plus and minus signs to obtain 40, e. g.

$12 + 34 - 5 + 6 - 7 = 40.$

Try and find another combination of these digits that would yield 55.

Nine Digits

Now write out the nine digits: 1 2 3 4 5 6 7 8 9.

You can as above arrive at 100 by inserting a plus or minus six times and get 100 thus:

$12 + 3 - 4 + 5 + 67 + 8 + 9 = 100.$

If you want to use only four plus or minus signs, you proceed thus:

$123 + 4 - 5 + 67 - 89 = 100.$

Now try and obtain 100 using only three plus or minus signs. It's much more difficult but possible.

With Ten Digits

Obtain 100 using all ten digits.

In how many ways can you do it? There are no less than four different ways.

Unity

Obtain unity using all ten digits.

With Five Twos

We only have five twos and all the basic mathematical operation signs at our disposal. Use them to obtain the following numbers: 15, 11, 12, 321.

Once More with Five Twos

Is it possible to obtain 28 using five twos?

250–251

With Four Twos

The problem is more involved. Use four twos to arrive at 111. Is that possible?

With Five Threes

To be sure, with the help of five threes and the mathematical operation signs we can represent 100 as follows:

$$33 \times 3 + \frac{3}{3} = 100.$$

But can you write 10 with five threes?

The Number 37

Repeat the above problem to obtain 37.

In Four Ways

Represent 100 in four various ways with five identical digits.

With Four Threes

The number 12 can be very easily expressed with four threes:

$12 = 3 + 3 + 3 + 3.$

It's more of a problem to obtain 15 and 18 using four threes:

$15 = (3 + 3) + (3 \times 3);$

$18 = (3 \times 3) + (3 \times 3).$

And if you were required to arrive at 5 in the same way, you might be not very quick to twig that $5 = \frac{3 + 3}{3} + 3.$

Now think of the ways to get the numbers 1, 2, 3, 4, 6, 7, 8, 9, 10.

With Four Fours

If you have done the previous problem and want some more in the same vein, try to arrive at all the numbers

from 1 to 10 with fours. This is no more difficult than getting the same numbers with the threes.

With Four Fives

Obtain 16 using four fives.

With Five Nines

Can you provide at least two ways of getting 10 with the help of five nines?

Twenty-Four

It's very easy to obtain 24 with three eights: $8+8+8$. Could you do this using other sets of three identical digits? The problem has several solutions.

Thirty

The number 30 can easily be expressed with three fives: $5 \times 5 + 5$. It's more difficult to do this with other sets of identical digits. Try it, you'll may be able to find several solutions.

One Thousand

Could you obtain 1,000 with the aid of eight identical digits?

Get Twenty

The following are three numbers written one below the other:

111

777

999

Try and cross out six digits so that the sum of the remaining numbers be 20.

Cross out Nine Digits

The following columns of five figures each contain 15 odd digits:

1 1 1
3 3 3
5 5 5
7 7 7
9 9 9

The problem is to cross out nine digits so that the numbers thus obtained add up to 1,111.

In a Mirror

The number corresponding to a year of the last century is increased 4 1/2 times if viewed in a mirror. Which year is it?

Which Year?

In this century, is there a year such that the number expressing it doesn't change if viewed "upside down"?

Which Numbers?

Which two integers, if multiplied together, give 7? Don't forget that both numbers should be *integers*, therefore answers like $3\,1/2 \times 2$ or $2\,1/3 \times 3$ won't do.

Add and Multiply

Which two integers, if added up, give more than if multiplied together?

The Same

Which two integers, if multiplied together, give the same as if added up?

Even Prime Numbers

You must know that prime numbers are those that are divisible without remainder by themselves only or by unity. Other numbers are called composite.

What do you think: are all the even numbers composite? Are there any even prime numbers?

Three Numbers

Which three integers, if multiplied together, give the same as if added up?

Addition and Multiplication

You've undoubtedly noticed the curious feature of the equalities:

$2+2=4,$

$2\times 2=4.$

This is the only case where the sum and product of two integers (equal to each other at that) are equal. You maybe are unaware that there are dissimilar numbers showing the same property. Think of examples of such numbers. So that you don't believe that the search would be in vain I assure you that there are many such number pairs, though none of them are integers.

Multiplication and Division

Which two integers yield the same result whether the larger of them is divided by the other or they are multiplied together?

The Two-Digit Number

There is a two-digit number such that if it is divided by the sum of its digits the answer is also the sum of the digits. Find the number.

Ten Times More

The numbers 12 and 60 have a fascinating property: if we multiply them together, we get exactly 10 times more than if we add them up:

$12\times 60=720,\quad 12+60=72.$

Try and find another pair like this. Maybe you can find several pairs with the same property.

Two Digits

What is the smallest positive integer that you could write with two digits?

The Largest Number

What is the largest number that you can write with four ones?

Unusual Fractions

Consider the fraction 6729/13,458. All the digits (save for 0) are used in it once. As is easily seen, the fraction is 1/2.

Use the nine digits to obtain the following fractions:

$$\frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{6},\ \frac{1}{7},\ \frac{1}{8},\ \frac{1}{9}.$$

What Was the Multiplier?

A schoolboy carried out a multiplication, then rubbed most of his figures from the blackboard so that only the first line of the figures and two digits in the last line survived. As to the other figures, only the following traces remained:

```
    235
×    **
   ----
   ****
+ ****
  -----
  **56*
```

Could you restore the multiplier?

Missing Figures

In this multiplication case more than half the figures are replaced by asterisks:

```
     *1*
×    3*2
     ---
     *3*
+   3*2*
   *2*5
  ------
  1*8*30
```

Can you restore the missing figures?

What Numbers?

A further problem of the same sort:

```
   **5
×
   1**
  ----
  2**5
+
 13*0
 ***
 -----
 4*77*
```

Strange Multiplication Cases

Consider the following case of the multiplication of two numbers:

48 × 159 = 7,632.

It's remarkable in that each of the nine digits is involved once here. Can you think of any other examples? If so, how many of them are there?

Mysterious Division

What is given below is nothing but an example of a long-division sum where all the digits are replaced by points:

```
             . . . 7 .
. . . ) . . . . . . . .
        . . . .
        ---------
          . . . .
          . . . .
        ---------
            . . .
            . . .
            -------
            . . . .
            . . . .
            -------
```

Not one digit in either the dividend or the divisor is known. It's only known that the *last but one* digit in the quotient is 7. Determine the result of the division.

The problem has only one answer.

Another Division Problem

Restore the missing figures in the division below:

```
       1**
325) *2*5*
     ***
     ----
     *0**
     *9**
     ----
       *5*
       *5*
       ---
```

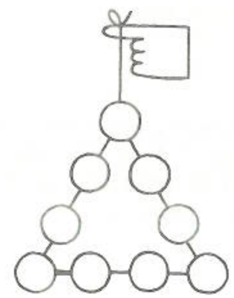
Division by 11

Write out a nine-digit number containing no repeated digits (all the digits are different), that divides by 11 without remainder.

What is the largest such number?

What is the smallest such number?

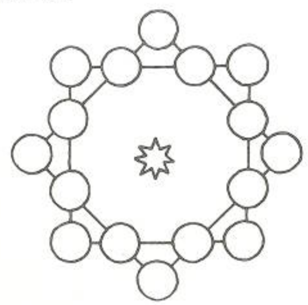
Figure 230



Triangle of Figures

Within the circles of the triangle of Fig. 230 arrange all the nine digits so that the sum of the digits on each side be 20.

Figure 231



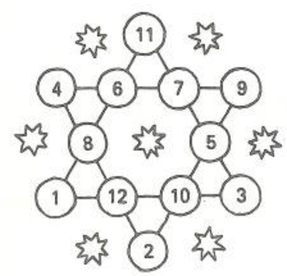
Another Triangle

Repeat the previous problem so that each side adds up to 17.

Eight-Pointed Star

Into the circles of the figure of Fig. 231 insert one of the numbers from 1 to 16 so that the sum of the numbers on the side of each square be 34 and the sum of the numbers at the corners of each square be 34, too.

Figure 232



Magic Star

The six-pointed star shown in Fig. 232 is "magic" because all the six lines of numbers have the same sum:

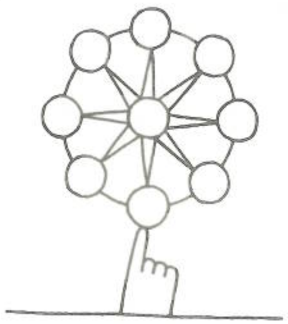
$4+6+7+9=26$, $11+6+8+1=26$,
$4+8+12+2=26$, $11+7+5+3=26$,
$9+5+10+2=26$, $1+12+10+3=26$.

However the numbers at the points of the star add up to another number:

$4 + 11 + 9 + 3 + 2 + 1 = 30.$

Couldn't you improve the star so that the numbers at the points also gave the same sum (26)?

Figure 233



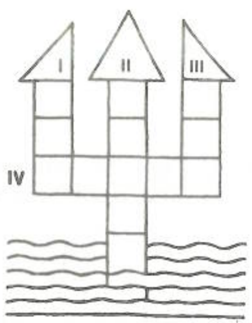
Wheel of Figures

The digits from 1 to 9 should be so arranged in the circles of the wheel of Fig. 233 that one digit is at the centre and the others elsewhere about the wheel so that the three figures in each line add up to 15.

Trident

It's required to arrange the numbers from 1 to 13 in the cells of the trident shown in Fig. 234 so that the sums of the figures in each of the three columns (I, II, and III) and in the line (IV) are the same.

Figure 234



Answers

Out of Seven Digits

There are three solutions:

$$123 + 4 - 5 - 67 = 55;$$
$$1 - 2 - 3 - 4 + 56 + 7 = 55;$$
$$12 - 3 + 45 - 6 + 7 = 55.$$

Nine Digits

$$123 - 45 - 67 + 89 = 100.$$

This is the only solution. It's impossible to arrive at the same result by using the plus and minus signs *less* than three times.

With Ten Digits

The following are the four solutions:

$$70 + 24\frac{9}{18} + 5\frac{3}{6} = 100;$$

$$80\frac{27}{54} + 19\frac{3}{6} = 100;$$

$$84 + 9\frac{4}{5} + 3\frac{12}{60} = 100;$$

$$50\frac{1}{2} + 49\frac{38}{76} = 100.$$

Unity

Represent unity as the sum of two fractions:

$$\frac{148}{296} + \frac{35}{70} = 1.$$

Those knowing more advanced mathematics may also give other answers:

$$123{,}456{,}789^0;\ 234{,}567^{9-8-1}, \text{ etc.,}$$

since any number to the zeroth power is unity.

With Five Twos

Write 15 as:

$$(2+2)^2 - \frac{2}{2} = 15; \quad \frac{22}{2} + 2 \times 2 = 15;$$

$$(2 \times 2)^2 - \frac{2}{2} = 15; \quad \frac{22}{2} + 2^2 = 15;$$

$$2^{(2+2)} - \frac{2}{2} = 15; \quad \frac{22}{2} + 2 + 2 = 15.$$

And 11 as:

$$\frac{22}{2} + 2 - 2 = 11.$$

Now the number 12,321. At first sight, it would seem impossible to write this five-digit number with five similar figures. The problem is manageable, however. Here is the solution:

$$\left(\frac{222}{2}\right)^2 = 111^2 = 111 \times 111 = 12{,}321.$$

Once More with Five Twos

$$22 + 2 + 2 + 2 = 28.$$

With Four Twos

$$\frac{222}{2} = 111.$$

With Five Threes

The solution is:

$$\frac{33}{3} - \frac{3}{3} = 10.$$

It's worth mentioning that the problem would have had exactly the same solution if we had had to express 10 with five ones, five fours, five sevens, five nines, or, in general, with any five identical digits. In fact:

$$\frac{11}{1} - \frac{1}{1} = \frac{22}{2} - \frac{2}{2} = \frac{44}{4} - \frac{4}{4} = \frac{99}{9} - \frac{9}{9}, \text{ etc.}$$

Also, there are other solutions to the problem:

$$\frac{3 \times 3 \times 3 + 3}{3} = 10;$$

$$\frac{3^3}{3} + \frac{3}{3} = 10.$$

The Number 37

There are two solutions:

$$33 + 3 + \frac{3}{3} = 37;$$

$$\frac{333}{3 \times 3} = 37.$$

In Four Ways

We can use ones, threes and (most conveniently) fives:

$$111 - 11 = 100;$$

$$33 \times 3 + \frac{3}{3} = 100;$$

$$5 \times 5 \times 5 - 5 \times 5 = 100;$$

$$(5 + 5 + 5 + 5) \times 5 = 100.$$

With Four Threes

$$1 = \frac{33}{33} \text{ (there are also other ways);}$$

$$2 = \frac{3}{3} + \frac{3}{3};$$

$$3 = \frac{3 + 3 + 3}{3};$$

$$4 = \frac{3 \times 3 + 3}{3};$$

$$6 = (3 + 3) \times \frac{3}{3}.$$

We've given the solutions through six only. Work out the remaining ones for yourselves. The above solutions, too, may be represented with other combinations of threes.

With Four Fours

$$1=\frac{44}{44},\ \text{or}\ \frac{4+4}{4+4},\ \text{or}\ \frac{4\times 4}{4\times 4},\ \text{etc.};$$

$$2=\frac{4}{4}+\frac{4}{4},\ \text{or}\ \frac{4\times 4}{4+4};$$

$$3=\frac{4+4+4}{4},\ \text{or}\ \frac{4\times 4-4}{4};$$

$$4=4+4\times(4-4);$$

$$5=\frac{4\times 4+4}{4};$$

$$6=\frac{4+4}{4}+4;$$

$$7=4+4-\frac{4}{4},\ \text{or}\ \frac{44}{4}-4;$$

$$8=4+4+4-4,\ \text{or}\ 4\times 4-4-4;$$

$$9=4+4+\frac{4}{4};$$

$$10=\frac{44-4}{4}.$$

With Four Fives

There is only one way:

$$\frac{55}{5}+5=16.$$

With Five Nines

The two ways are as follows:

$$9+\frac{99}{99}=10,$$

$$\frac{99}{9}-\frac{9}{9}=10.$$

Those knowing more mathematics may add several other solutions, e. g.

$$\left(9+\frac{9}{9}\right)^{\frac{9}{9}}=10,\ \text{or}\ 9+99^{9-9}=10.$$

Twenty-Four

The two solutions are:

$$22+2=24;\quad 3^3-3=24.$$

Thirty

The three solutions are

$$6\times 6-6=30;\quad 3^3+3=30;\quad 33-3=30.$$

One Thousand

$$888+88+8+8+8=1{,}000.$$

Get Twenty

The crossed out digits are replaced by zeros:

011
000
009

because $11+9=20$.

Cross Out Nine Digits

The problem permits of several solutions. We furnish four:

100	111	011	101
000	030	330	303
005	000	000	000
007	070	770	707
999	900	000	000
1111	1111	1111	1111

In a Mirror

The only figures that are not inverted in a mirror are 1, 0, and 8. Accordingly, the year we seek can only contain these figures. Besides, we know that the year is in the 19th century, hence the first two figures are 18.

It's easily seen now that the year is 1818 because in a mirror we obtain 8181, which is 4 1/2 times more than 1818. In fact,

$$1818 \times 4\,1/2 = 8181.$$

The problem has no other solutions.

Which Year?

In the 20th century there is only one such year, viz. 1961.

Which Numbers?

The answer is simple: 1 and 7. There are no other numbers.

Add and Multiply

There are many such numbers, e. g.

$$3 \times 1 = 3; \qquad 3 + 1 = 4;$$

or

$$10 \times 1 = 10; \quad 10 + 1 = 11.$$

In general, any pair of integers of which one is unity will work.

This is because adding one increases a number and multiplying by one does not change it.

The Same

The numbers are 2 and 2. There are no other integers.

Even Prime Numbers

There is one even prime number – 2. It only divides by itself and 1.

Three Numbers

Multiplying 1, 2, and 3 gives the same as adding them up:

$$1 + 2 + 3 = 6, \quad 1 \times 2 \times 3 = 6.$$

Addition and Multiplication

There are a lot of such pairs. Examples are:

$$3 + 1\ 1/2 = 4\ 1/2, \qquad 3 \times 1\ 1/2 = 4\ 1/2,$$
$$5 + 1\ 1/4 = 6\ 1/4, \qquad 5 \times 1\ 1/4 = 6\ 1/4,$$
$$9 + 1\ 1/8 = 10\ 1/8, \qquad 9 \times 1\ 1/8 = 10\ 1/8,$$
$$11 + 1.1 = 12.1, \qquad 11 \times 1.1 = 12.1$$
$$21 + 1\ 1/20 = 22\ 1/20, \qquad 21 \times 1\ 1/20 = 22\ 1/20,$$
$$101 + 1.01 = 102.01, \qquad 101 \times 1.01 = 102.01, \text{ etc.}$$

Multiplication and Division

There are many correct number pairs. For example,

$$2 \div 1 = 2, \qquad 2 \times 1 = 2,$$
$$7 \div 1 = 7, \qquad 7 \times 1 = 7,$$
$$43 \div 1 = 43, \quad 43 \times 1 = 43.$$

The Two-Digit Number

The number we seek should clearly be a square. As among the two-digit numbers there are only *six* squares, then by trial-and-error method we readily find the unique solution, namely 81:

$$\frac{81}{8+1} = 8 + 1,$$

Ten Times More

The following are the four other pairs of such numbers: 11 and 110; 14 and 35; 15 and 30; 20 and 20.

In fact,

$$11 \times 110 = 1{,}210; \quad 11 + 110 = 121;$$
$$14 \times 35 = 490; \quad 14 + 35 = 49;$$
$$15 \times 30 = 450; \quad 15 + 30 = 45;$$
$$20 \times 20 = 400; \quad 20 + 20 = 40.$$

The problem has no other solutions. Searching for the solutions by trial and error is tiresome and a knowledge of the ABC of algebra would make the process easier and enable us not only to find all the solutions, but also to make sure that the problem doesn't have more than five solutions.

Two Digits

Many may believe that the number is 10. No, it's 1, expressed as follows:

$$\frac{1}{1},\ \frac{2}{2},\ \frac{3}{3},\ \frac{4}{4},\ \text{etc., up to } \frac{9}{9}.$$

Those who know some more mathematics may add to these answers a number of others:

$$1^0,\ 2^0,\ 3^0,\ 4^0,\ \text{etc., up to } 9^0,$$

because any number to the zeroth power is unity*.

The Largest Number

The commonest answer is 1,111. But the number is far from being the largest, 11^{11} is much more, 250,000,000 times more.

Unusual Fractions

There are several solutions. One of them is

$$\frac{1}{3}=\frac{5{,}823}{17{,}469};\quad \frac{1}{4}=\frac{3{,}942}{15{,}768};$$

$$\frac{1}{5}=\frac{2{,}697}{13{,}485};\quad \frac{1}{6}=\frac{2{,}943}{17{,}658};$$

$$\frac{1}{7}=\frac{2{,}394}{16{,}758};\quad \frac{1}{8}=\frac{3{,}187}{25{,}496};\quad \frac{1}{9}=\frac{6{,}381}{57{,}429}.$$

There are many versions, especially the ones for 1/8 of which there are more than 40.

What Was the Multiplier?

We argue as follows. The figure 6 is the result of the addition of two figures, one of which may be either 0 or 5. But, if the lower one is 0, then the upper is 6. But, may the upper figure be 6? Testing shows that whatever the second figure of the multiplier, 6 cannot be in the last but one place of the first partial product. Accordingly, the lower figure of the last but one column must be 5 and above it 1.

---

* But the solutions $\frac{0}{0}$ or 0^0 would be wrong because they are meaningless.

We can now easily restore some of the missing figures:

$$\begin{array}{r} 235 \\ \times\ \ {**} \\ \hline {**1*} \\ +\ {***5}\ \ \\ \hline {**56*} \end{array}$$

The last figure in the multiplier must be more than 4, otherwise the first partial product will not consist of four figures. It cannot be 5 as we won't then get 1 in the last but one place. Let's try 6 and this works out all right. We have:

$$\begin{array}{r} 235 \\ \times\ \ {*6} \\ \hline 1\,410 \\ +\ {***5}\ \ \\ \hline {**560} \end{array}$$

Reasoning further along the same lines we find that the multiplier is 96.

Missing Figures

The missing figures are restored one after another if we use the following argument.

For convenience we assign numbers to the lines

$$\begin{array}{rl} {*1*} & \text{I} \\ \times\ {3*2} & \text{II} \\ \hline {*3*} & \text{III} \\ {3*2*}\ \ & \text{IV} \\ +\ {*2*5}\ \ \ \ & \text{V} \\ \hline {1*8*30} & \text{VI} \end{array}$$

It's easily seen that the last asterisk in line III is 0, which follows from the fact that 0 is at the end of line VI.

Now we determine the value of the last asterisk in line I. This figure must give a number ending in zero when multiplied by 2, and a number ending in 5 (line V) when multiplied by 3. There is only one such figure – 5.

We can now guess the meaning of the asterisk in line II. It's 8 because only when multiplied by 8 the number 15 gives a result ending in 20 (IV).

Eventually, the meaning of the first asterisk in line I becomes clear: it is 4, because only 4, when multiplied by 8, gives a result beginning with 3 (line IV).

The remaining figures now present no problem. Suffice it to multiply together the

numbers of the first two lines that have now been completely determined. We'll end up with the multiplication:

$$\begin{array}{r} 415 \\ \times\ 382 \\ \hline 830 \\ 3320 \\ 1245 \\ \hline 158530 \end{array}$$

What Numbers?

Arguing as above we uncover the meaning of the asterisks in this case, too. We get

$$\begin{array}{r} \times\ 325 \\ 147 \\ \hline 2275 \\ +\ 1300 \\ 325 \\ \hline 47775 \end{array}$$

Strange Multiplication Cases

The patient reader can find the following nine cases where the multiplication calculations meet the question's demands. They are:

$$\begin{array}{ll} 12 \times 483 = 5{,}796, & 48 \times 159 = 7{,}632, \\ 42 \times 138 = 5{,}796, & 28 \times 157 = 4{,}396, \\ 18 \times 297 = 5{,}346, & 4 \times 1738 = 6{,}952, \\ 27 \times 198 = 5{,}346, & 4 \times 1963 = 7{,}852. \\ 39 \times 186 = 7{,}254, & \end{array}$$

Mysterious Division

For convenience, we'll number the lines in the arrangement thus:

$$\begin{array}{lr} & . \; . \; . \; 7 \; . \\ & . \; . \; . \;)\overline{. \; . \; . \; . \; . \; . \; . \; . \; .} \\ \text{I} & . \; . \; . \; . \\ \text{II} & \overline{. \; . \; . \; .} \\ \text{III} & . \; . \; . \\ \text{IV} & \overline{. \; . \; .} \\ \text{V} & . \; . \; . \\ \text{VI} & \overline{. \; . \; . \; .} \\ \text{VII} & . \; . \; . \; . \\ & \overline{} \end{array}$$

Looking at line II we conclude that the second figure of the quotient is 0 as it was necessary to borrow two figures from the dividend. Denote the divisor by x. Lines IV and V indicate that $7x$ (the product of the last but one digit in the quotient and the divisor), when it was subtracted from a number not larger than 999, gave a difference not less than 100. Clearly, $7x$ cannot exceed $999 - 100$, i.e. 899, hence x is not larger than 128. Further, we see that the number in line III is more than 900, otherwise, it wouldn't give a two-digit difference when subtracted from a four-digit number. Thus the third digit of the quotient should be $900 \div 128$, i.e. more than 7.03. Accordingly, it is either 8 or 9. The numbers in I and VII are four digits long hence the third digit in the quotient is 8 and the extreme left and right digits are 9.

This actually completes the problem as the desired result (the quotient) has now been found. It is 90,879.

It's not necessary to go on with the argument to find the dividend and divisor, as we wanted the *quotient* only. The problem doesn't require us to decipher the whole of the arrangement. Besides, there are 11 pairs of numbers that satisfy the given arrangement of points and give 7 in the fourth place of the quotient, viz.

$$\left.\begin{array}{r} 10{,}360{,}206 \div 114 \\ 10{,}451{,}085 \div 115 \\ 10{,}541{,}964 \div 116 \\ 10{,}632{,}843 \div 117 \\ 10{,}723{,}722 \div 118 \\ 10{,}814{,}601 \div 119 \\ 10{,}905{,}480 \div 120 \\ 10{,}996{,}359 \div 121 \\ 11{,}087{,}238 \div 122 \\ 11{,}178{,}117 \div 123 \\ 11{,}268{,}996 \div 124 \end{array}\right\} = 90{,}879.$$

Another Division Problem

The answer is:

$$\begin{array}{r} 162 \\ 325)\overline{52650} \\ -\ 325 \\ \hline 2015 \\ -\ 1950 \\ \hline 650 \\ -\ 650 \\ \hline \end{array}$$

Division by 11

To solve the problem requires the knowledge of the criterion of divisibility by 11. A number is divisible by 11 if the difference between the sum of the even digits and the sum of the odd digits either is divisible by 11 or is zero.

By way of example, test the number 23,658,904.

The sum of the digits in the even places is

$$3+5+9+4=21$$

and the sum of the digits in the odd places is

$$2+6+8+0=16.$$

Their difference (we subtract from the largest) is

$$21-16=5.$$

The difference (5) doesn't divide by 11 nor is it zero, hence the number under consideration doesn't divide by 11.

Take another number – 7,344,535:

$$3+4+3=10;$$
$$7+4+5+5=21;$$
$$21-10=11.$$

As 11 divides by 11, the number in question is a multiple of 11.

Now we can easily work out the order in which we should write the nine digits so as to arrive at a number that is a multiple of 11 and meets the conditions of the problem.

Yet another example is 352,049,786. Let's test it:

$$3+2+4+7+6=22;$$
$$5+0+9+8=22.$$

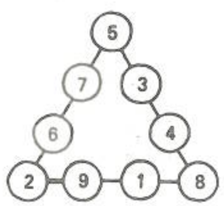
The difference is $22-22=0$, hence the number divides by 11.

Of these numbers the largest is 987,652,413, and the smallest is 102,347,586.

Triangle of Figures

The solution is shown in Fig. 235. The figures in the middle of each line can be interchanged to obtain further solutions.

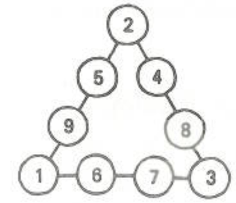
Figure 235



Another Triangle

Again, the solution is given in Fig. 236. Also, the figures in the middle of each line can be interchanged to obtain further solutions.

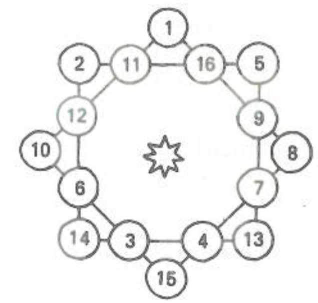
Figure 236



Eight-Point-Star

The solution is in Fig. 237.

Figure 237



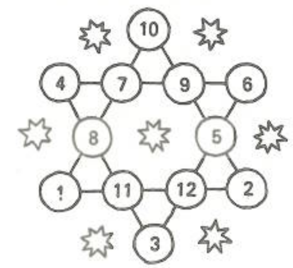
Magic Star

To make our life easier we'll start off with the following considerations.

The numbers at the points must add up to 26, but the sum of all the numbers in the star is 78. Accordingly, the numbers on the internal hexagon is $78-26=52$.

Now let's look at one of the large triangles. On each side we have 26, and for all the three we get $26\times 3=78$, with each number at the vertex entering twice. But as the sum of the three internal pairs (i. e. of the internal hexagon) is known to be 52, then the double sum of the numbers at the vertices of each triangle is $78-52=26$, the single sum is thus 13.

Figure 238



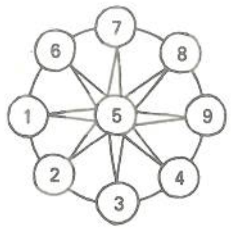
The field of search has now been narrowed markedly. We know, for instance, that neither 12 nor 11 can be at the star points (why?). Hence the tests might be begun with 10. In that case we immediately determine the two numbers that must occupy the remaining vertices of the triangle: 1 and 2.

Moving on after this manner we eventually arrive at the desired arrangement. It's shown in Fig. 238.

Wheel of Figures

The solution is given in Fig. 239.

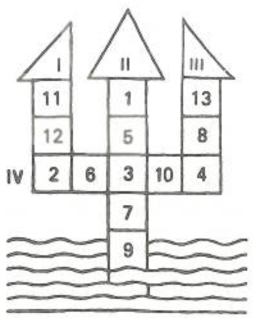
Figure 239



Trident

The following is the desired arrangement (Fig. 240). The sum of the numbers in each of the four lines is 25.

Figure 240



272–273

Merry Arithmetic

Simple Multiplication

If you don't remember the multiplication table properly and have difficulty in multiplying by 9, then your own fingers might be of help.

Place both hands on a table, your 10 fingers will be your computer.

Suppose you want to multiply 4 by 9.

Your *fourth* finger gives the answer: on the left of it there are three fingers, on the right, six. So you read: 36, hence $4 \times 9 = 36$.

Further examples: how much is 7×9?

Your *seventh* finger has six fingers on its left and three on its right. The answer: 63.

What is 9×9? On the left of the *ninth* finger there are eight fingers, on the right, one. The answer is 81.

This living computer will remind you, for example, that 6×9 is 54, not 56.

Cats and Mats

Once some cats
found some mats.
But if each mat
had but one cat
there's be a cat
without a mat.
Should each mat
now have two cats
there'd be a mat
without a cat.
How many cats
and how many mats?

Figure 241

Sisters and Brothers

I have an equal number of sisters and brothers. But my sister has two times more brothers than sisters. How many are we?

How Many Children?

I have six sons. Each son has a sister. How many children have I?

18—975

Breakfast

Two fathers and two sons breakfasted on three eggs, each having a whole egg. How do you account for it?

Three Quarters of a Man

A team leader was asked how many people there were in his team. He answered in a rather involved way: "Not many: three quarters of us plus three quarters of a man, that's all."
Could you compute the number of people in his team?

How Old Are They?

"Gran'pa, please tell me how old your son is?"
"His is as many weeks old as my grandson is days old."
"And your grandson, how old is he?"
"His age is as many months old, as I am years old."
"How old are you, then?"
"Together the three of us are 100 years old. Now guess how old each of us is."

Figure 242

Who is Older?

In two years my boy will be twice as old as he was two years ago. And my girl in three years will be three times as old as she was three years ago.
Who is older, my boy or my girl?

The Age of the Son

Now my son is a third my age. But five years ago he was a quarter my age.
How old is he?

His Age

A witty person was asked about his age. The answer was: "If you take thrice my age in three years and subtract thrice my age three years back, then you'll have my age precisely."
How old is he now?

Three Daughters and Two Sons

An uncle visited his two nieces and three nephews.
The first to greet him were little Johnny and his

sister Anne. The little chap proudly declared that he was twice as old as his sister. Then Nadine ran out to meet the uncle and her father said that both girls together were twice as old as the boy.
When Alexis came from school the father reckoned that both boys together are twice as old as both girls.
The latest to come was Libby, she saw the guest and exclaimed happily: "Uncle, you just arrived on my birthday. Today I'm 21."
"And you know what?" the father added, "It just occurred to me that my three daughters together are two times older than my sons."
How old was each son and daughter?

Two Trade Unionists

I remember hearing a conversation between two trade unionists:
"So you've been a trade union member twice as long as me?"
"Yes, exactly twice."
"But last we met you said that you'd been a member three times longer."
"Two years back? Then that was so, but now I've only been twice as long as you."
How many years has each of them been a trade union member?

How Many Games

Three persons were playing draughts. They had played three games. How many games had each of them played?

Figure 243

Snail

A snail was climbing up a 15-m tree. Each day it climbed 5 metres, but each night as it slept it slid back down 4 metres.
How many days did it take the snail to reach the summit?

To the Town

A farmer was travelling to a town. The first half of the route he went by train, 15 times faster than if he had gone by foot. However, the second half of the route he

rode on an oxcart, at half the speed of a walker.
How much time did he save as compared with walking all the way to the town?

Figure 244

To the Village

From a town to a village the road first goes uphill for 8 kilometres, then 24 kilometres downhill. John went there on a bicycle and the non-stop journey there took him 2 hours 50 minutes. He also bicycled back, again non-stop, and spent 4 hours 30 minutes.
How fast could John ride uphill and how fast downhill?

Two Schoolboys

A schoolboy said to his mate, "Give me an apple, and I'll have twice as many as you."
"That would be unfair," replied the mate, "You give me one then we'll even."
How many apples had each initially?

Binding

Here is an insidious problem. A bound book costs 2 roubles 50 kopecks. The book is 2 roubles more expensive than the binding.
How much does the binding cost?

The Cost of Buckle

A belt with a buckle costs 68 kopecks. The belt costs 60 kopecks more than the buckle.
How much does the buckle cost?

276–277

Figure 245

Casks of Honey

In store there were seven casks brim full of honey, seven half-full ones, and seven empty casks, all belonging to three firms that wanted to divide both the honey and casks into equal shares.
How can they divide without transferring the honey from one cask into another?
If you think that various ways of doing so are available, indicate all those which occur to you.

Postage Stamps

A man bought a 5 roubles worth of postage stamps of three kinds: 50-kopeck stamps, 10-kopeck stamps and 1-kopeck stamps – 100 pieces, all told.
How many stamps of each kind did he buy?

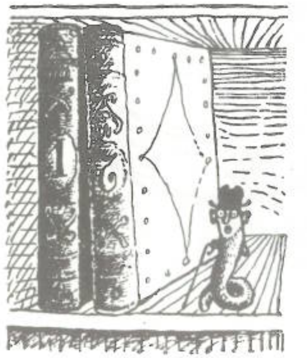
How Many Coins?

A customer got his change of 4 roubles 65 kopecks, all in 10 and 1-kopeck coins – in all, 42 coins.
How many coins of each worth was he given?
How many solutions has the problem?

Socks and Gloves

One box contains 10 pairs of brown and 10 pairs of black socks, another contains 10 pairs of brown and 10 pairs of black gloves.
How many single socks and gloves is it sufficient to take from each box to have any one pair of socks and gloves?

Figure 246



Book Worm

There are insects that feed on books eating their way through leaves and thus ruining the bulk of the book. One such book worm has eaten through from the first page of the first volume to the last page of the second volume on a bookshelf, as shown in the accompanying figure.

There are 800 pages in each volume so, how many pages has the worm ruined?

The problem is not difficult but it has a catch.

Spiders and Beetles

A boy collects spiders and beetles in a box and he now has 8 insects in all. There are 54 legs in the box in total.

How many spiders and how many beetles are there?

Seven Friends

A man has seven friends. The first visits him every night, the second every other night, the third every third night, the fourth every fourth night, etc., through to the seventh friend who comes every seventh night.

Figure 247

Is it often the case that all the seven friends get together there on the same night?

The Same Problem Continued

On those nights when the seven friends get together the host treats them to some wine, and all touch glasses in pairs.

How many times do the glasses ring as they touch one another?

278–279 **Answers**

Cats and Mats

This problem is solved in this way. Ask the question: How many more cats would be needed to occupy all the places on the mats the second time than to get the situation we had the first time? We can easily figure that out: in the first case one cat was left without a place, whilst in the second case all the cats were seated and there were even places for two more. Hence for all the mats to have been occupied in the second case there should have been $1+2$, i.e. three, more cats than there were in the first case. But then each mat would have one more cat. Clearly there were three mats all in all. Now we seat one cat on each mat and add one more to obtain the number of cats, four.

Thus, the answer is four cats and three mats.

Sisters and Brothers

Seven: four brothers and three sisters. Each brother has three brothers and three sisters; each sister has four brothers and two sisters.

How Many Children?

Seven: six sons and one daughter. (The common answer is twelve, but each son would then have six sisters, not one.)

Breakfast

The situation is very simple. Seating at the table were three, not four people: a grandfather, his son, and his grandson. The grandfather and his son are fathers, and the son and grandson are sons.

Three Quarters of a Man

Note that the three-quarters-of-a-man is the last quarter of the team. So the whole of the team is four times the three quarters of a man, i.e. three. In consequence, the team consisted of three men.

How Old Are They?

We know thus that the son is 7 times, and the grandfather 12 times, older than the grandson. If the grandson were one year old, the son would be seven, and the grandfather 12. Adding the three together gives 20, which is exactly a fifth of the real figure. It follows that in actuality the grandson is five, the son 35, and the grandfather 60.

Check: $5+35+60=100$.

Who is Older?

Neither is: they are twins and each of them at the time is six.

The calculation is simple: two years hence the boy will be four years older than he was two years ago, and twice as old as he was then. Hence he was four years old two years ago. Accordingly, now he is $4+2=6$ years old.

The girl's age is the same.

The Age of the Son

If now a son is one third the age of his father, then the father is older in years by twice the son's age. Five years ago the father was, of course, also twice his son's *present* age older than his son was *then*. On the other hand, since at that time the father was four times older than the son, he was older by the triple then age of the son. Consequently, the double present age of the son equals the triple then age of the son. Thus, the son is now 1 1/2 times older than he was five years ago. It follows that five years is a half of the son's previous age, and hence five years ago the son was 10, and now he is 15 years old.

Thus, the son is now 15 years and the father 45. Checking this, five years ago the father was 40 and the son was 10, i.e. a quarter the father's present age.

His Age

Arithmetically the problem has a rather involved solution, but the situation simplifies considerably if we draw on the services of algebra and set up an equation. We'll denote the number of years we're after by x. The wit's age in three years will then be $x+3$, and three years ago, $x-3$. We'll thus have

$$3(x+3)-3(x-3)=x$$

The equation gives $x=18$. So the witty person's age is at present 18 years.

Let's check. In three years time he'll be 21 and three years ago he was 15. The difference is

$$3\times 21-3\times 15=63-45=18,$$

which equals the present age of the man.

Three Daughters and Two Sons

We know that Johnny is twice as old as Anne, and Nadine and Anne together are twice as old as Johnny. Accordingly, the sum of the ages of Nadine and Anne is four times more than Anne's age. It follows directly that *Nadine is three times as old as Anne.*

We also know that the ages of Alexis and Johnny combined are twice the combined age of Nadine and Anne. But Johnny's age is double Anne's age, and the ages of Nadine and Anne put together give the fourfold age of Anne. Accordingly, Alexis's age plus the double age of Anne are equal to the eightfold age of Anne. Thus, *Alexis is six times older than Anne.*

Lastly, as stated, the ages of Libby, Nadine, and Anne combined equal the sum of the ages of Johnny and Alexis.

For convenience, we'll compile the table:

Libby	21 years
Nadine	three times older than Anne
Johnny	two times older than Anne
Alexis	six times older than Anne

We can now say that 21 years plus the trebled Anne's age plus an Anne's age are equal to the fourfold Anne's age plus the twelvefold Anne's age.

Or, 21 years plus the fourfold age of Anne are equal to the sixteen-fold age of Anne.

In consequence, 21 years are equal to the twelve-fold age of Anne and Anne is thus $21/12=1\ 3/4$ years.

We can now easily determine that Johnny is 3 1/2, Nadine 5 1/4, and Alexis 10 1/2.

Two Trade Unionists

One has been with the trade union for eight years, the other for four years. Two years ago the first one had been with the trade union six years, the second two years, i.e. three times as long. The problem is readily solvable using an equation.

How Many Games?

The commonest answer is that each played once, ignoring the fact that three (and any odd number in general) players cannot each play only once for who then did the third player play? It takes two partners to have a game. If we denote the players by A, B, and C, the three games will be

A with B
A with C
B with C

We see that each played twice, not once:

A with B and C
B with A and C
C with A and B

So the answer is: each of the three played twice, although three games had been played in all.

Snail

In 11 days. During the first 10 days the snail had crawled up 10 metres, 1 metre a day. The next one day it climbed the remaining 5 metres, i.e. it reached the summit. (The common answer is 15 days.)

To the Town

The farmer lost time, he did not save it. The second half of the route took as much time as he would have spent travelling to the town on foot. He thus could not save

time, he was bound to lose time.

His loss amounted to 1/15 of the time required to cover half a route on foot.

To the Village

The solution of the problem follows from the following calculation:

24 kilometres uphill and 8 kilometres downhill took 4 hours 30 minutes;
8 kilometres uphill and 24 kilometres downhill took 2 hours 50 minutes.

If we multiply out the second line by three, we obtain: 24 kilometres uphill and 72 kilometres downhill takes 8 hours 30 minutes.

A bit of algebra gives that the bicyclist covers 64 kilometres downhill in 4 hours. Hence, downhill he travelled at $64/4 = 16$ kilometres an hour.

We'll find in much the same way that he travelled uphill at 6 kilometres an hour. Testing the answer is an easy exercise.

Two Schoolboys

Transferring an apple balances out the number of apples, thus suggesting that one had two apples more than the other. If we subtract one apple from the smaller number and add it to the larger number, then the difference will increase by two and become four. We know that then the larger number will be equal to double the smaller one. Accordingly, the smaller number is 4 and the larger 8.

Before the transfer one schoolboy had $8 - 1 = 7$ apples, and the other $4 + 1 = 5$ apples.

Let's check whether or not the numbers become equal if we subtract an apple from the larger and add it to the smaller:

$$7 - 1 = 6; \quad 5 + 1 = 6.$$

Thus, one schoolboy has 7 apples and the other 5 apples.

Binding

The off-the-cuff answer is usually: the binding costs 50 kopecks. But then the book would cost 2 roubles, i.e. it would be only 1 rouble 50 kopecks more expensive than the binding.

The true answer is: the binding costs 25 kopecks, the book 2 roubles 25 kopecks with the result that the book costs 2 roubles more than the binding.

The Cost of Buckle

Perhaps you've decided that the buckle costs 8 kopecks. If so, you're mistaken, as the belt would then cost only 52 kopecks more than the buckle, not 60 kopecks more.

The correct answer is the buckle costs 4 kopecks, then the belt costs $68 - 4 = 64$ kopecks, i.e. 60 kopecks more than buckle.

Casks of Honey

The problem becomes a fairly easy exercise if we note that in the 21 casks bought there were $7 + 3\ 1/2 = 10\ 1/2$ caskfuls of honey. Each firm must then get 3 1/2 caskfuls of honey and seven casks.

We could divide them in the following two ways:

First way

1st firm	3 full, 1 half-full, 3 empty	Total: 3 1/2 caskfuls of honey
2nd firm	2 full, 3 half-full, 2 empty	Total: 3 1/2 caskfuls of honey
3rd firm	2 full, 3 half-full, 2 empty	Total: 3 1/2 caskfuls of honey

Second way

1st firm	3 full, 1 half-full, 3 empty	Total: 3 1/2 caskfuls of honey
2nd firm	3 full, 1 half-full, 3 empty	Total: 3 1/2 caskfuls of honey
3rd firm	1 full, 5 half-full, 1 empty	Total: 3 1/2 caskfuls of honey

Postage Stamps

There is only one answer: the customer bought

1 × 50-kopeck stamp
39 × 10-kopeck stamps
60 × 1-kopeck stamps

Really, there were $1 + 39 + 60 = 100$ pieces all in all, and the total cost was $50 + 390 + 60 = 500$ kopecks.

How Many Coins

The problem has four solutions:

	I	II	III	IV
Roubles	1	2	3	4
10-kopeck pieces	36	25	14	3
1-kopeck pieces	5	15	25	35
Total	42	42	42	42

Socks and Gloves

Three socks will be enough, as two of them are bound to be of the same colour. But with the gloves the situation is not that simple. These differ from one another not only in their colour, but also in that half of them are right-handed and half left-handed.

Here 21 gloves will be sufficient. With a smaller number it might appear that all of them would be right-handed, or left-handed for that matter (10 pairs of brown left and 10 pairs of black left).

Book Worm

The common answer is that the worm went through 800 + 800 pages plus two covers. But this is not so. Stand two books side by side as shown in Fig. 246, and see how many pages there are between the first page of the first book and the last page of the second book. You'll discover that there is nothing but *two covers* between them.

Thus the book worm had only destroyed the two covers without touching their leaves.

Spiders and Beetles

To tackle the problem we should first of all remember from your nature lessons how many legs beetles have and how many spiders have. In fact, the numbers are six and eight, respectively. With this in view we suppose that the box only contains beetles, either all told. Their legs would then add up to $6 \times 8 = 48$, six fewer than given in the problem. Let's now try and replace one beetle with one spider. This will increase the number of legs by two because the spider has two more legs.

Clearly three such replacements will bring the total number of legs in the box up to the desired 54. But in that case there will only be five beetles, the rest being spiders.

The box thus contained five beetles and three spiders.

Let's check: five beetles give 30 legs, the three spiders 24, the total being $30 + 24 = 54$, as required.

The problem can also be solved in another way. We may start off assuming that the box only contains spiders, eight of them. The number of legs will then be $8 \times 8 = 64$, 10 legs more than what was stated. Replacing one spider with one beetle reduces the number of legs by two. We'll have to make five such substitutions in order that to arrive at 54. Put another way, we'll retain only three spiders, with the rest being replaced by beetles.

Seven Friends

You should easily twig that the seven can only come together in a number of days that divides by 2, 3, 4, 5, 6, and 7. The smallest such number is 420.

Consequently, the friends will only get together once every 420 days.

The Same Problem Continued

All of those present (the host and his seven friends) touch their glasses with those of the remaining seven. The number of combinations 2 at a time totals $8 \times 7 = 56$. But this counts each pair twice (e.g. the third guest with the fifth and the fifth with the third are considered as different pairs). Hence the glasses ring $56/2 = 28$ times.

Can You Count?

The question might seem to be even insulting for a person more than three years old. Who can't? To utter the words "one", "two", "three" etc., in succession doesn't take much genius. And still I'm sure that you're not always equal to this seemingly simple task. Everything depends on what is to be counted. It's no problem to count, say, the nails in a box. But suppose the box contains screws as well as nails. It's required to find out how many of each there are. How could you go about it? Would you separate the heap into nails and screws and then count them?

This sort of a problem comes up for a housewife when she has to count the washing for laundry. She first sorts the washing out: shirts go to one heap, towels to another, etc. And it's only after she had done this tedious job that she begins to count the items in each heap.

That's what is called not knowing how to count! This way of handling dissimilar objects is utterly inconvenient, labour consuming and occasionally even completely impossible. It's all well and good if you have to count nails or washing: they are fairly easily sorted out into separate heaps. But try to place yourself into a forester's shoes who wants to count all the pines, spruces, birches and aspens in the same hectare. He cannot sort out all the trees according to their species. Well, should you first count all the pines, then all the spruses, then all the birches and then all the aspens? Would you go all round the whole area four times?

Couldn't the job be done in a simpler way, perhaps by a *single* tour of the area? Yes, there is such a way and it has been used since time immemorial by foresters. I'll illustrate its principle essence referring to our nails and screws.

To count the nails and screw at one go, without sorting them out, get a pencil and a sheet of paper marked out as shown below:

Nails	Screws

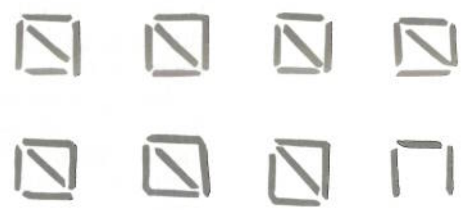
Now begin counting. Take out of the box whatever comes first. If it's a nail you make a dash in "Nails", if it's a screw, mark a dash in "Screws". Take out a second piece and repeat the procedure, then a third, a fourth, etc., until the box is finished. In the end, in the "Nails" column you'll have as many dashes as there are nails in the box, and in the "Screws" column as many dashes as there are screws. It only remains to count up the dashes.

We could simplify the counting procedure. To do so we should not just dispose our dashes one under another, but group them as shown in Fig. 248 with five dashes in each group.

Figure 248

It's convenient to arrange these squares into pairs, i.e. after the first 10 dashes begin a new row, then the third, and so on. The arrangement will be approximately as shown in Fig. 249.

Figure 249



It's easy to count the dashes thus arranged: you see at once that here we have three complete tens, one five plus three dashes, i.e. $30 + 5 + 3 = 38$.

Other figures are also possible. So they often use figures in which each complete square means 10 (Fig. 250).

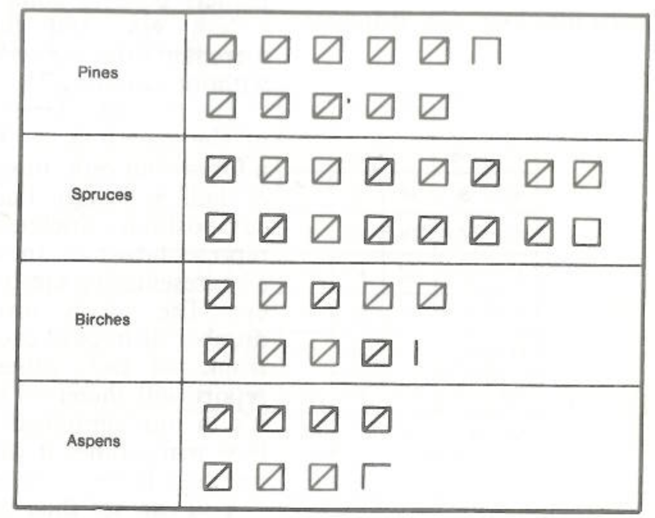
Figure 250

When counting trees of different species in a forest area you should proceed in much the same way, only now you'll have four columns or lines, not two, lines being more convenient here. So you should begin with a sheet like this:

286–287

Pines	
Spruces	
Birches	
Aspens	

Figure 251



You'll end up with about what is shown in Fig. 251.

It's a straightforward exercise here to work out the totals:

Pines 53 Birches 46
Spruces 79 Aspens 37

The same procedure is used by a medical worker who counts under the microscope red and white blood corpuscles in a blood specimen.

Should you need to count the plants of various species in a meadow you'll now know how to handle the job and do it in the shortest time possible. First write down the names of the plants found and allot a line to each, leaving several lines for other plants you may come across. Start off, for example, with an arrangement like the one in Fig. 251, and proceed as if it were the forest survey.

Why Count Trees in a Forest?

Why is it actually necessary to count the trees in a forest? Some town dwellers even think that it's impossible. In the novel *Anna Karenina* by L. N. Tolstoy an agriculture expert, Levin, asked his naive relative who wanted to sell his forest:

"Have you counted the trees?"

"How can one count trees?" he answers in bewilderment. "Count sands or rays from distant planets perhaps some lofty mind could..."

"Oh yes? But the lofty mind of Ryabinin (a merchant—*the author*) can. And no peasant will buy, without counting."

The trees in a forest are counted to assess the volume of the wood in it. They count not all the trees in a forest, but only those in a definite area, say a quarter or half a hectare that is so chosen that the density, composition, thickness and height of its trees were representative of those in the entire forest. Selecting a representative sample area requires, of course, a good eye. The survey involves determining not only the number of trees of each species, but also the number of trunks of each gauge (say, 30 cm, 35 cm, etc.). The report will therefore include more than four entries as it's in our simplified example. Now you can imagine how many times it would be necessary to go all round the area if the trees were counted in some other way.

You can see thus that counting is only a simple and easy business when handling similar objects. When handling dissimilar things the just described procedures are needed.

288–289 Fast Reckoning

(Simple tricks of mental arithmetic)

The following is a collection of simple and easily grasped tricks to speed up your mental arithmetic. If you want to master them you should realize that to be used fully they need to be approached conscientiously, not mechanically. But it pays to master them as they'll enable you to do calculations in your head without error, as with written calculations.

Multiplication by Simple Number

1. When multiplying by a simple number (for example, 27×8) don't begin by multiplying the ones, as you would do in a written operation. First, multiply the tens of the multiplicand ($20 \times 8 = 160$), then the ones ($7 \times 8 = 56$) and add up the results ($160 + 56 = 216$).

Further examples:

$$34 \times 7 = 30 \times 7 + 4 \times 7 = 210 + 28 = 238,$$
$$47 \times 6 = 40 \times 6 + 7 \times 6 = 240 + 42 = 282.$$

2. It would also pay to remember the multiplication table up to 19×9:

	2	3	4	5	6	7	8	9
11	22	33	44	55	66	77	88	99
12	24	36	48	60	72	84	96	108
13	26	39	52	65	78	91	104	117
14	28	42	56	70	84	98	112	126
15	30	45	60	75	90	105	120	135
16	32	48	64	80	96	112	128	144
17	34	51	68	85	102	119	136	153
18	36	54	72	90	108	126	144	162
19	38	57	76	95	114	133	152	171

Knowing the table you could multiply say 147×8 in your head as follows:

$$147 \times 8 = 140 \times 8 + 7 \times 8 = 1120 + 56 = 1176.$$

3. If one of the numbers to be multiplied together is representable in the form of two factors, it may be

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convenient to multiply in succession by three factors. For example:

$225 \times 6 = 225 \times 2 \times 3 = 450 \times 3 = 1350.$

Multiplication by Two-Digit Number

4. This kind of multiplication can be made simpler by reducing it to the conventional multiplication by a simple number.

When the multiplicand is simple, it and the multiplier are interchanged and then the procedure of item 1 can be followed. For example:

$6 \times 28 = 28 \times 6 = 120 + 48 = 168.$

5. When both multipliers are two-digit, one of them is mentally broken down into tens and ones. For example:

$29 \times 12 = 29 \times 10 + 29 \times 2 = 290 + 58 = 348,$
$41 \times 16 = 41 \times 10 + 41 \times 6 = 410 + 246 = 656,$
(or $41 \times 16 = 16 \times 41 = 16 \times 40 + 16 = 640 + 16 = 656$).

It's more convenient to break the multiplier down into tens and ones and so get smaller figures.

6. If the multiplicand or multiplier are more readily representable in the head in the form of two simple factors (e.g. $14 = 2 \times 7$), then this trick is used to reduce one of the initial factors while increasing the other accordingly (cf. item 3). For example:

$45 \times 14 = 90 \times 7 = 630.$

Multiplication and Division by 4 and 8

7. To multiply in your head a number by 4, you double the number twice. For example,

$112 \times 4 = 224 \times 2 = 448,$
$335 \times 4 = 670 \times 2 = 1{,}340.$

8. When multiplying by 8, the number is doubled three times. For example,

$217 \times 8 = 434 \times 4 = 868 \times 2 = 1{,}736.$

Another way of multiplying mentally by 8 is to multiply the multiplicand by ten and subtract double the multiplicand (that is, multiply by $10 - 2$ in the long run):

$217 \times 8 = 2{,}170 - 434 = 1{,}736.$

Or more convenient still:

$217 \times 8 = 200 \times 8 + 17 \times 8 = 1{,}600 + 136 = 1{,}736.$

9. For a number to be mentally divided by 4, the number is halved twice. For example,

$76 \div 4 = 38 \div 2 = 19,$
$236 \div 4 = 118 \div 2 = 59.$

10. To divide mentally by 8, the number is halved three times. For example,

$464 \div 8 = 232 \div 4 = 116 \div 2 = 58,$
$516 \div 8 = 258 \div 4 = 129 \div 2 = 64\ 1/2.$

Multiplication by 5 and 25

11. Multiplying by 5 is actually multiplying by 10/2. Thus a zero is ascribed to the number and the result is divided by two. For example,

$74 \times 5 = 740 \div 2 = 370.$
$243 \times 5 = 2{,}430 \div 2 = 1{,}215.$

If our number is even, it's more convenient to halve it first and then add the zero. For example,

$$74 \times 5 = \frac{74}{2} \times 10 = 370.$$

12. In the case of 25, a number is multiplied by 100/4, i.e. if the number is divisible by 4, it is divided by 4 first and two zeros are then ascribed to the result. For example,

$$72 \times 25 = \frac{72}{4} \times 100 = 1{,}800.$$

But if the division yields a remainder, then if it's 1 we add 25 to the quotient, if 2 we add 50, and if 3 we add 75. This follows from the fact that $100 \div 4 = 25$, $200 \div 4 = 50$, and $300 \div 4 = 75$.

Multiplication by 1 1/2, 1 1/4, 2 1/2, 3/4

13. When multiplying by 1 1/2, add to the multiplicand its half. For example,

$34 \times 1\ 1/2 = 34 + 17 = 51.$
$23 \times 1\ 1/2 = 23 + 11\ 1/2 = 34\ 1/2$ (or 34.5).

14. When multiplying by 1 1/4, add to the multiplicand its quarter. For example,

$48 \times 1\,1/4 = 48 + 12 = 60.$

$58 \times 1\,1/4 = 58 + 14\,1/2 = 72\,1/2$ (or 72.5).

15. To multiply by 2 1/2, add to the doubled number its half. For example,

$18 \times 2\,1/2 = 36 + 9 = 45.$

$39 \times 2\,1/2 = 78 + 19\,1/2 = 97\,1/2$ (or 97.5).

Another technique consists in multiplying by 5 and dividing by two:

$18 \times 2\,1/2 = 90 \div 2 = 45.$

16. To multiply by 3/4 (that is, to find 3/4 of a number), the number is multiplied by 1 1/2 and divided by two. For example,

$$30 \times 3/4 = \frac{30 + 15}{2} = 22\,1/2 \text{ (or 22.5).}$$

Another form of the technique consists in subtracting from the multiplicand its quarter or adding to a half of the multiplicand a half of the half of the multiplicand.

Multiplication by 15, 125, 75

17. Multiplication by 15 is replaced by multiplying by 10 and then by 1 1/2 (because $10 \times 1\,1/2) = 15$). For example,

$18 \times 15 = 18 \times 1\,1/2 \times 10 = 270.$

$45 \times 15 = 450 + 225 = 675.$

18. Multiplication by 125 is replaced by multiplying by 100 and by 1 1/4 (because $100 \times 1\,1/4 = 125$). For example,

$26 \times 125 = 26 \times 100 \times 1\,1/4 = 2{,}600 + 650 = 3{,}250.$

$$47 \times 125 = 47 \times 100 \times 1\,1/4 = 470 + \frac{4{,}700}{4} = 4700 +$$
$$+ 1{,}175 = 5{,}875.$$

19. Multiplication by 75 is replaced by multiplying by 100 and by 3/4 (because $100 \times 3/4 = 75$). For example,

$$18 \times 75 = 18 \times 100 \times 3/4 = 1800 \times 3/4 = \frac{1800 + 900}{2} =$$
$$= 1{,}350.$$

Note: Some of the above examples can be conveniently handled using the technique of item 6:

$18 \times 15 = 90 \times 3 = 270.$

$26 \times 125 = 130 \times 25 = 3{,}250.$

Multiplication by 9 and 11

20. When multiplying by 9, add a zero to the number and subtract the multiplicand from the result. For example,

$62 \times 9 = 620 - 62 = 600 - 42 = 558.$

$73 \times 9 = 730 - 73 = 700 - 43 = 657.$

21. When multiplying by 11, add a zero to the number and add the multiplicand to the result. For example,

$87 \times 11 = 870 + 87 = 957.$

Division by 5, 1 1/2, 15

22. To divide by 5 double the number and move the decimal point one place to the left. For example,

$$68 \div 5 = \frac{136}{10} = 13.6.$$

$$237 \div 5 = \frac{474}{10} = 47.4.$$

23. Dividing by 1 1/2 consists in doubling the number and dividing the result by 3. For example,

$36 \div 1\,1/2 = 72 \div 3 = 24.$

$53 \div 1\,1/2 = 106 \div 3 = 35\,1/3.$

24. Dividing by 15 consists in doubling the number and dividing the result by 30. For example,

$240 \div 15 = 480 \div 30 = 48 \div 3 = 16.$

$462 \div 15 = 924 \div 30 = 30\,24/30 = 30\,4/5 = 30.8.$

(or $924 \div 30 = 308 \div 10 = 30.8$).

Squaring

25. To square a number ending in 5 (e.g. 85) the number of tens (8) is multiplied by itself plus one ($8 \times \times 9 = 72$) and on the right of the result 25 is ascribed (in our example this yields 7,225). Some more examples:

25^2; $2 \times 3 = 6$; 625.

45^2; $4 \times 5 = 20$; 2,025.

145^2; $14 \times 15 = 210$; 21,025.

The procedure follows from the formula:

$(10x + 5)^2 = 100x^2 + 100x + 25 = 100x(x + 1) + 25.$

26. This technique can also be applied to decimal fractions ending in 5:

$8.5^2 = 72.25$; $14.5^2 = 210.25$; $0.35^2 = 0.1225$, etc.

27. As $0.5 = 1/2$ and $0.25 = 1/4$, the procedure of item 25 can also be used to square numbers ending in the fraction 1/2:

$(8\ 1/2)^2 = 72\ 1/4.$
$(14\ 1/2)^2 = 210\ 1/4$, etc.

28. Mental squaring can often be simplified using the formula

$(a \pm b)^2 = a^2 + b^2 \pm 2ab.$

For example,

$41^2 = 40^2 + 1 + 2 \times 40 = 1{,}601 + 80 = 1{,}681.$
$69^2 = 70^2 + 1 - 2 \times 70 = 4{,}901 - 140 = 4{,}761.$
$36^2 = (35 + 1)^2 = 1{,}225 + 1 + 2 \times 35 = 1{,}296.$

The procedure is also convenient for numbers ending in 1, 4, 6, and 9.

Calculations by Formula $(a + b)(a - b) = a^2 - b^2$

29. Let's multiply 52×48. We mentally represent the multipliers as $(50 + 2)(50 - 2)$ and use the formula:

$(50 + 2)(50 - 2) = 50^2 - 2^2 = 2{,}496.$

This technique can be used whenever one multiplier can be conveniently represented as a sum of two numbers and the other as a difference of the same numbers, e. g.

$69 \times 71 = (70 - 1)(70 + 1) = 4{,}899.$
$33 \times 27 = (30 + 3)(30 - 3) = 891.$
$53 \times 57 = (55 - 2)(55 + 2) = 3{,}021.$
$84 \times 86 = (85 - 1)(85 + 1) = 7{,}224.$

30. The procedure may conveniently be used for calculations of the following type:

$1\ 1/2 \times 6\ 1/2 = (7 + 1/2)(7 - 1/2) = 48\ 3/4.$
$11\ 3/4 \times 12\ 1/4 = (12 - 1/4)(12 + 1/4) = 143\ 15/16.$

It Pays to Remember:

$37 \times 3 = 111$

With this in mind we can easily carry out the multiplication of 37 by 6, 9, 12, etc.

$37 \times 6 = 37 \times 3 \times 2 = 222.$
$37 \times 9 = 37 \times 3 \times 3 = 333.$
$37 \times 12 = 37 \times 3 \times 4 = 444.$
$37 \times 15 = 37 \times 3 \times 5 = 555$, etc.

$7 \times 11 \times 13 = 1{,}001$

With this in mind we can easily carry out the multiplications of the following type:

$77 \times 13 = 1{,}001.$
$77 \times 26 = 2{,}002.$
$77 \times 39 = 3{,}003$, etc.

$91 \times 11 = 1{,}001.$
$91 \times 22 = 2{,}002.$
$91 \times 33 = 3{,}003$, etc.

$143 \times 7 = 1{,}001.$
$143 \times 14 = 2{,}002.$
$143 \times 21 = 3{,}003$, etc.

We have only discussed the simplest and most convenient techniques of mental arithmetic. An inquiring mind can, through practice, work out further procedures to simplify calculations.

The Smallest Magic Square

Since time immemorial people have amused themselves by constructing magic squares. The problem consists in arranging successive numbers (beginning with 1) over the cells of a divided square so that the numbers in all the lines, columns and diagonals add up to the same number.

The smallest magic square has nine cells. It can easily be shown by trials that a four-cell magic square is impossible. The following is an example of a 9-cell magic square:

Figure 252

4	3	8
9	5	1
2	7	6

In this square we might add up either $4+3+8$, or $2+$ $+7+6$, or $3+5+7$, or $4+5+6$, or any other line of three numbers, the result is always 15. The result could be envisaged beforehand, without constructing the square as such: the three lines of the square should contain all the nine numbers and they add up to

$1+2+3+4+5+6+7+8+9=45.$

On the other hand, this sum must clearly be equal to thrice the sum of a single line. Hence for each line

$45 \div 3 = 15.$

Using the same argument we can determine in advance the sum of the numbers in a line or column of any magic square consisting of an arbitrary number of cells. We only have to divide the sum of all its numbers by the number of its lines.

Turns and Reflections

Having constructed one magic square we can readily derive its modifications, i.e. a series of new magic squares. If, for instance, we have the square given in Fig. 253, then by mentally turning it by 90° we'll obtain another magic square (Fig. 254):

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Figure 253

6	1	8
7	5	3
2	9	4

Figure 254

8	3	4
1	5	9
6	7	2

Further turns – by 180° and 270° – will give two more modifications of the initial square.

Each of the new magic squares can in turn be modified by reflecting it in a mirror. Figure 255 depicts the initial square with one of its mirror reflections.

Figure 255

6	1	8
7	5	3
2	9	4

2	9	4
7	5	3
6	8	1

All the turns and reflections possible with the 9-cell square yield the following versions (Fig. 256):

Figure 256 (1-3)

6	1	8
7	5	3
2	9	4

1

8	1	6
3	5	7
4	9	2

2

2	7	6
9	5	1
4	3	8

3

Figure 256 (4-8)

6	7	2
1	5	9
8	3	4

4

4	9	2
3	5	7
8	1	6

5

2	9	4
7	5	3
6	1	8

6

8	3	4
1	5	9
6	7	2

7

4	3	8
9	5	1
2	7	6

8

This is the complete collection of magic squares that could be compiled of the first nine figures.

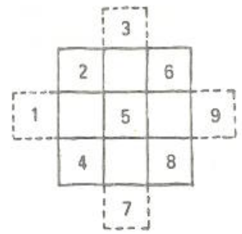
Bachet's Method

Here's an ancient method of constructing odd magic squares, i.e. squares with any odd number of cells: 3 ×

× 3, 5 × 5, 7 × 7, etc. The method was suggested in the 17th century by the French mathematician Claude-Gaspar Bachet (1581-1638). The method being suitable for the 9-cell square, it'll be convenient to begin discussing the method with this, the simplest case.

So, having drawn a square divided into nine cells we'll write the numbers from 1 to 9 in succession arranging them in the oblique lines as shown in Fig. 257.

Figure 257



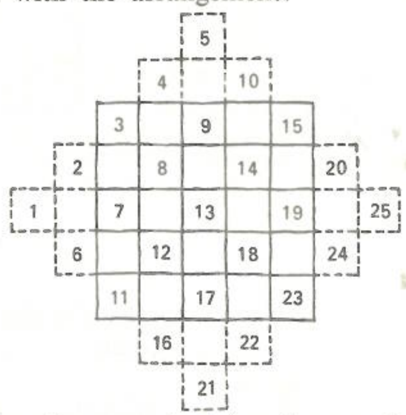
We transfer the numbers that appear to lie beyond the confines of the square into the square so that they join the lines at the opposite sides of the square. We thus obtain:

Figure 258

2	7	6
9	5	1
4	3	8

Let's apply Bachet's technique to a 5 × 5 square. We'll begin with the arrangement:

Figure 259



It only remains now to bring the numbers outside the confines of the square into it. To do so, we'll need to bring the three-number arrangements from beyond

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the square ("terraces") to the *opposite* sides of the square. This will give a 25-cell magic square (Fig. 260).

The idea behind this simple technique is fairly complicated, though the reader can check it practically.

Now that we have this magic square with 25 cells we can obtain its modifications by using turns and mirror reflections.

Figure 260

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

The Indian Method

The Bachet (or "terrace") method is not the only approach to constructing squares with an odd number of cells. Worth mentioning among other techniques is a relatively easy procedure that is thought to have been devised in India about two thousand years ago. It can be briefly couched in six rules. Read them carefully and then see them applied to a magic square with 49 cells (Fig. 261).

Figure 261

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

1. In the middle of the upper line we write 1, and at the bottom in the next column on the right we write 2.
2. We write the next numbers successively along the diagonal upwards and to the right.
3. Having reached the right edge of the square we go over to the extreme left cell in the next line up.
4. Having reached the upper edge of the square we go over to the lowest cell of the next column to the right.

Note: Having reached the upper right corner cell we go over to the leftmost lower corner cell.

5. Having reached an occupied cell we skip over it.
6. If the last occupied cell belongs to the lowermost line, we proceed from the uppermost cell in the column.

Observing these rules we can quickly construct magic squares with any odd number of cells.

If the number of cells in the square *doesn't divide* into 3 we may begin the square following a rule other than 1.

We may start from any cell along the diagonal line passing between the middle cell of the leftmost column and the middle cell of the uppermost line. All the other numbers are placed according to rules 2-5.

It is possible therefore to construct several squares by this method. By way of example, we provide the following 49-cell magic square (Fig. 262).

Figure 262

32	41	43	3	12	21	23
40	49	2	11	20	22	31
48	1	10	19	28	30	39
7	9	18	27	29	38	47
8	17	26	35	37	46	6
16	25	34	36	45	5	14
24	33	42	44	4	13	15

Exercise. Use the Indian method to construct several magic squares with 25 and 49 cells. Obtain other squares by turns and mirror reflections.

Figure 263

			×		
0					
					0
		×			

Figure 264

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Squares with an Even Number of Cells

These magic squares can't be constructed using any common or convenient rule. There is one relatively simple procedure for even squares, the number of whose cells is divisible by 16. This means that one side of these squares has a number of cells that is a multiple of 4, i.e. it consists of 4, 8, or 12, etc., cells.

We'll now agree as to what we'll call "opposite" cells. As an example, Fig. 263 presents two pairs of opposite cells: one marked by crosses, the other by circles.

We see that if a cell lies in the second line from the top and fourth from the left, then the respective opposite cell will lie in the penultimate line and fourth from the right. (The reader is recommended to practise finding some other "opposite" cells.) Note that the opposites of cells in a diagonal also lie on the same diagonal.

We'll explain the procedure referring to 8×8 square. To begin with, we'll place all the numbers from 1 to 64 in the cells in succession (Fig. 264).

All the diagonal lines in the resultant square have the same sum – 260, just what we want for a 8×8 magic square (Check!). But the lines and columns of the square give different sums. So, the upper line adds up to 36, i.e. 224 less than required, and the last line adds up to 484, i.e. 224 more than required. Noting that each number in the last line is 56 more than the number in the same column but in the first line and that $224 = 4 \times 56$, we come to the conclusion that the sums of these two lines can be equalized if we replace a half of the numbers in the first line by the corresponding number in the last line. For instance, the numbers 1, 2, 3, and 4 are replaced by 57, 58, 59, and 60, respectively.

What has been said about the first and eighth lines is also true of the second and seventh, the third and sixth, and in general for any pair of lines equidistant from their respective extremum lines (i.e. first and last). After the numbers in all the lines have been interchanged we'll obtain a square in which lines have equal sums.

It is, however, necessary that the columns, too, give the same sum. With the initial arrangement we could have achieved this by the same kind of exchange that we used with the lines. But after rearranging the lines, the situation has become more complex. To identify the numbers to be exchanged we make use of the following

Figure 265

1 ×	2	3	4 ×	5 ×	6	7	8 ×
9 ×	10 ×	11	12	13	14	15 ×	16 ×
17	18 ×	19 ×	20	21	22 ×	23 ×	24
25	26	27 ×	28 ×	29 ×	30 ×	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Figure 266

64	2	3	61	60	6	7	57
56	55	11	12	13	14	50	49
17	47	46	20	21	43	42	24
25	26	38	37	36	35	31	32
33	34	30	29	28	27	39	40
41	23	22	44	45	19	18	48
16	15	51	52	53	54	10	9
8	58	59	5	4	62	63	1

technique, which could have been applied from the very beginning. Then, instead of a double rearrangement (of the lines and columns) we exchange "opposite" numbers. But this rule is not sufficient in itself, however, since we've found that only a *half* of the numbers need to be exchanged, the remaining numbers staying in their previous places. Which of the opposite pairs then are to be exchanged?

The following four rules are an answer to this question:

1. We divide the magic square into four squares as shown in Fig. 265.

2. In the upper left corner we mark with crosses a half of all the cells so that each column and line has exactly one half of its cells marked. This can be done in a variety of ways, an example is shown in the above figure.

3. In the right upper square we mark with crosses the cells that are *symmetrical* about those marked in the upper left corner.

4. We now only have to replace the numbers in the cells marked by those in the opposite cells.

As a result of the permutation we will have obtained a 64-cell magic square presented in Fig. 266.

We could, however, use many other ways of marking the cells in the left upper square so that rule 2 would be fulfilled, for example, as shown in Fig. 267.

Figure 267

	×	×	
×	×		
×			×
		×	×

1

	×	×	
×			×
×			×
	×	×	

2

×			×
	×	×	
	×	×	
×			×

3

×	×		
×	×		
		×	×
		×	×

4

The reader will undoubtedly find many other ways of arranging the crosses in the cells of the upper left square.

Using then rules 3 and 4 we can readily derive other magic squares with 64 cells.

Arguing along these lines we can construct magic squares consisting of 12 × 12, 16 × 16, etc., cells. The reader can do this on his own.

Whence the Name?

The first recorded evidence of the magic square comes from an ancient oriental book referring to 4,000–5,000 B.C. Indians in ancient times had a better understanding of magic squares, from where the passion for magic squares was taken over by ancient Arabs, who would assign mysterious qualities to these combinations of numbers.

In medieval Western Europe magic squares were the stock-in-trade of representatives of pseudosciences, such as alchemistry and astrology. It is from the medieval superstitious perceptions that these squares have derived their unusual name – "magic". Astrologists and alchemists believed that a magic square drawn on a piece of wood was able to deliver a man from misfortune.

* * *

The construction of magic squares is not just a pastime. Many famous mathematicians have been interested in their theory and it has been applied in some of the important problems of mathematics. So, there is a way of solving sets of equations in many unknowns that uses results from the theory of magic squares.

Dominoes

A Chain of 28 Bones. Why is it possible to have a continuous chain of 28 bones (a domino piece is also sometimes called a bone) constructed without breaking the rules of the game?

Beginning and End of the Chain. The 28-bone chain ends in 5 points. How many points are there at the other end?

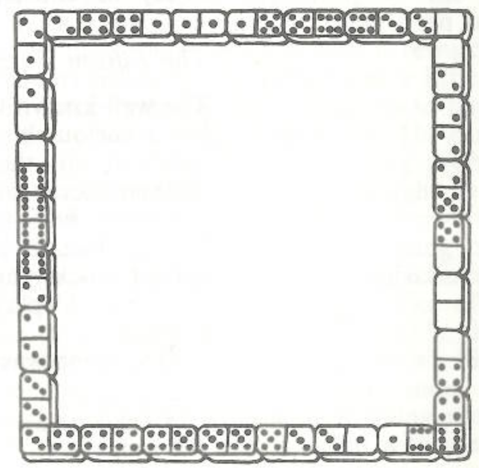
Trick with Dominoes. Your friend takes one of the dominoes and tells you to build a continuous chain out of the remaining 27 bones. He insists that it's always possible whichever bone he takes. He leaves you on your own and goes to another room.

You begin working and see that your friend is right: the 27 bones produced a chain. What is more surprising is that your friend, although remaining in the other room, calls out the number of points at each end of the chain.

How does he know? Why is he confident that any 27 bones can produce a continuous chain?

Frame. Figure 268 shows a square frame made from dominoes whilst observing the rules. The sides of the frame may be equal in length but not in the total number of points: the upper and left sides contain 44 points each and the other two sides contain 59 and 32.

Figure 268



23

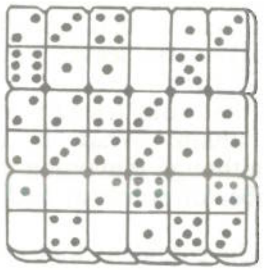
Can you produce a square frame such that each side contains the same sum total of points – 44?

Figure 269

Seven Squares. We can select four bones so that these will form a square with the same sum of points on each side. An example is given in Fig. 269 in which the points on each side add up to 11.

Using a complete set of dominoes, can you build *seven* such squares? They do not have to have the same common sum of points of their sides.

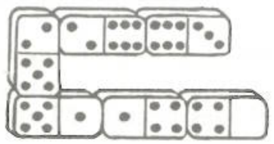
Figure 270



Domino Magic Squares. Figure 270 shows a square of 18 dominoes that is remarkable in that the sum of the points on any of its lines (be it longitudinal, transverse or diagonal) is the same, namely 13.

You are asked to construct several such 18-bone magic squares, but now with another line sum. For an 18-bone square 13 is the smallest sum whilst 23 is the largest.

Figure 271



Domino Progression. In Fig. 271 you see six bones arranged according to the rules of the game but note that the number of points on each (both halves) increases by one. Beginning with 4, the series consists of the following numbers of points 4, 5, 6, 7, 8, and 9.

A series of numbers increasing (or decreasing) by the same amount each time is called an "arithmetic progression". In our case each number is one more than the previous one, but the difference may be arbitrary.

Try to construct some other 6-bone progressions.

*The Fifteen Puzzle**

The well-known tray with 15 numbered square counters has a curious history few people even suspect of. We'll recall it in the words of W. Arens, a German mathematician and investigator:

"About half a century ago, in the late 1870s, the Fifteen Puzzle bobbed up in the United States; it spread quickly and owing to the uncountable number of devoted players it had conquered, it became a plague.

"The same was observed on this side of the Ocean, in

* Other names are the Boss Puzzle, Jeu de Taquin, and Diablotin.

Europe. Here you could even see the passengers in horse trams with the game in their hands. In offices and shops bosses were horrified by their employees being completely absorbed by the game and they were forced to ban the game during office and class hours. Owners of entertainment establishments were quick to latch onto the rage and organized large contests. The game had even made its way into solemn halls of the German Reichstag. 'I can still visualize quite clearly the grey-haired people in the Reichstag intent on a square small box in their hands,' recalls the geographer and mathematician Sigmund Günter who was a deputy during the puzzle epidemic.

"In Paris the puzzle flourished in the open air, in the boulevards, and proliferated speedily from the capital all over the provinces. A French author of the day wrote, 'There was hardly one country cottage where this spider hand't made its nest lying in wait for a victim to flounder in its web.'

"In 1880 the puzzle fever seems to have reached its climax. But soon the tyrant was overthrown and defeated by the weapon of mathematics. The mathematical theory of the puzzle showed that of the many problems that might be offered only a half were solvable, the other half were impossible, however ingenious the technique applied to solve them.

"It thus became clear why some problems wouldn't yield under any conditions and why the organizers of the contests had dared offer such enormous rewards for solving the problems. The inventor of the puzzle took the cake in this respect suggesting to the editor of a New York newspaper that he publish an unsolvable problem in the Sunday edition with a reward of 1,000 dollars for its solution. The editor was a little reluctant so the inventor expressed his willingness to pay his own money. The inventor was Sam Loyd. He came to be widely known as an author of amusing problems and a multitude of puzzles. Curiously enough, he failed to patent his Fifteen Puzzle in the USA. According to the regulations, he had to submit a "working model" so that a prototype batch could be manufactured from it. He posed the problem to a Patent Office official, but when the latter enquired if it were solvable, the answer was 'No, it is mathematically impossible'. The official therefore reasoned: 'In which case there can't be a working model and without a working model there can be no patent.' Loyd was satisfied with the decision.

He would perhaps have been more insistent had he foreseen the unprecedented success of his invention*."

We'll continue the story of the puzzle using the inventor's own words:

Figure 272

The normal arrangement of counters (I)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

"The old dwellers of the realm of aptitude will remember how in the early 1870s I made the whole world rack its brains over a tray of movable counters, that came to be known as the Fifteen Puzzle. The fifteen counters were arranged in order in the tray with only 14 and 15 counters inverted as shown in the accompanying illustration (Fig. 273). The puzzle was to get the counters into the normal arrangement by individually sliding them so that the 14 and 15 were permutated.

Figure 273

The unsolvable case (II)

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

"The 1000-dollar reward offered for the first correct solution remained unretrieved although everybody was busy on it. Funny stories were told of shop-keepers who forget for this reason to open their shops, of respectful officials who stood throughout the night under a street lamp seeking a way to solve it. Nobody wanted to give up as everyone was confident of imminent success. It was said that navigators allowed their ships to run aground, engine drivers took their trains past stations, and farmers neglected their ploughs."

* * *

We'll now introduce the reader to the beginnings of the game. In its complete form it's very complicated and closely related to one of the branches of higher algebra (the theory of determinants). We'll just confine our discussion to some of the elements as presented by W. Arens.

The task of the game is normally as follows: using successive movements made possible by the presence of the blank space the arbitrarily arranged squares should be brought to the normal arrangement, i. e. the counters are in numerical order with the 1 in the upper left

* The episode was used by Mark Twain in his novel *The American Claimant.*

corner, followed by the 2 on the right, then the 3 and the 4 in the upper right corner; in the next row there should be from left to right the 5, 6, 7, 8, etc., with the blank space ending up back in the lower right corner. The normal arrangement is given in Fig. 272.

Now think of an arrangement with the 15 counters scattered arbitrarily. A number of movements can always bring the 1 to the place occupied by it in the figure. In exactly the same way we can, without touching counter 1 move counter 2 to the adjacent place on the right. Next, without touching either the 1 or 2, we can move the 3 and 4 to their normal places. If these occasionally are not in the two last columns, we can bring them there and through a number of movements achieve the arrangement sought. Now the upper line is in the normal order and we'll leave it as it is in later manipulations. In the same way we'll also bring the second line into the normal order. We'll easily find that it's always possible. Further, within the space of the two last lines we'll need to arrange counters 9 and 13, which is always possible, too. It now only remains to arrange a small patch of six spaces, of which one is free and the other five are occupied by the 10, 11, 12, 14, and 15, arbitrarily arranged. Within this patch we can always bring the 10, 11, and 12 into the normal arrangement. This done, the 14 and 15 will be arranged in the last line either in the normal or inverted order (Fig. 273). This procedure, which the reader can easily test in practice, will always yield the following result.

Any initial arrangement can be brought into either the Fig. 272 (*I*) form or the Fig. 273 (*II*) form.

If an arrangement, we'll denote it by *S*, can be brought to *I*, then the opposite is clearly possible, i. e. *I* can be brought to *S*. After all, the movements are all reversible. If, for instance we can push the 12 in arrangement into the blank space, then we can always restore the previous arrangement by the reverse move.

We thus have two series of arrangements such that the arrangements in one series can be brought to normal arrangement *I*, and those in the other series can be brought to arrangement *II*. Conversely, from the normal arrangement we can obtain any arrangement in the first series, and from *II* any arrangement in the second series. Lastly, any two arrangements in the same series are transferable one to the other.

Could we go further and combine the two

arrangements? We could rigorously prove (we are not going to here) that these arrangements cannot be interchanged, however many moves are used. Therefore, the formidable variety of arrangements break down into two separate series: (1) those that can be brought into the normal arrangement, and (2) those that can't and it was for these arrangements that the enormous rewards were promised.

How are we to know whether or not a given arrangement belongs to the first series? An example will clarify this.

Let's consider the arrangement shown in Fig. 274. The first line is in perfect order, as is the second save for the last counter (9). Counter 9 comes *before* 8. This sort of violation of the order is called inversion. Concerning counter 9 we'll say that here we have one inversion. Further examination reveals that the 14 precedes three counters (12, 13, and 11), thus giving three inversions (14 before 12, 14 before 13, and 14 before 11). This amounts to $1+3=4$ inversions. Further, the 12 precedes the 11 and the 13 precedes the 11. This adds two more inversions bringing the total to six. This procedure is used to determine the total number of inversions for any arrangement with the blank space in the lower right corner. If, as in the case in hand, the total number of inversions is *even*, then the arrangement can be brought to the normal one; in other words, it's solvable. If the number of inversions is *odd*, the arrangement belongs to the second series, i.e. it is an insolvable arrangement.

Figure 274

1	2	3	4
5	6	7	9
8	10	14	12
13	11	15	

Thanks to the new light shed on the puzzle by mathematics the earlier morbid passion that was shown for the game is now unthinkable. Mathematics has produced an exhaustive explanation of the game, one that leaves no loophole. The outcome of the game is dependent not on chance nor on aptitude, as in other games, but on purely mathematical factors that predetermine it unconditionally.

We'll now consider some of the solvable problems with the game that were produced by the resourceful Loyd.

Figure 275

	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Problem I. Starting off the arrangement in Fig. 273 bring the counters into the numerical order with the blank space in the upper left corner (Fig. 275).

Problem II. Starting off with the arrangement in Fig. 273 turn the tray 90° to the right and obtain the arrangement of Fig. 276.

Figure 276

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Problem III. By moving the counters according to the rules turn the tray into a magic square, i.e. arrange the counters so that the sum of the numbers in all directions is 30.

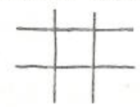
The "11" Game

This is a game for two. Eleven matches (or other objects) are placed on a table. One player takes one, two or three matches, just as he likes. Then the other also takes one, two or three matches. Now again the first, and so on. It's forbidden to take more than three matches at a time. He who takes the last match loses.

How must you play so that you can always win?

The "15" Game

This game is not to be confused with the Fifteen Puzzle. It's more like the well-known "noughts and crosses" game. It's played by two people taking turns. Each player writes a number from 1 to 9 in one of the cells of the network shown below.



Each player selects his cell so that his opponent couldn't complete a row of three figures (the row may be transverse or diagonal) that add up to 15.

The player completing such a row or filling in the last cell of the network is the winner.

Is there any way of winning the game with certainty?

The "32" Game

First 32 matches are arranged on a table. Two people play alternately. The beginner draws one, two, three, or four matches, and then the other player also takes as many matches as he chooses, but again not more than four. And so on. The player taking the last match wins.

The game, you see, is very simple but it is curious in that the beginner can always win if he plays correctly.

Could you indicate the "right" way to win?

The Reverse of the Last Game

The previous game can be modified so that the player taking the last match loses, not wins.

How must you play then to win with certainty?

The "27" Game

The game is similar to the previous ones. It's also played by two people and also requires that the players alternately take no more than four matches. But the object of the game is different: the winner is the one who ends up with an even number of matches.

The beginner here is at advantage, too. He can so calculate his draws that he always will win.

What is the secret of his fail-safe strategy?

The Reverse of the Last Game

The object of the "27" game can be reversed so that the winner is the one ending up with an odd number of matches.

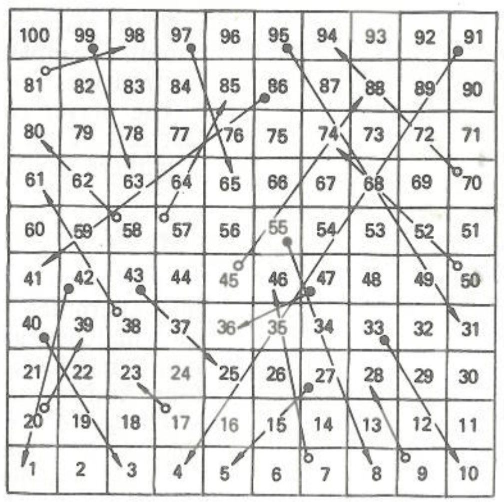
In this case, what is the fail-safe procedure?

Arithmetic Travel

Several people may take part in this game. You'll need:

(1) a board (of cardboard),
(2) a die (of wood),
(3) several counters (as many as there are players).

Figure 277



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The board is a cardboard square, preferably a large one, divided into 10 × 10 cells that are numbered from 1 to 100 as shown in Fig. 277.

The die, about 1 cm on side, is made of wood. The faces are sandpapered and numbered from 1 to 6 (or marked with points as dominoes).

The counters may be various coloured disks, squares, etc.

Taking turns, the players throw the die. If the die shows, say, 6, the player moves his counter 6 squares forward, his next throw takes his counter forward by as many cells as there are points on the die. When the player's counter comes to a cell where an arrow begins, the counter must follow the arrow to its end either forwards, or backwards.

The player whose counter first reaches 100 is the winner.

Think of a Number

Think of a number, follow the procedure given below, and I'll guess the result of your calculations.

Should the result differ, check through your calculations since you will have been in error, not I.

No. 1

The number must be less than 10 though not zero
Multiply it by 3;
Add 2;
Multiply by 3;
Add the number thought of;
Cross out the first digit;
Add 2;
Divide by 4;
Add 19.

The result is 21

No. 2

The number must be less than 10 though not zero
Multiply it by 5;
Multiply by 2;
Add 14;
Subtract 8;
Cross out the first digit;
Divide by 3;
Add 10.

The result is 12

No. 3

The number must be less than 10 though not zero
Add 29 to it;
Discard the last digit;
Multiply by 10;
Add 4;
Multiply by 3;
Subtract 2.

The result is 100

No. 4
The number must be less than 10 though not zero
Multiply it by 5;
Multiply by 2;
Subtract the number thought of;
Add up the digits;
Add 2;
Square it;
Subtract 10;
Divide by 3.

The result is 37

No. 5
The number must be less than 10 though not zero
Multiply it by 25;
Add 3;
Multiply by 4;
Cross out the first digit;
Square it;
Add up the digits;
Add 7.

The result is 16

No. 6
The number must have two digits
Add 7;
Subtract it from 110;
Add 15;
Add the number thought of;
Divide by 2;
Subtract 9;
Multiply by 3.

The result is 150

No. 7
The number must be less than 100
Add 12 to it;
Subtract it from 130;
Add 5;
Add the number thought of;
Subtract 120;
Multiply by 7;
Subtract 1;
Divide by 2;
Add 30.

The result is 40

No. 8
Any number (besides zero)
Multiply it by 2;
Add 1;
Multiply by 5;
Discard all the digits but the last;
Multiply it by itself;
Add up the digits.

The result is 7

No. 9
The number must be less than 100
Add to it 20;
Subtract from 170;
Subtract 6;
Add the number thought of;
Add up the digits;
Multiply it by itself;
Subtract 1;
Divide by 2;
Add 8.

The result is 48

No. 10
The number must be three digits long
Write the same number on its right;
Divide by 7;
Divide by the number thought of;

Divide by 11;
Multiply it by 2;
Add up the digits.

The result is 8

No. 11
The number must be less than 10
Multiply it by 2;
Multiply by 2;
Multiply by 2;
Add the number thought of;
Add the number thought of;
Add 8;
Discard all the digits but the last;
Subtract 3;
Add 7.

The result is 12

Guessing a Three-Digit Number

Think of a three-digit number. Leave aside the last two digits and double the first one. Add 5 to the result, then multiply by 5, add the second digit and multiply by 10. Add the third digit to the new result and tell me what you've arrived at. I'll immediately guess the number you've thought of.

Let's take an example. Say your number is 387. It goes through the following sequence of operations.
You double the first digit: $3 \times 2 = 6$;
Add 5: $6 + 5 = 11$;
Multiply by 5: $11 \times 5 = 55$;
Add the second digit: $55 + 8 = 63$;
Multiply by 10: $63 \times 10 = 630$;
Add the third digit: $630 + 7 = 637$.
So you tell me the final result (637) and I tell you the initial number. Explain how.

Another Number Trick

Think of a number;
Add 1;
Multiply by 3;
Add 1 again;
Add the number thought of;
Tell me the result.

When you tell me the result I subtract 4 from it, divide the difference by 4 and obtain the number you thought of.

For instance, suppose you thought of 12.
Add 1, we get 13.
Multiplied by 3, we get 39.
Added 1, we get 40.
Added the number thought of: $40 + 12 = 52$.
When you tell me the number, 52, I subtract 4 from

23

it, and divide the difference, 48, by 4. I thus get 12, the number you thought of.

How does the procedure work?

Guessing the Crossed-Out Digit

Ask your friend to think a multidigit number and then ask him to do the following:

Write the number down;
Transpose its digits in an arbitrary order;
Subtract the smaller number from the larger;
Cross out one of the digits (but not a zero);
Name the remaining digits in any order;
You will then tell your friend the crossed-out digit.

Example. Your friend thought of 3857.
He performed the following:

3857,

8735,

$8735 - 3857 = 4878.$

Your friend crosses out the 7 and tells you the remaining digits in the following order, say:

8, 4, 8.

From these digits you can determine the crossed digit.

How can this be done?

Guessing the Day and Month of Birth

Get your friend to write down the day and month of his (or her) birth and to carry out the following operation:

Double the day;
Multiply by 10;
Add 73;
Multiply by 5;
Add the serial number of the month of birth.

When he (or she) tells you the final result of his (or her) calculations, you can tell him (or her) the day and month of his (or her) birth.

Example. Suppose your friend was born on the 17 of August, i.e. on the 17th of the 8th month. He does the following:

$17 \times 2 = 34;$
$34 \times 10 = 340;$
$340 + 73 = 413;$

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$413 \times 5 = 2065;$
$2065 + 8 = 2073.$

Your friend tells you the number 2073 and you tell him his birthday.

How can you do this?

Guessing Someone's Age

You can guess the age of a friend if you ask him (or her) to do the following:

Write down side by side any two digits that differ in more than 1;
Write any digit between them;
Reverse the order of the three-digit number obtained;
Subtract the smaller number from the larger;
Reverse the digits of the difference;
Add the result to the difference;
Finally add his age to the sum.
Your friend tells you the final result of the operations
and then you can tell him his age.

Example. Your friend is 23. He performs the following:

25;
275;
572;
$572 - 275 = 297;$

$297 + 792 = 1089;$

$1089 + 23 = 1112.$

The number 1112 is the final result and from it you determine the age. How?

How Many Sisters? How Many Brothers?

You can guess how many brothers and sisters your friend has, if you ask him to do the following:

Add 3 to the number of brothers;
Multiply by 5;
Add 20;
Multiply by 2;
Add the number of sisters;
Add 5.

The friend tells you the final result of his computations and you can tell him how many brothers and sisters he has.

Example. Your friend has four brothers and seven sisters. He thus does the following:

$4+3=7;$

$7\times 5=35;$

$35+20=55;$

$55\times 2=110;$

$110+7=117;$

$117+5=122.$

The friend tells you the number 122 and you can tell him how many brothers and sisters he has.

How can you do this?

Trick with a Telephone Directory

Here is another impressive trick. Get your friend to write down any number with three different digits. Suppose he writes 648. Ask him to reverse the digits in the number he has chosen and subtract the smaller one from the larger*. He will thus write:

$$\begin{array}{r} 846 \\ -\ 648 \\ \hline 198 \end{array}$$

Ask to rearrange the digit of the difference in the reverse order and add both numbers up. Your friend will write:

$$\begin{array}{r} 198 \\ +\ 891 \\ \hline 1089 \end{array}$$

These calculations should be done in secret so your friend thinks that the final result must be unknown to you.

Now give your friend a telephone directory and ask him to open it on the page whose number is equal to the first three digits of the final result. He does so and waits for further instructions. You then ask him to count the telephone subscribers (down from the top or up from the bottom) until he gets to the one given by

* If the difference is a two-digit number (99), it is written with a zero in front (099).

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the last digit of the number (1089). He thus finds the ninth subscriber and you tell him the name of the man and his telephone number.

This naturally amazes your friend: he selected a number at random and you can tell him the subscriber's name and number.

What is the trickery here?

Guessing Domino Points

The trick is arithmetic, based on calculation.

Let your friend put a domino piece into his pocket. You promise to guess the number of points if he makes some simple calculations. Let his bone be the 6–3. Ask him to double one of the numbers (e. g. 6)

$6\times 2=12,$

and add 7

$12+7=19.$

Ask him to multiply the result by 5

$19\times 5=95$

and to add the other number of points of the domino piece (i. e. 3)

$95+3=98.$

He tells you the final result and you in your head subtract 35 to find the points on the piece: $98-35=$ $=63$, i. e. the piece was the 6–3.

Why is it so and why one must always subtract 35?

Formidable Memory

Conjurers sometimes amaze the public by their striking memory: they memorize long series of words, numbers, etc. Each of you can also surprise friends with such a trick.

On 50 small paper cards write the numbers and letters shown below: a long number and in the left corner a letter or a combination of a letter and a figure. Distribute these cards among your friends and claim that you remember exactly which number is on which card. They need only tell you the number of the card and you'll immediately tell them the number written on it. You are told, say, "E4" and you can say at once "10,128,224".

A 24,020	B 36,030	C 48,040	D 510,050	E 612,060
A1 34,212	B1 46,223	C1 58,234	D1 610,245	E1 712,256
A2 44,404	B2 56,416	C2 68,428	D2 7,104,310	E2 3,124,412
A3 54,616	B3 66,609	C3 786,112	D3 8,106,215	E3 9,126,318
A4 64,828	B4 768,112	C4 888,016	D4 9,108,120	E4 10,128,224
A5 750,310	B5 870,215	C5 990,120	D5 10,110,025	E5 11,130,130
A6 852,412	B6 972,318	C6 1,092,224	D6 11,112,130	E6 12,132,036
A7 954,514	B7 1,074,421	C7 1,194,328	D7 12,114,235	E7 13,134,142
A8 1,056,616	B8 1,176,524	C8 1,296,432	D8 13,116,340	E8 14,136,248
A9 1,158,718	B9 1,278,627	C9 1,398,536	D9 14,118,445	E9 15,138,354

The numbers being very long and 50 in all, your power will shock all those present. But... you didn't think you had to learn the 50 long numbers by heart. Everything is much simpler.

What is the trickery here?

Another Memory Thriller

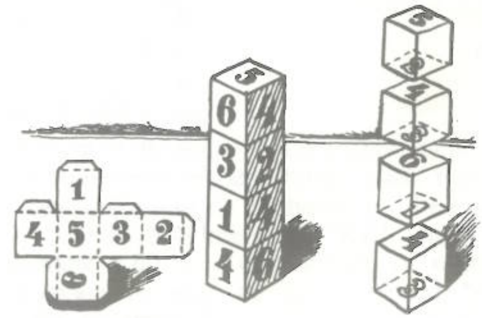
Having written a long series of figures (20 or more), you proclaim that you can without mistake repeat the whole series, figure by figure. And really, you put up a brilliant performance, despite the fact that the sequence of figures shows no pattern.

How can you do it?

Mysterious Cubes

Make several cubes of paper (e.g. four) and write figures on their faces arranging them as shown in Fig. 278. With these cubes you can show an interesting arithmetic trick.

Figure 278



Ask your friends to put the cubes in your absence one on top of another in any arrangement to form a column. On entering the room you need only cast a glance at the column and immediately determine the sum of the figures on the closed faces of the four cubes. For example, in the case shown in Fig. 278 you would call out the sum 23. You can easily see that it is so.

Trick with Cards

Make seven cards as shown in Fig. 279. Write the numbers on them and cut them exactly as shown. One of the cards is left blank, but is cut.

Now give the six cards with the numbers to your friend and ask him to remember one of the numbers written on the cards, and then give you back only those cards on which there is this number.

Having received the cards, stack them neatly, put the clean card on the top, and add up in your head those figures that are seen through the cuts. The result will be the number sought.

You will hardly crack this nut. The trick is based on a special selection of numbers in the cards. The idea behind it is rather complicated and I am not going to dwell on it here.

How to Find the Sum of Unwritten Numbers

You undertake to guess the sum of three numbers of which only one is written. The trick is performed as follows. Ask your friend to write down any multidigit number – the first summand.

Suppose he writes 84,706. Then you leave enough room for the second and third summands and write

23

Figure 279

39	63	54	38	45	61	49	33
53	□	57	46	43	41	□	62
34	40	□	55	42	51	59	35
60	32	44	59	□	58	□	58
36	48	50	56	52	47	42	37

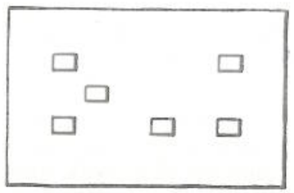
45	63	27	10	58	9	61	42
29	8	11	57	30	59	□	62
13	24	□	60	40	47	14	56
46	□	12	44	□	25	□	27
43	15	41	31	26	62	12	28

33	49	27	17	21	55	61	39
3	□	31	51	63	43	□	13
15	7	1	19	15	23	59	41
57	□	29	9	□	35	□	51
53	5	47	25	45	33	11	37

54	23	18	58	63	31	26	51
29	□	61	50	20	27	□	62
56	28	□	17	59	48	21	60
31	□	19	55	□	30	16	53
63	49	24	57	22	52	27	25

5	47	28	53	61	13	20	52
37	□	44	30	46	55	4	7
22	63	□	12	62	14	60	31
23	□	29	54	□	15	□	6
46	36	39	21	45	28	63	38

11	38	62	51	43	26	55	15
10	□	63	35	31	19	□	46
14	3	□	59	27	7	58	18
26	□	6	47	2	39	□	22
54	23	50	30	35	42	11	34



down in advance the sum total of the three numbers:

1st summand	84,706
2nd summand	
3rd summand	
Sum total	184,705

Then your friend writes the second summand (it must have the same number of digits as the first), and you write the third summand yourself:

1st summand	84,706
2nd summand	30,485
3rd summand	69,514
Sum total	184,705

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You can see that the sum was predicted accurately. Explain.

To Foresee a Sum

In earlier times number superstitions were no less widespread than other superstitions. What the result of such number fads might be is shown by the example of Ilya Teglev, the hero of the story "Rat!...tat!...tat!" by Ivan Turgenev. A chance coincidence of numbers led him to imagine he was an unrecognized Napoleon. After he had committed suicide a sheet of paper was found in his pocket with the following calculations:

Napoleon	Ilya Teglev
Napoleon was born on August 15, 1769	Ilya Teglev was born on January 7, 1811
1769	1811
15	7
8 (August is the 8th month)	1 (January is the 1st month)
Total 1792	Total 1819
1	1
7	8
9	1
2	9
Total 19 (!)	Total 19 (!)
Napoleon died on May 5, 1825	Ilya Teglev died on July 21, 1834
1825	1834
5	21
5 (May is the 5th month)	7 (July is the 7th month)
Total 1835	Total 1862
1	1
8	8
3	6
5	2
Total 17 (!)	Total 17 (!)

Such number fortune-telling was widespread at the beginning of World War I, when it was hoped the outcome could be foreseen using the method. In 1916 Swiss newspapers initiated their readers into the "mysteries" by the following revelation about the fate of Emperors of Germany and Austro-Hungary:

	Wilhelm II	Franz-Joseph
Year of birth	1859	1830
Year of accession	1888	1848
Age	57	86
Years reigned	28	68
Total	3832	3832

The sums, you see, are equal, each representing the double of 1916, whence it was concluded that the year would be fatal for both emperors.

But here we have not just a chance coincidence, but human stupidity. Blinded by superstition, the prophets didn't twig that if you so much as slightly changed the lines in the calculations, their mysterious character would go up in smoke.

Arrange the lines as follows:

Year of birth

Age

Year of accession

Years reigned

Now what year would you obtain if you add a man's age to the year of his birth? Of course, the year when you make your calculation. The same year will result if to the year of an emperor's accession you add the years he has reigned. We can easily see now why the adding up of the four numbers yielded the same result for both emperors, double 1916. What else could they arrive at?

You can use this idea for a funny trick. Ask a friend who doesn't know the trick to write the following four numbers on a sheet of paper and add them up:

Year of birth

Year of entering school (factory, etc.)

Age

Years he's been studying (working, etc.)

Although you may not know any of the four numbers it's a simple matter for you to guess their sum: you only have to double the current year.

Repeating the trick may well expose the secret. To muddle up the situation, introduce several additional numbers you know between the ones you don't. If you play your cards right, each time the result will be different and the secret will thus be more difficult to perceive.

A Chain of 28 Bones. To simplify the task we'll set aside all the seven doubles: 0–0, 1–1, 2–2, 3–3, 4–4, 5–5, 6–6. The remaining 21 bones have each of the point numbers repeated six times. For example, the 4-point pattern (on one end) is on the following six pieces:

4–0, 4–1, 4–2, 4–3, 4–5, 4–6.

So each number, as we see, occurs an *even* number of times. Clearly, the pieces of such a set can be matched to the ends of other pieces until all the set is exhausted. This done, when the 21 bones are arranged in a continuous line, we insert the doublets between the butts of 0–0, 1–1, etc. Thus, all the 28 dominoes appear to be arranged in a line, the rules of the game observed.

Beginning and End of the Chain. We can without difficulty show that the chain of 28 bones must end in the number with which it began. In fact, if this were not the case, the numbers of points at the ends of the chain would appear an *odd* number of times (inside the chain the numbers must occur in pairs). We know, however, that in a complete set of bones each number occurs eight, i.e. even number of, times. Consequently, our assumption of unequal point-patterns at the ends of the chain isn't valid. (An argument like this in mathematics is termed "proof by contradiction".)

By the way, the property we have just proven suggests the following curious consequence: a 28-bone chain can always be joined at the ends to yield a ring. Accordingly, a complete set of dominoes can be arranged not only in a chain with free ends, but also in a closed ring, all the rules being observed.

The reader might ask how many ways can such a chain or ring be achieved? Without launching into the tiresome details of the computation, we will here only say that this number is enormous – 7,959,229,931,520. It represents the product of the following seven factors: $2^{13} \times 3^8 \times 5 \times 7 \times 4{,}231$.

Trick with Dominoes. The answer follows from what has just been said. We know that 28 dominoes always make a closed ring, hence if we remove a bone from this ring, then:

(1) the remaining 27 dominoes will make a continuous chain with open ends;

(2) the end numbers in this chain will be those that are on the bone removed.

Having hidden a bone, we can always predict the point-patterns at the ends of the chain made up of the remaining bones.

Frame. The points on the sides of the square sought will add up to $44 \times 4 = 176$, i.e. 8 more than the total in the complete set (168). This, of course, occurs because the points at the corners of the square are included twice. This yields the sum of points at the square vertices, i.e. 8. This makes the search for the desired arrangement somewhat easier, but only slightly. The solution is shown in Fig. 280.

Seven Squares. We'll give two of the many solutions possible. In the first one (at the top of Fig. 281) we have:

1 square with sum 3,	2 squares with sum 9,
1 square with sum 6,	1 square with sum 10,
1 square with sum 8,	1 square with sum 16.

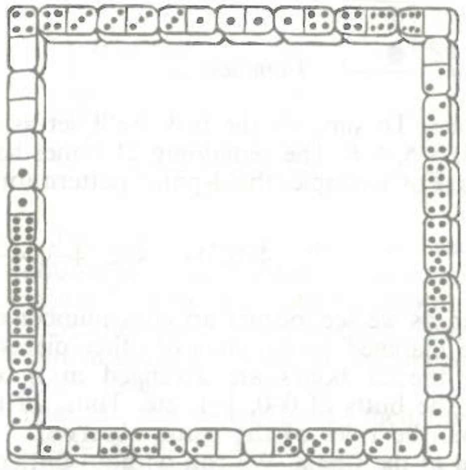
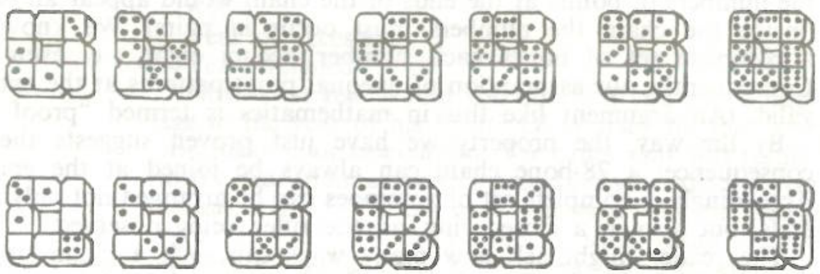


Figure 280

Figure 281

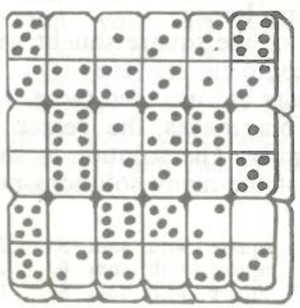


In the second solution (at the bottom of Fig. 281):

2 squares with sum 4, 2 squares with sum 10,
1 square with sum 8, 2 squares with sum 12.

Domino Magic Squares. Figure 282 shows an example of the magic square with 18 points in a line.

Figure 282



Domino Progression. By way of example, we'll consider two progressions with differences equal to 2:

(a) 0–0; 0–2; 2–2; 2–4; 4–4; 4–6.
(b) 0–1; 1–2; 2–3; 3–4; 4–5; 5–6.

All told, there are 23 progressions for 6 bones. The initial bones are as follows:

(a) For unit-difference progressions:

0–0, 1–1, 2–1, 2–2, 3–2,
0–1, 2–0, 3–0, 3–1, 2–4,
1–0, 0–3, 0–4, 1–4, 3–5,
0–2, 1–2, 1–3, 2–3, 3–4.

(b) For progressions with differences of 2:

0–0, 0–2, 0–1.

The Fifteen Puzzle

Problem I. The arrangement can be arrived at in the following 44 moves:

14, 11, 12, 8, 7, 6, 10, 12, 8, 7,
4, 3, 6, 4, 7, 14, 11, 15, 13, 9,
12, 8, 4, 10, 8, 4, 14, 11, 15, 13,
9, 12, 4, 8, 5, 4, 8, 9, 13, 14,
10, 6, 2, 1.

Problem II. The aim is achieved by 39 moves:

14, 15, 10, 6, 7, 11, 15, 10, 13, 9,
5, 1, 2, 3, 4, 8, 12, 15, 10, 13,
9, 5, 1, 2, 3, 4, 8, 12, 15, 14,
13, 9, 5, 1, 2, 3, 4, 8, 12.

Problem III. The moves are as follows:

12, 8, 4, 3, 2, 6, 10, 9, 13, 15,
14, 12, 8, 4, 7, 10, 9, 14, 12, 8,
4, 7, 10, 9, 6, 2, 3, 10, 9, 6,
5, 1, 2, 3, 6, 5, 3, 2, 1, 13,
14, 3, 2, 1, 13, 14, 3, 12, 15, 3.

The "11" Game

If you start, you have to take two matches leaving nine. No matter how many your partner takes next, you then have to leave only five matches on the table. You should easily see that you can always do this. And no matter how many of these five your partner takes, you can leave one match and win.

If your partner begins, then the outcome of the game depends on whether or not your partner knows the secret of the fail-safe play.

The "15" Game

If you want to win for sure, begin with 5. But in which cell? Let's consider the three possibilities one by one.

1. The 5 is written in the middle cell. Which ever cell your opponent chooses, you can write in the vacant cell in the same row 15–5–x (where x is your opponent's number). The number 15–5–x, i.e. 10–x, is clearly less than 9.

		x
	5	
10–x		

2. The 5 is written in a corner cell. Your partner will take either x or y. If he writes x, you will have to fill in cell y; if he writes y, you respond with x. Either way you win using the above rule

5		
		x
	y	

3. The 5 is in the middle of the right column. Your partner may occupy one of the cells: x, y, z, or t.

	x	z
5		
	y	t

Your answer to x is t; to y, z; to z, y; to t, x. In all the cases you win.

The "32" Game

It's fairly easy to find the way to win in this game, if you take the trouble to play it backwards from the end. You'll figure out that if your last-but-one draw leaves five matches on the table, then your win is a sure thing since your partner may not take more than four matches, hence you can take after him all the remaining matches. But can you contrive so that you could make your the last but one move leave five matches on the table? You'll have, by your previous draw, to leave exactly 10 matches, then, whatever his choice, he can't leave you less than six, so that you will always be able to leave to him five. Further, how can you contrive so that your partner will have to draw from 10? To achieve this your previous draw must leave 15 matches on the table.

In this way, by subtracting five each time you'll find that earlier you would have to leave 20 matches on the table and before that 25 matches, and so at the beginning, 30 matches, i.e. you must begin by drawing 2 matches.

Thus, for the game to be a success begin by drawing 2 matches, then after your partner has taken some matches, take as many as are required to leave 25, next go leave 20, then 10, and finally five. The last match will be yours without fail.

The Reverse of the Last Game

Your last-but-one draw must now leave six matches on the table. Then any draw your partner may make will leave from two to five matches, i.e. your last draw can leave the last match to him in any event. Thus, your last-but-two draw must leave 11 matches on the table, and on your earlier draw you should leave 16, 21, 26, and 31 matches, respectively.

You thus begin by taking 1 match and your later draws leave 26, 21, 16, 11, and 6 matches. This will unfailingly leave the last match to your partner.

The "27" Game

The way to win here is somewhat more difficult than in the previous game. You must start off with the following two considerations:

1. If before the final draw you have an *odd* number of matches, you must leave five matches to your partner and your win is a cinch. In fact, the next draw of your partner will leave you with four, three, two or one matches. If four are left take three and win, if three take them and win, if two take one and win, and if one take it and win.

2. If before the final draw you have an *even* number of matches, you must leave six or seven matches to your partner. In fact, the game will proceed as follows. If your partner's next draw leaves six matches to you, you must take one and, now with an odd number of matches, you can safely leave five matches to your partner in which case he loses all right (see above). If he leaves five matches, not six, you must take four of them and win; if four take them all and win; if three take two and win; and finally, if he leaves two you also win. He cannot leave less than two.

Now you should be able to find the sure way to win without difficulty. If you have an odd number of matches, you must leave on the table a number of matches that is a multiple of 6 minus one, i.e. 5, 11, 17, or 23. If you have an even number of matches, you must leave a multiple of 6 or the multiple plus one, i.e. 6 or 7, 12 or 13, 18 or 19, 24 or 25. Zero is considered an even number, therefore in the beginning you must take two or three, and then follow the previous procedure.

If you abide by this rule you will win always. Only you must see to it that your partner doesn't take the initiative.

The Reverse of the Last Game

If you have an *even* number of matches, you must leave to your partner a multiple of 6 minus one; if you have an *odd* number, you leave a multiple of 6 or the multiple plus one. This ensures for your win. At the beginning you have zero matches (i.e. an even number), therefore your first draw must be to take four matches and leave 23 to your partner.

Think of a Number

No. 1. If the number thought of is a, then the operations are as follows

$$(3a + 2) \times 3 + a = 10a + 6.$$

The result is a two-digit number, the first digit being the number you first thought of, the second being 6.

Crossing out the first digit eliminates the number first thought of.

The rest is self-explanatory.

Nos. 2, 3, 5 and 8 are modifications of what has just been described.

Nos. 4, 6, 7 and 9 use another way of eliminating the number thought of. In No. 9, for instance, the operations are as follows

$$170 - (a + 20) - 6 + a = 144.$$

The rest is self-explanatory.

No. 10 requires a special procedure. To write a three-digit number on the right of itself is equivalent to multiplying it by 1,001 (e.g. $356 \times 1{,}001 = 356{,}356$). But $1{,}001 = 7 \times 11 \times 13$. Therefore, if you think of a three-digit number a, then the operations are

$$\frac{a \times 1{,}001}{7 \times a \times 11} = 13.$$

The rest is clear.

* * *

You thus see that in each of the above cases the guessing is based on eliminating the number thought of. Now try and devise some new examples of your own.

Guessing a Three-Digit Number

The first digit was first multiplied by 2, then by 5 and by 10, i.e. by $2 \times 5 \times 10 = 100$. The second digit was multiplied by 10. The third one is added as it is. Besides, we add $5 \times 5 \times 10$, i.e. 250, to the result.

Thus, if we subtract 250 from the result, we'll have the first digit multiplied by 100 plus the second digit multiplied by 10 plus the third digit. In short, we'll end up with the number thought of.

We thus conclude that to guess the number thought of we must subtract 250 from the result of our calculations.

Another Number Trick

A close examination of the procedure shows that the result must be the four times the number thought of plus 4. If we thus subtract 4 and divide the rest by 4, we'll arrive at the number we seek.

Guessing the Crossed-Out Digit

Those who know the criterion for divisibility by 9 will know that dividing the sum of the digits of any number by 9 gives the same remainder as the number itself. Any two

numbers composed of the same digits must therefore give equal remainders when divided by 9. So if we subtract one of the numbers from the other, the difference will be exactly divisible by 9 as the subtraction will cancel out the remainders.

This thus suggests that the digits of the difference your friend obtained add up to a number divisible by 9. Since the digits 8, 4, 8 that were told to you add up to 20 and you can infer that the nearest number divisible by 9 is 27 you can find the digit needed to get 27, and hence the digit crossed-out, which is 7.

Guessing the Day and Month of Birth

To work out the date sought we must subtract 365 from the final result. The last two digits of the difference will then be the number of the month, and the preceding digits the number of the day. In our example

$$2073 - 365 = 1708.$$

From 1708 we determine the date: 17.08. Why?

Let K be the number of the day, and N the number of the month. We obtain

$$(2K \times 10 + 73) \times 5 + N = 100K + N + 365.$$

Clearly, subtracting 365 gives a number with K hundreds and N ones.

Guessing Someone's Age

If you go through the procedure several times, you should notice that at all times you add the age to the same number, namely 1,089. Therefore, if you subtract 1,089 from the result, you'll obtain the age sought.

Demonstrating the trick several times you might change the procedure so as not to expose the secret. For example, by requesting 1,089 be divided by 9 and then add the age to the result.

How Many Sisters? How Many Brothers?

Subtract 75 from the final result. In our example

$$122 - 75 = 47.$$

The first digit of the result gives the number of brothers, the second the number of sisters. In fact, if the number of brothers is a, and the number of sisters is b, then

$$[(a + 3) \times (5 \times 20)] \times 2 + b + 5 = 10a + b + 75$$

and we arrive at a two-digit number ab.

The trick can only be a success if the number of sisters is not larger than nine.

Trick with a Telephone Directory

The point is that you know the final result beforehand. Whatever the three-digit number, the outcome is always the same – 1,089. You can easily test this. Thus beforehand you remember the name and number of the subscriber in the ninth line (from the top or bottom) on page 108.

Guessing Domino Points

Let's trace through the operations to which we subject the first number. We first multiplied it by 2 and then by 5, i.e. by 10. In addition, we added $7 \times 5 = 35$.

Consequently, if we subtract 35 from the result, we'll obtain as many tens as there are points at one end of the domino. Adding the points at the other end gives the second digit of the result.

It's now clear why the figures of the result give the numbers of points at once.

Formidable Memory

The alpha-numerical code of a card indicates the number written on it.

Above all, you must remember that A stands for 20, B for 30, C for 40, D for 50, E for 60. Therefore the code means some number. For example A1–21, C3–43, and E5–65.

From this number you arrive at the long number written on the card following a definite rule. Let's discuss it referring to an example.

Suppose you are told the code E4, i.e. 64. You handle this number as follows: First, add up its digits:

$$6 + 4 = 10.$$

Second, double it:

$$64 \times 2 = 128.$$

Third, subtract the larger digit from the smaller one:

$$6 - 4 = 2.$$

Finally, multiply both digits together:

$$6 \times 4 = 24.$$

Write all the results you obtain in a line

10,128,224,

to obtain the number written on the card.

The operations performed could be represented symbolically as

$$+, 2, -, \times,$$

i.e. adding, doubling, subtracting, multiplying.

Some more examples:

Code D3:	D3 = 53	Code B8:	B8 = 38,
	$5 + 3 = 8,$		$3 + 8 = 11,$
	$53 \times 2 = 106,$		$38 \times 2 = 76,$
	$5 - 3 = 2,$		$8 - 3 = 5,$
	$5 \times 3 = 15.$		$8 \times 3 = 24.$
	8,106,215		1,176,524.

In order not to strain your memory you can name the numbers as you work them out or else write them slowly on a blackboard.

The pattern is rather difficult to discover, therefore the trick generally amazes people.

Another Memory Thriller

The answer is ridiculously simple: write down the telephone numbers of your acquaintances

Mysterious Cubes

The answer lies in the arrangement of the numbers on the faces of each cube: the sum of the numbers on the opposite faces of a cube is seven in all cases (check in Fig. 278). Therefore, the numbers on the top and bottom faces of all the four cubes stacked in a column add up to $7 \times 4 = 28$. If you subtract the number on the top face of the upper cube from 28, you'll always get the sum of the numbers on all the seven closed faces of the column.

How to Find the Sum of Unwritten Numbers

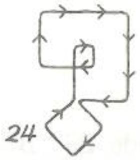
If you add 99,999, i.e. 100,000 – 1, to a five-digit number, then another digit, 1 appears on the left of the number, and the last digit is reduced by 1. The trick is based on this. So if you mentally add 99,999 to the first number

$$\begin{array}{r} 84{,}706 \\ + \\ 99{,}999 \end{array}$$

you can immediately write the future sum of all three numbers, i.e. 184,705. Now you have only to ensure that the second and third numbers on their own add up to 99,999. This is achieved by subtracting mentally each digit of the second number from nine when writing the third number. In our example the second number is 30,485, so you write 69,514. Since

$$\begin{array}{r} 30{,}485 \\ + \\ 69{,}514 \\ \hline 99{,}999 \end{array}$$

then the result you've written beforehand will work out without fail.



24

With a Stroke of the Pen

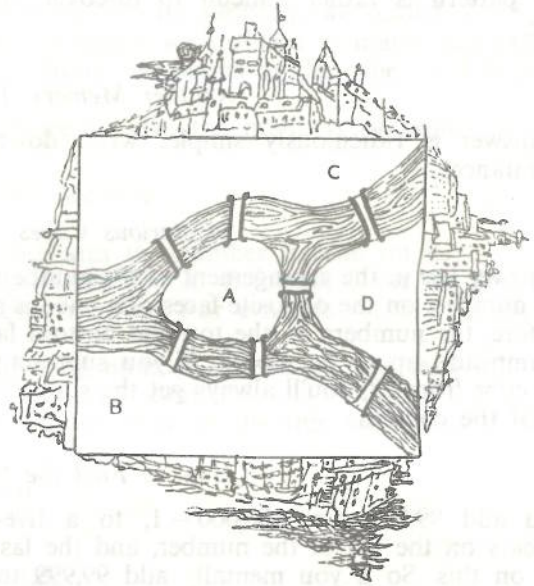
(Drawing figures with one continuous line)

The Königsberg Bridge Problem

The great mathematician Euler was once interested in a curious problem that he described thus:

"There is an island called Kneiphof in Königsberg*. The river flowing around it is split into two branches which are spanned by seven bridges (Fig. 283).

Figure 283



"Is it possible to visit all of these bridges, crossing each once only?

"Some people believe that it is possible. Others think this is impossible."

What do you make of it?

What is Topology?

Euler devoted to the Königsberg bridge problem a whole mathematical investigation that in 1736 he presented to the St. Petersburg Academy of Sciences. The work was begun with the following words defining the branch of mathematics to which similar questions might be referred:

---

* Now Kaliningrad in the USSR.

332–333

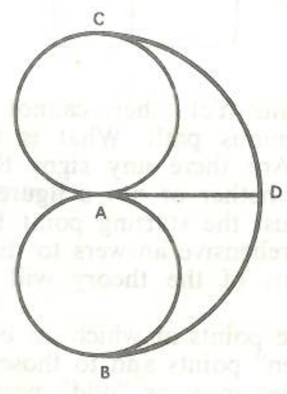
"Besides the aspect of geometry that treats of the quantities and measuring techniques, an aspect that has been developed since ancient times, Leibnitz was the first to mention another aspect that he called the 'geometry of position'. This branch of geometry is only interested in the arrangement of the parts of a figure, ignoring their sizes*.

"Recently, I heard of a problem referring to the geometry of position, and so I have decided to present here by way of example the method I have found of solving the problem."

Euler was referring to the Königsberg bridge problem. We're not now going to discuss the reasoning of this eminent mathematician but will only confine ourselves to some brief remarks that support his final derivation. His conclusion was that it was impossible to meet the condition of the problem.

Examination

Figure 284



For simplicity we'll replace the river's branches by the scheme in Fig. 284. The size of the island and the lengths of the bridges are of no consequence and now we know this is the characteristic feature of all the topological problems.

Therefore, the localities *A*, *B*, *C*, and *D* in Fig. 283 can be replaced by points marked by the same letters where the paths meet. Now the problem is seen to reduce to tracing the figure in Fig. 284 with one continuous path so that no line is drawn twice.

Let's show that it's impossible to do so. We must arrive at each of the node points (*A*, *B*, *C*, and *D*) by one of the paths and then leave by another path. The only exception are the initial and final points: since you don't come from anywhere to start and you don't go anywhere when you leave. Thus, for our figure to be "unicursal" every point, save for two, must meet either two, or four (in general any even number) of paths. At each of the points *A*, *B*, *C*, and *D* in the figure odd numbers of paths meet. It's thus impossible to trace the figure with one continuous path and so it is not possible to cross all the Königsberg bridges as required.

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* Nowadays this branch of higher geometry is generally termed "topology" and it has developed into an extensive field of mathematics. The problems in this section of the book belong to only a small part of the branch of topology.

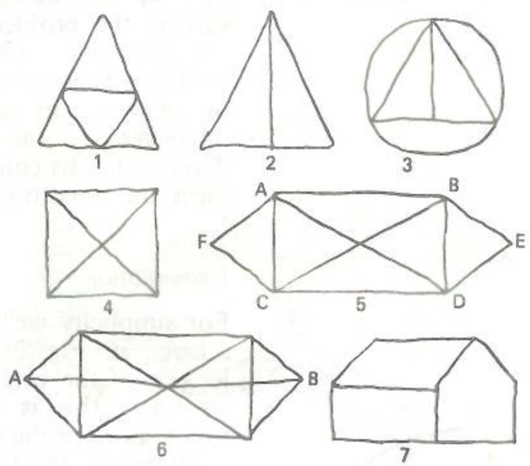
Seven Problems

Try and draw each of the following seven figures with one continuous path.

A Bit of Theory

Attempts to trace figures *1-6* in Fig. 285 yield different results. Some of the figures can be drawn regardless of where the path begins. Others can only be drawn if the

Figure 285



path starts from definite points. Yet others cannot be drawn at all by one continuous path. What is the reason for this difference? Are there any signs that would enable us to predict whether or not a figure is unicursal, and if so what must the starting point be?

The theory provides comprehensive answers to these questions, and some elements of the theory will be presented below.

We will now refer to those points at which an *even* number of lines meet as "even" points and to those at which an *odd* number of lines meet as "odd" points.

It can be shown (we'll omit the proof) that any figure only has either zero, or two, or four (in general an even number) odd points in it.

If there are no odd points in the figure, it can always be drawn with a single stroke of the pen, wherever you start from. Examples are figures *1* and *5* in Fig. 285.

If there are two odd points in the figure, then it can also be drawn in this way, you must only begin from one of the odd points (either one). You will find that

you'll always finish your drawing at the other odd point. Examples are figures *2*, *3*, and *6*. In *6*, for example, you must begin either from point *A* or from *B*.

If a figure has more than two odd points, it's noncursal. Examples are figures *4* and *7*, which both contain four odd points.

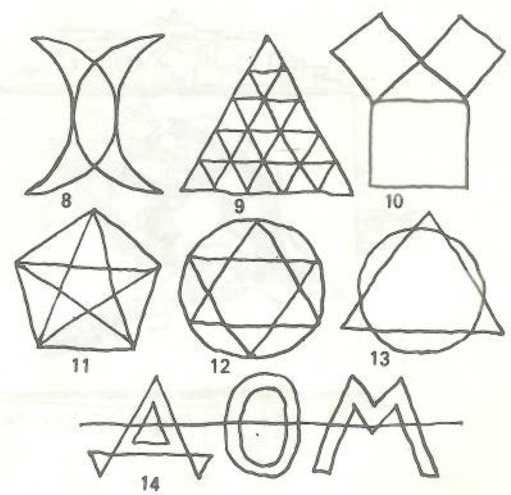
Now you know enough to identify which figures are unicursal and the points from which you could start your drawing. Professor W. Arens suggests you should be guided by another rule, namely "All the lines that have already been drawn in a given figure should be regarded as absent and when selecting the next line see to it that the figure remains complete (doesn't disintegrate) if the line you've chosen is removed from it."

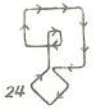
Suppose, for instance, that in figure *5* we've followed the path *ABCD*. If now we draw in line *DA*, we'll have to deal with two figures, *ACF* and *BDE*, and they are *not connected* (figure *5* falls apart). Thus, having completed figure *AFC* we won't be able to go over to *BDE* since there'll be no undrawn lines connecting them. Therefore, having covered *ABCD*, you mustn't go along *DA* but should first trace the path *DBED* and then follow the remaining line, *DA*, over to *AFC*.

Seven More Problems

Trace figures *8-14* with a continuous line (Fig. 286).

Figure 286

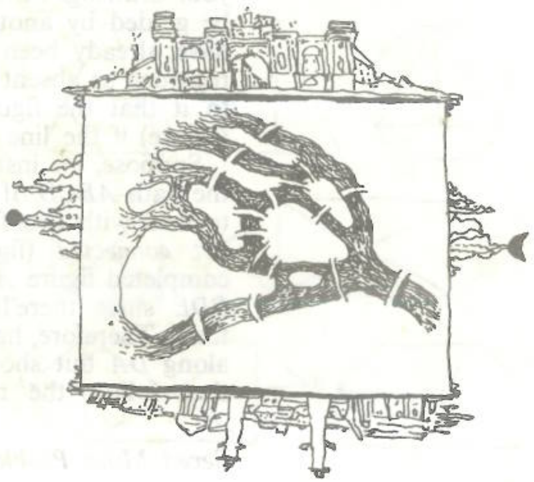


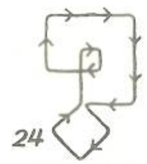


The Leningrad Bridges

The puzzle is to take a walk around the region of Leningrad shown in the figure and come back at the starting point whilst crossing each bridge just once. Unlike the Königsberg bridge problem, the task is feasible and the reader should now be sufficiently armed with knowledge to handle the problem on his own.

Figure 287





Answers

The figures below give the solutions of respective problems in this section.

Figure 288

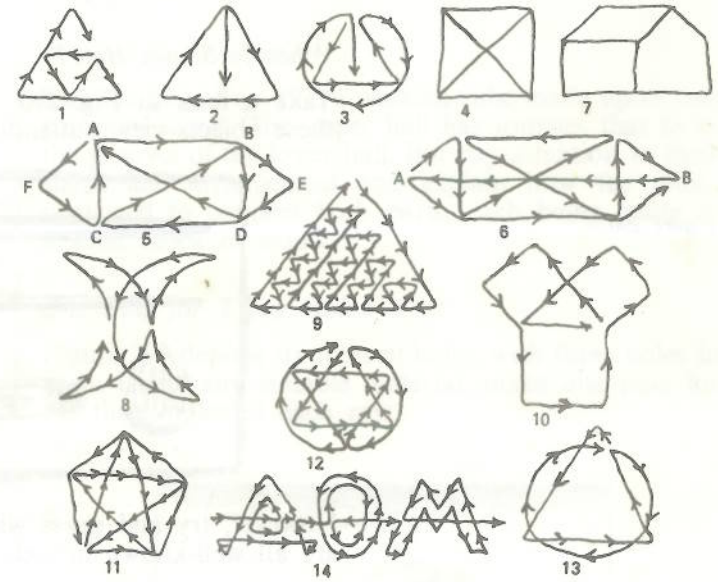
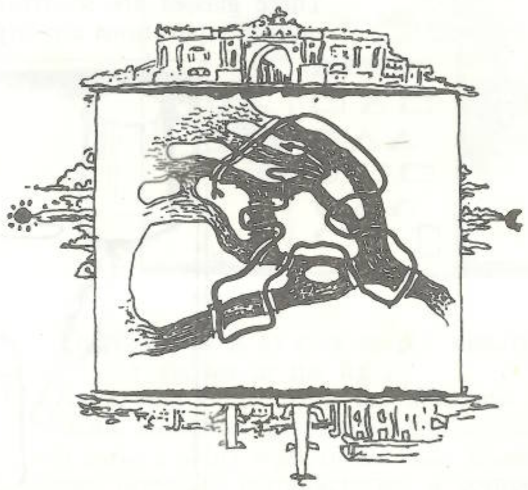


Figure 289



25

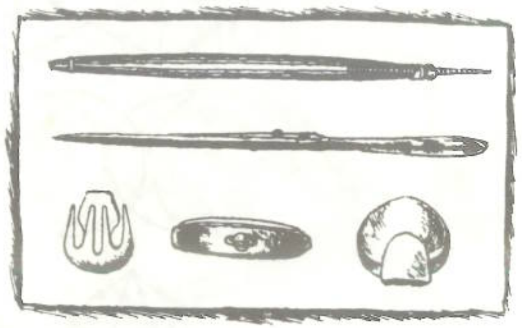
How Many Faces?

How many faces has a hexahedral pencil?
On the face of it the question is naive, or... intricate. Think hard before you look the answer up.

What Is Shown Here?

Take a look at Fig. 290. The unusual aspects make these objects view outlandish and recognition difficult.

Figure 290

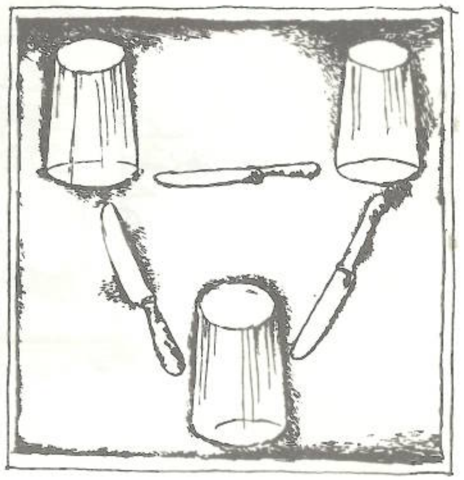


However, try and guess what the figure shows. These are all well-known household things.

Glasses and Knives

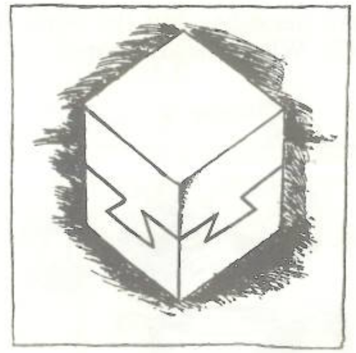
Three glasses are so arranged on the table that their mutual separations are larger than the length of a knife

Figure 291



(Fig. 291). Nevertheless, you are asked to contrive bridges of these knives such that they connect all three glasses. It goes without saying that dislodging the glasses is forbidden, as is the use of anything besides the three glasses and three knives.

Figure 292



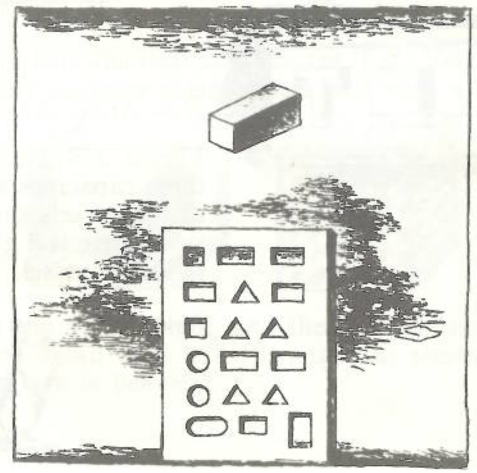
How Is It Achieved?

You see here (Fig. 292) a wooden cube made up of two pieces of wood. The upper half has tongues that fit in the grooves of the lower half. But pay attention to their shape and arrangement and explain how the joiner contrived to connect both parts, each being made of a solid block of wood.

One Plug for Three Holes

Figure 293 depicts six rows of holes, with three holes in each. Using any suitable material, make *one* plug for the three holes of each row.

Figure 293



The first row is as easy as pie: clearly the answer is the block shown in the figure.
As to the other rows the situation is a bit more difficult. However, anyone good at engineering drawing will make a short work of the task. Essentially, the task comes down to manufacturing a component from its three views.

Further "Plug" Puzzles

The accompanying figures show three more boards. Again, find a plug to close the three holes in each board.

Figure 294

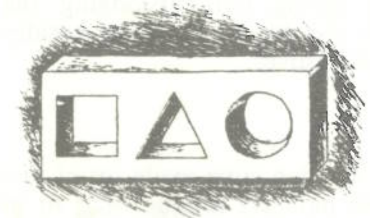


Figure 295

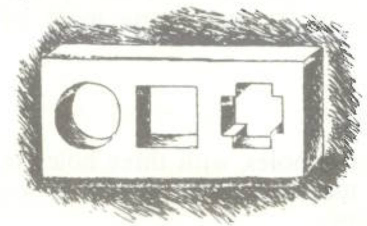


Figure 296

Two Cups

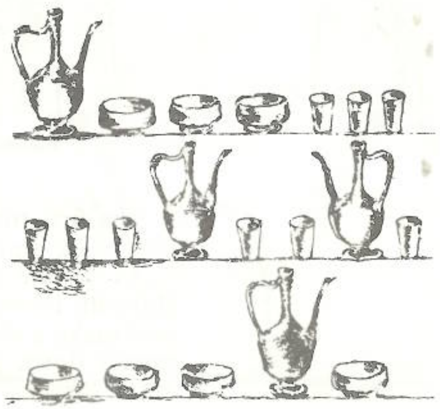
One cup is twice higher than the other, but the other is 1 1/2 times wider (Fig. 297). Which holds more?

Figure 297

How Many Glasses?

Figure 298 depicts three shelves on which vessels of three capacities are arranged so that the total capacity of the vessels on each shelf is the same. The smallest vessel here is a glass. Find the capacity of the other two kinds of vessel.

Figure 298



Two Saucepans

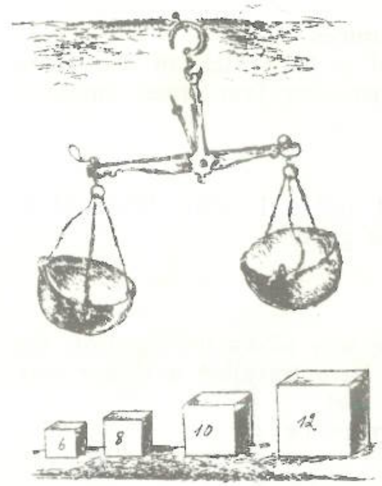
Consider two similar saucepans. Their walls are equally thick but one is eight times more capacious than the other.

How many times heavier is it?

Four Cubes

Four solid cubes of the same material have different heights: 6 cm, 8 cm, 10 cm, and 12 cm (Fig. 299). Arrange them on the pans of a balance for it to be in equilibrium.

Figure 299



Half-Full

An open barrel contains some water, seemingly half its capacity. But you want to know it for certain and you don't have a stick or any other measuring device to measure the contents of the barrel.

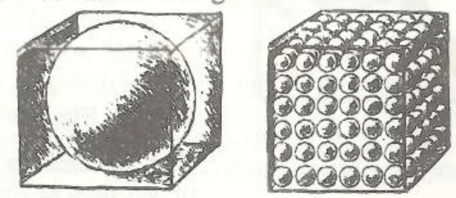
Find a way out.

Which Is Heavier?

There are two identical cubic boxes (Fig. 300): the one on the left contains a large steel ball with a diameter equal to the box's height, and the one on the right is filled with small steel balls arranged as shown.

Which box is heavier?

Figure 300



Tripod

It's believed that a tripod never rocks, even if its legs have different lengths.

Is that so?

How Many Rectangles?

How many rectangles can you identify in this figure (Fig. 301)? Not squares but rectangles, of any size.

Figure 301

Chessboard

How many differently arranged squares could you identify on the chessboard?

A Brick

A brick weighs 4 kilogrammes.
What is the weight of a toy brick of the same material with all its dimensions four times smaller?

A Giant and a Dwarf

Consider a 2-metre giant and a 1-metre dwarf. How many times heavier is the giant?

Along the Equator

If you could walk all the way along the equator, the top of your head would have travelled a longer way than each point on your feet.
What would this difference be?

Figure 302

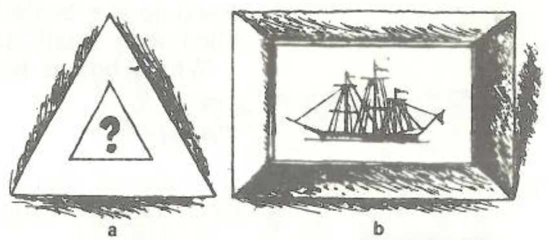
Through a Magnifying Glass

An angle of 1 1/2° is viewed through a 4 × power magnifying glass (Fig. 302).
What will its apparent magnitude be?

Similar Figures

This problem is for those who know about the concept of geometric similarity. Referring to Fig. 303, answer

Figure 303



the following questions:
1. Are the internal and external triangles similar (Fig. 303*a*)?
2. In the frame of the picture (Fig. 303*b*), are the internal and external rectangles similar?

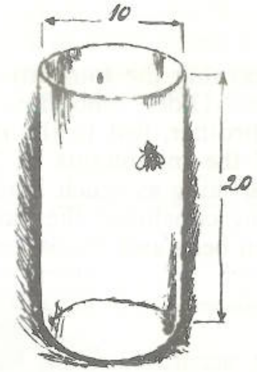
The Height of a Tower

Suppose there is a tourist attraction in your town, a tower whose height you don't know, however. But you have got a photograph of the tower on a picture postcard.
How could this picture help you to determine the height?

A Strip

A bit of mental arithmetic: if a square metre is divided into 1-mm squares, and all of them are arranged side by side on a straight line, how long will be the strip obtained?

Figure 304



A Column

Now imagine a column produced by stacking all the 1-mm cubes contained in 1 cubic metre. How high would this column be?

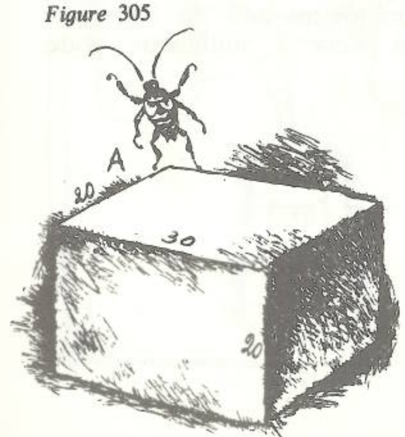
Sugar

Which is heavier: a glassful of granulated sugar or pressed sugar?

The Path of a Fly

Consider a cylindrical glass jar 20 centimetres high and 10 centimetres in diameter. On the inner wall, 3 centimetres from the top, there is a drop of honey, and on the outer wall, the diametrically opposite, there is a fly (Fig. 304).
Trace the shortest path for the fly to get to the honey.
Don't hope that the fly could find the shortest way on its own, thereby simplifying the problem. This would require a knowledge of geometry on its part, that would be "superflyish".

Figure 305



The Path of a Beatle

At the roadside lies a granite block 30 centimetres long, 20 centimetres high and 20 centimetres thick (Fig. 305). A beatle is sitting at point *A* and wants to find the shortest way to *B*.
Trace the path and find out how long it is.

A Bumble-Bee's Travels

A bumble-bee sets out on a long journey. From its nest it flies due south, crosses a river and after an hour's travel alights on a hill covered with clover. Fluttering from a flower to flower it spends half-hour here.

Now it goes to the orchard where the bumble-bee yesterday saw gooseberry-bushes in blossom. The orchard lies due west of the hill and it makes a "bumble-bee" line there, where it arrives 3/4 of an hour later. The bushes being in full blossom, the insect takes 1 1/2 hours to visit all of them.

At last, the bumble-bee starts on its return journey and takes the shortest route possible.

How long has the bumble-bee been away from its nest?

The Foundation of Carthage

According to a tradition concerning the foundation of Carthage the Tyrian princess Dido, who lost her husband at the hands of her brother, fled to the north coast of Africa with many of the inhabitants of Tyre. She bought from the Numidian king as much land "as an oxen hide occupies". Having concluded the bargain Dido had the hide cut into thin belts and thanks to this trick she got a site big enough for a fortress to be erected. So the citadel of Carthage was built, and later developed into a city.

How calculate the area that, according to the legend, the fortress could occupy, given that the oxen hide had a surface area of 4 square metres and the belts into which Dido had it cut were 1 millimetre wide.

How Many Faces?

The problem reveals an incorrect usage of words. A hexagonal pencil doesn't have six faces, as may well be believed. If it isn't sharpened, it has eight faces, all in all: six lateral faces and two small "end" faces. If it really had six faces, it would have quite another shape, namely a block with a rectangular cross section.

The habit of only counting side faces in prisms, and ignoring the end faces is widespread. Many people say: trihedral prisms, tetrahedral prisms, etc., whereas these prisms should be referred to as triangular prisms, quadrangular prisms, etc., according to their cross section. What is more, a trihedral prism (i.e. having three faces) cannot exist.

The pencil mentioned in the problem should be referred to as "hexagonal", not "hexahedral".

What Is Shown Here?

The objects are a razor, a pair of scissors, a fork, a pocket watch, and a spoon. When we look at some object we, generally speaking, see it projected onto a plane normal to the line of sight. Here you were not shown the views that you see habitually and this is enough to render an object almost unrecognizable.

Glasses and Knives

This is quite possible to achieve by arranging the knives as shown in Fig. 306.

Figure 306

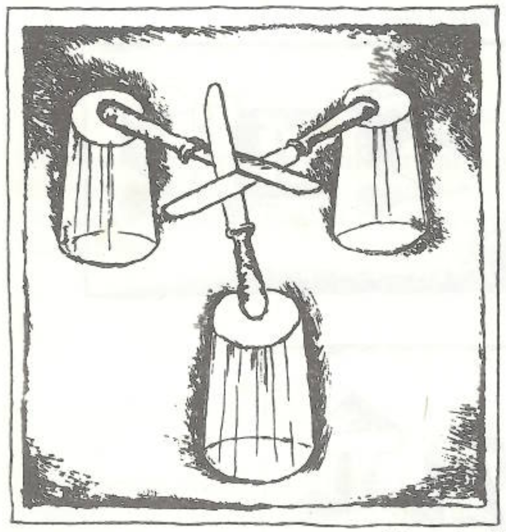
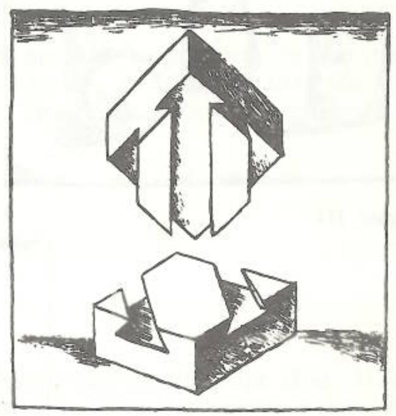


Figure 307



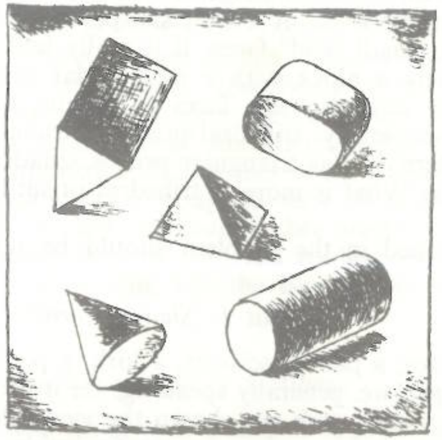
How Is It Achieved?

The way out is very simple, as can be seen from Fig. 307.

One Plug for Three Holes

The suitable plugs are shown in Fig. 308.

Figure 308



Further "Plug" Puzzles

In this case, the plugs are more complicated (Figs. 309, 310, 311).

Figure 309

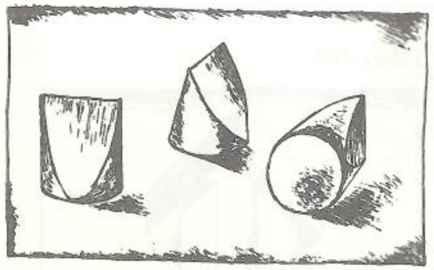


Figure 310

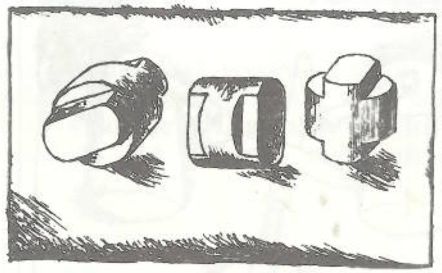


Figure 311

Two Cups

The cup that is 1 1/2 times wider would (with the same height) have $(1\ 1/2)^2$, i.e. 2 1/4 times more volume. Since it is only half the height of the other cup, in the final analysis it still holds more than the taller cup.

How Many Glasses?

A comparison of the first and third shelves shows that they differ only in that the third shelf contains one more middle-sized vessel whilst the three small vessels are missing. The total capacity of the vessels on each shelf being the same, it's obvious that the capacity of one middle-sized vessel equals that of the three small ones. The middle-sized vessel thus equals three glasses. It only remains now to determine the capacity of a large vessel. By replacing all the middle-sized vessels on the first shelf by the appropriate number of glasses we get one large vessel and 12 glasses on the top shelf.

Comparison with the second half yields that one large vessel holds 6 glasses.

Two Saucepans

The two saucepans are geometrically similar. Given that the larger saucepan holds 8 times more, all of its dimensions are twice larger: it's twice higher and wider. Its surface area must then be 2×2 times larger, because the surfaces of similar bodies relate to each other as the squares of their linear dimensions. The thickness of walls being the same, the weight of a saucepan depends on its surface area. The answer is therefore that the larger pan is *four times* heavier.

Four Cubes

We must place the three smaller cubes on one pan, and the largest one on the other. It's easily verified that the balance will be in equilibrium. Let's show that the total volume of the three smaller cubes equals that of the largest one. This follows from the relationship

$$6^3 + 8^3 + 10^3 = 12^3,$$

i.e.

$$216 + 512 + 1{,}000 = 1{,}728.$$

Half-Full

The simplest way is to tilt the barrel so that the water reaches the edge (Fig. 312). If some of the bottom shows above the surface, however little, the barrel is less than half-full. If, on the contrary, the bottom is well below the surface of the water, the barrel is more than half-full. Finally, if the upper edge of the bottom is exactly on the water level, the barrel is exactly half-full.

Figure 312

Which Is Heavier?

Let's imagine the right cube as consisting of small cubes, each containing a ball. It's easily seen that the large ball occupies the same proportion of the large cube's volume as each small ball occupies of the smaller cube's volume. We can readily work out the number of these small balls and cubes: $6 \times 6 \times 6 = 216$. The total volume of the 216 balls accounts for the same share of the 216 cubes as the big ball relative to the big cube. It follows that both boxes contain the same amount of metal, and hence their weight is the same.

Tripod

A tripod can always touch the floor with each of its three legs, because through any three points in space one can draw a plane, and only one at that. This explains why a tripod doesn't rock. This problem, you see, is purely geometrical and not physical.

That is why tripods are so convenient as supports for field instruments and cameras. A fourth leg wouldn't make the support any more stable.

How Many Rectangles?

225.

Chessboard

The chessboard contains more than 64 squares. Apart from the small black and white squares there are the larger squares consisting of 4, 9, 16, 25, 36, 49, and 64 unit squares. These must be taken into account as well.

Unit Squares	Number on Chessboard
1	64
4	49
9	36
16	25
25	16
36	9
49	4
64	1
Total	204

Thus, the chessboard contains 204 differently arranged squares of different sizes.

A Brick

If you thought the toy brick weighs 1 kilogramme, i.e. only a quarter lighter, you would be wrong. It's not only a quarter the length, but also a quarter the width and a quarter the height of a standard brick, therefore its volume and weight is $4 \times 4 \times 4 = 64$ times less.

Consequently, the correct answer is: the toy brick weighs $4{,}000 \div 64 = 62.5$ grammes.

A Giant and a Dwarf

Now you are well equipped to solve this problem correctly. Since human bodies are approximately similar, the giant would be eight times heavier, not twice as heavy.

The tallest giant ever recorded was a man from Alsace in Germany. He was 275 centimetres high, a metre higher than an average man. The smallest dwarf was under 40 centimetres, i.e. he was seven times smaller than the Alsatian giant. Therefore, if the giant were to stand on one pan of a balance, $7 \times 7 \times 7 = 343$ or a whole crowd of dwarfs would have to stand on the other to balance.

Along the Equator

We take the man to be 175 centimetres high and denote the Earth's radius by R. We thus have

$$2 \times 3.14 \times (R + 175) - 2 \times 3.14 \times R = 2 \times 3.14 \times 175 = 1{,}100 \text{ cm},$$

i.e. about 11 metres. Interestingly enough, the result is independent of the globe's radius.

Through a Magnifying Glass

If you believe that the magnifying glass will make the angle look as if it were $1\,1/2° \times 4 = 6°$, you put your foot in it. Viewing through a magnifying glass doesn't make

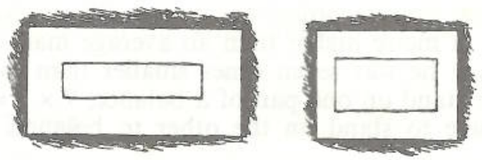
the angle any larger. True, the arc subtending the angle increases, but the radius of the arc increases as much, with the result that the central angle remains the same (Fig. 313).

Figure 313

Similar Figures

Not infrequently, both questions are answered in the affirmative. In actual fact, only the triangles are similar. For triangles to be similar it's sufficient for the angles to be equal and since the sides of the inner triangle are parallel to those of the external one, the angles are equal. With other polygons it's not sufficient only to have equal angles (or parallel sides which is mathematically the same). It is also necessary that their *sides* be *proportional.* For the internal and external rectangles of the frame this is only the case for squares (more generally, for rhombs). In any other cases, however, the sides of the external rectangle are not proportional to the sides of the internal rectangle, and hence the figures are not similar. This stands out especially for rectangular frames with

Figure 314



wide planks as shown in Fig. 314. In the left frame the ratio of the external sides is 2 : 1, and of the inner sides 4 : 1. In the right frame the ratio of the external sides is 4 : 3, and of the internal sides 2 : 1.

The Height of a Tower

To work out the height of the tower we should at first measure as accurately as possible the height of the tower and the length of its base in the picture. Suppose the height in the picture is 95 centimetres and the base is 19 centimetres. Now measure the base of the real tower, which is, say, 14 metres.

Then you argue as follows. The picture and the real tower are similar, the height-to-base ratio in reality and in the picture are the same. The first ratio is $95:19=5$, then you conclude that the height of the real tower is five times larger than its base: $14 \times 5 = 70$ metres, i.e. the tower is 70 metres high. It's worth noting that the method

only works with pictures that don't distort proportions which is often the case with inexperienced cameramen.

A Strip

There are 1,000,000 square millimetres in 1 metre. Each thousand 1-mm squares arrange along a line span 1 metre, and a thousand thousand 1-mm squares give 1,000 metres, i.e. 1 kilometre. Thus the strip will be 1 kilometre long.

A Column

The answer is striking: the column will be... 1,000 kilometres high.

Let's test it mentally. There are $1000 \times 1000 \times 1000$ cubic millimetres in 1 cubic metre. Each thousand 1-mm cubes stacked one upon another gives a 1-metre column. Multiplied by 1,000 this gives 1,000 metres = 1 kilometre. Multiplying by the last 1,000, we obtain 1,000 kilometres.

Sugar

Use your imagination. Suppose for simplicity that the lumps of pressed sugar are 100 times larger across than the particles of granulated sugar. Now imagine that all the granules in the granulated sugar were enlarged 100 times together with the glass containing them. The capacity of the glass would be increased $100 \times 100 \times 100$, i.e. one million times, as would the weight of the sugar. Let's take a normal glassful of this enlarged granulated sugar, i.e. one millionth part of the contents of the giant glass. Clearly, it will weigh as much as a normal glassful of conventional granulated sugar. But then, what is the enlarged granulated sugar? It's nothing but pressed sugar. Accordingly, a glassful of pressed sugar has the same weight as that of granulated sugar.

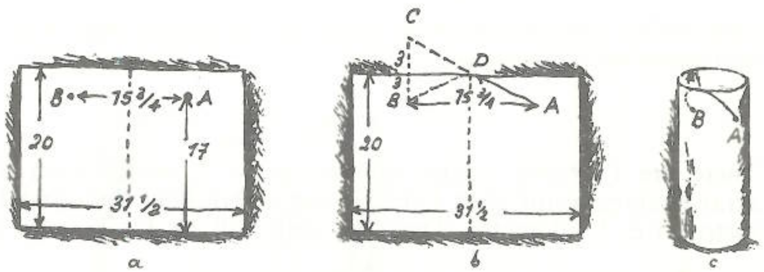
If we had made the magnification 60-fold instead of 100-fold or any other magnification, the situation wouldn't have changed in the least. The key thing here is that the pieces of pressed sugar are regarded here as being geometrically similar to the particles of granulated sugar and are at that arranged in a similar manner. The assumption is not strict, but it's fairly close to reality if the lumps are irregular.

The Path of a Fly

Let's make the sides of the cylinder jar into a flat surface. We'll obtain a rectangle (Fig. 315*a*) 20 centimetres high with the base equal to the circumference of the jar, i.e. $10 \times 3\,1/7 = 31\,1/2$ centimetres (approximately). On this rectangle we now can mark the positions of the fly (*A*) and honey drop (*B*).

Now to find the point at which the fly must cross the edge we'll proceed as follows. We'll draw a line from *B* (Fig. 315*b*) at a right angle to the upper side of the rectangle and continue it an equal distance beyond the edge. We obtain point *C* which we connect with a line to *A*. Point *D* will be where the fly must cross the edge to the other side, the path *ADB* being the shortest.

Figure 315

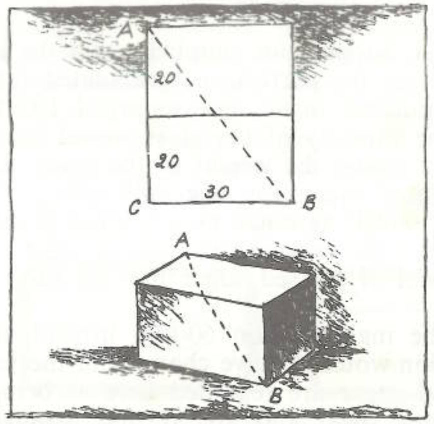


Having found the shortest way on the rectangle, we'll again make it into a cylinder and find out how our fly must walk to get to the honey drop in the shortest time possible (Fig. 315*b*).

The Path of a Beatle

We'll mentally turn the upper face of the stone so that it lies in the same plane as the front face (Fig. 316). The shortest route then is the line connecting *A* and *B*. What is

Figure 316



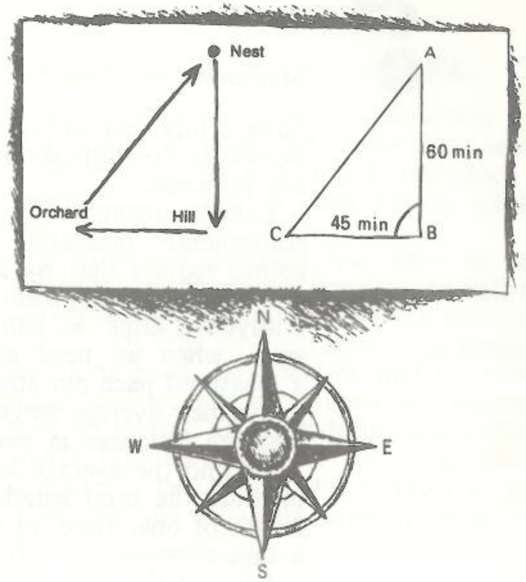
its length? We have the right triangle *ABC*, where $AC = 40$ cm, $CB = 30$ cm. According to the Pythagorean theorem, *AB* must be 50 cm, because $30^2 + 40^2 = 50^2$. So the shortest path $AB = 50$ cm.

A Bumble-Bee's Travels

The problem would be a "piece of cake", if we knew the time taken by the bumble-bee to cover the distance from the orchard to its nest. Geometry will help us work this out.

Let's draw the path of the insect. We know that it flew at first "due south" for 60 minutes. Then it flew for 45 minutes "due west", i.e. at right angles to the first leg, and finally it flew back to its nest by the shortest path possible, i.e. along a straight line. We thus obtain the right triangle *ABC* with two known legs, *AB* and *BC*, the hypotenuse *AC* remaining to be determined.

Figure 317



Geometry teaches that if one leg of a right-angled triangle is three units long, the other leg four units long, then the hypotenuse is exactly five units long.

For example, if legs are 3 metres and 4 metres, then the hypotenuse is 5 metres; if 9 and 12 kilometres, then the hypotenuse is 15 kilometres, and so forth. In our case one leg is 3×15 minutes of flight long and the other 4×15 minutes long, hence the hypotenuse $AC = 5 \times 15$ minutes of flight long. We have thus found that the bumble-bee took 75 minutes, i.e. 1 1/4 hour, to cover the distance from the orchard to its nest.

Now it's child's play to figure out how long our bumble-bee had been away from its nest:

Flights: $1 + 3/4 + 1\ 1/4 = 3$ hours.
Stops: $1/2 + 1\ 1/2 = 2$ hours.
Total: $3 + 2 = 5$ hours.

The Foundation of Carthage

Since the surface area of the hide was 4 square metres, or 4 million square millimetres, and the belt thickness was 1 millimetre, the total length of the belt (clearly, Dido had it cut in a spiral) was 4 million millimetres, or 4,000 metres, i.e. 4 kilometres long. A belt this long can encircle a square area of 1 square kilometre, or a round area of 1.3 square kilometres.

Without a Tape-Measure

26

*Measuring by Paces**

Since a tape-measure is not always at hand, it pays to be able to do without one where approximate estimates are sufficient.

Longish distances, for instance during hikes, can be conveniently measured by paces. But this does of course require that we know how long our paces are and could count them. Admittedly, paces are not always the same: we can walk with short steps or long steps, when we need to. But still when we walk at a measured pace our steps are about similar, and if we know their average length, we can without much error measure distances in paces.

To find the average length of your pace you should measure the total length of many paces and find the length of one. Here, of course, we cannot do without a tape-measure.

Lay out the tape on a smooth piece of ground and measure a 20-metre stretch. Draw in the line and remove the tape. Now walk along the line in your normal way and count the number of paces you have. It's possible that the stretch of ground you measured does not contain an integer number of paces. Then, if the remainder is shorter than a step, it can be simply discarded; if it's longer than a step, the remainder is taken to be a whole step. Dividing the total length of 20 metres by the number of paces gives the average length of one pace. This number should be remembered so that, if necessary, it might be used for measuring.

In order not to lose count of paces you can, especially over long distance, use the following trick. Count up to 10 and then tick off a finger of your left hand. After the five fingers of the left hand have been ticked off, i.e. 50 paces covered, start ticking off the fingers on the right hand. We can in this way count up to 250, and then start from the very beginning remembering how many times have we ticked off all the fingers of the right hand. For example, if having covered a certain distance you've ticked off all the fingers of the right hand twice and you end up with three fingers more ticked off on the right hand, and

* We will call two steps 1 pace.

four on the left, you've made

$2 \times 250 + 3 \times 50 + 4 \times 10 = 690$ paces.

You should also add the paces you made after the last finger of your left hand has been ticked off.

By the way, there is an old rule which says that the length of an average step of an adult equals the distance from floor to his eyes.

Another old practical rule refers to walking speed: a man covers as many kilometres in half an hour as he makes paces in 3 seconds. We can easily show that the rule is only true for one rather large length of pace. Let the pace length be x metres, and the number of paces made in 3 seconds be n. Then in 3 seconds the walker goes nx metres, and in half an hour (1,800 seconds) he travels 600 nx metres, or 0.6 nx kilometres. For this distance to be equal to the number of paces made in 3 seconds the following equality should hold

$0.6\ nx = n$, or $0.6\ nx = 1$.

Hence $x = 1.66$ metres.

If the first old rule relating pace length to man's height, then the second rule is only valid for people of average height, about 175 centimetres.

Living Scales

To measure objects appropriately without a tampe-measure, you can proceed as follows. Measure by

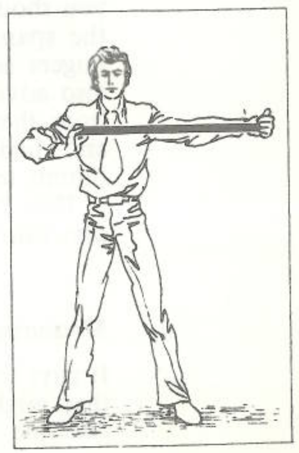


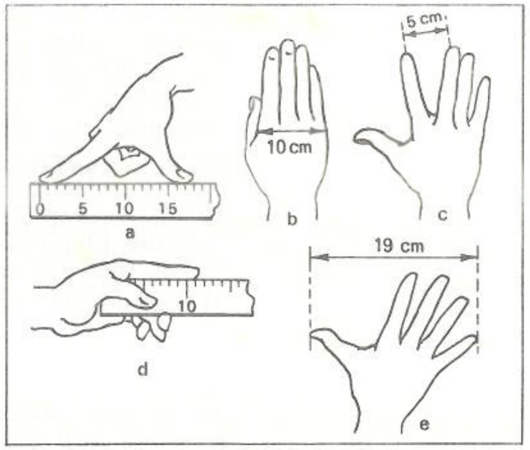
Figure 318

a stick or a rope the length from the end of your outstretched arm to your opposite shoulder (Fig. 318) – in adult it's about a metre. Another way of getting an approximate metre is to mark off six times the distance from your thumb to forefinger, separated as widely as possible (Fig. 319*a*).

The latter piece of advice introduces us to the art of measuring "with bare hands". Initially, you should only measure parts of your hand and remember the results.

Which parts then should be measured? Above all, its width, as shown in Fig. 319*b*. In adults it's about 10 centimetres, but it varies from person to person and

Figure 319



you should know its exact value. Then it pays to know the span between the ends of your middle and index fingers separated as wide as possible (Fig. 319*c*). It is also advisable to know the length of your index finger from the base of your thumb, as shown in Fig. 319*d* and, finally, the width of your outspread palm from thumb to little finger, as shown in Fig. 319*e*.

This "live scale" will enable you to estimate the dimensions of small objects.

Measuring with Coins

It pays to know the size of your national coins, because they might be of help in measuring objects. By way of example, the Soviet coins have convenient dimensions:

1 kopeck piece is exactly 1 1/2 centimetres across, 5 kopeck piece is 2 1/2 centimetres across, and so on. Remember the diameters of your coins!

Figure 320

Guessing Domino Points

The trick is based on a dodge that can't be guessed.

You could surprise your friends by saying that you'll guess the drawn domino's points from an adjoining room. For better effect suggest they blindfold you. Really, one of your friends draws the piece and asks you to guess its points and you, from the adjacent room, give them the right numbers straight away and without so much as a glance at it or your friends.

What is the idea behind the trick?

Disappearing Line

Copy out the figure in Fig. 321 very accurately. Cut the ring out, apply it to the ring in the figure and turn it counterclockwise so that the severed part of each line registers with the remains of a neighbouring one. You'll witness something enigmatic: instead of the 13 lines that were there before the figure will only show 12. One line will have disappeared. Where to?

The reverse operation brings the line back. Where from?

Figure 321

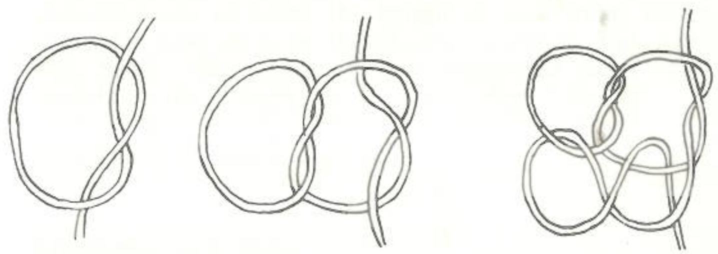
A Mysterious Knot

We'll now turn to trick with things.

Here is a curious trick that could surprise your friends.

Take a piece of string about 30 centimetres long (Fig. 322) and make a loose knot on it as shown on the

Figure 322



left of the accompanying figure. Add a second loop (see the knot in the middle). You're sure to expect that tightening the string now will give you a good double knot. But to be on the safe side we'll make the knot

358–359

smarter by threading one of the loose ends through both loops as shown on the right.

All the preparations over, we can proceed to the main part of the trick. Take hold of one end of the string and offer the other to your friend. Pull! You'll discover something neither you nor your friend expected: instead of an involved knot you'll have a smooth piece of string! The knot will have gone.

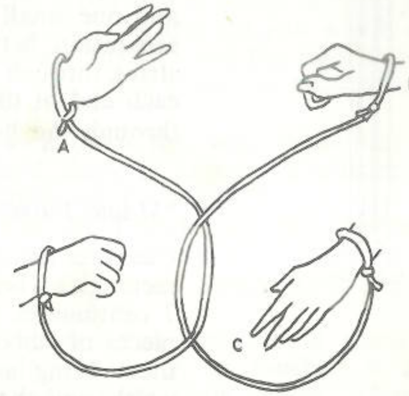
The trick is a success if only you make the third loop exactly as shown. So examine the knots in the figure carefully.

Escaping

Bind your friends (A and B) as shown in Fig. 323.

Is it possible to set the friends free without cutting the strings?

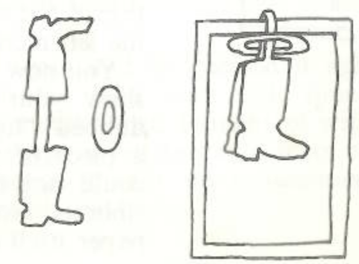
Figure 323



A Pair of Boots

Take a sheet of strong paper and cut out a frame, a pair of boots and an oval ring as shown in Fig. 324. The hole in the oval ring is the size of the width of the

Figure 324



frame, but narrower than the legs of the boots. Therefore, if you are asked to hang the boots on the frame as shown in the figure, you'll obviously think that it's impossible.

But it is possible. How?

Figure 325

Corks on a Ring

There is a ring of strong paper, on which two corks hang suspended from a short piece of string with a wire ring slung on the string as shown in Fig. 325.

Remove the corks from the paper ring.

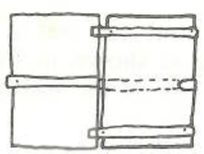
Figure 326

Two Buttons

Figure 326 shows a sheet of paper with two long cuts and one small oval hole that is a bit smaller than the separation between the long cuts. Thread a piece of string through the hole and the cuts and tie a button to each end of the string so that the buttons won't pass through the hole.

"*Magic Purse*"

Figure 327



Cut two rectangles out of a sheet of cardboard, the rectangles being the size of a notebook, say 7 centimetres long and 5 centimetrės wide. Get three pieces of ribbon (paper strips will do as well), two of them being a centimetre longer than the rectangles' width, and the third a centimetre longer than twice the width of the rectangles. Glue the ribbons to the rectangles as shown in Fig. 327. In so doing, bend the ends of the shorter ribbons under the right rectangle and glue them to it, and glue the other ends to the back side of the left rectangle. Glue the end of the longer ribbon to the outside of the right rectangle, thread the ribbon under it, then round the outside of the left rectangle and glue its end under this rectangle.

You now have your "magic" purse. Using it, you can show your friends a fascinating trick that can be dubbed "live paper" or something like that. Take a piece of paper signed by your friend so that you could not replace it. Stick the paper under the two ribbons. Now close the purse, reopen it. Presto! The paper itself emerged from under both papers but (what

is beyond belief!) it got under the centre ribbon on the opposite side of the purse.

Explain.

Guessing Matches

In my childhood I was much amazed by a trick shown to me by my elder brother. Going about my business in my room once I heard in the adjacent room some laughter that wetted my curiosity. I peered in and saw my brother and his student friend laughing.

"Come in, boy! We'll show you an interesting trick."

That was exactly what I wanted. My brother was a great wag.

"Look here," said Alex arranging matches on a table in a random manner, "I put ten matches down at random. Now I'll go into the kitchen and you think of a match here. When you're ready, call me. I'll just take a look at the matches and tell you which one it was."

"And he'll say that it's not the right one," the guest interrupted, "No, some control is needed here, we can't do without it!"

"Okay, we'll do it this way: when he's thought of a match he'll show it to you. You'll be a witness."

"That's different. Let's start."

My brother left. I made sure that he was gone and couldn't peer into the keyhole. Then I thought of a match, showed it to the student without touching and called out: "Ready!"

I didn't believe that Alex would guess the match since I hadn't so much as touched it and all the matches remained in their places. How could he possibly guess?

But he did! He just came up to the table and without a moment's hesitation pointed to the match. I even tried hard not to look at it in order not to betray myself. My brother even didn't glance at me and still guessed... Well I never!

"Want another go?"

"Of course!"

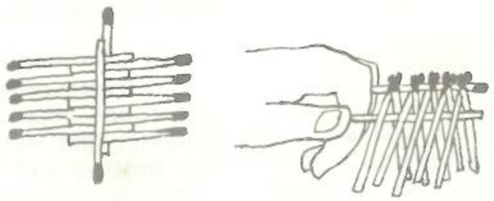
We did it again and he guessed it again! A dozen times we did the trick and each time my brother indicated the match I had thought of without mistake. I was on the verge of bursting into tears as I was dying to know the secret. Finally, my tormentors took pity on me and revealed the trick.

What was it?

Eleven Matches on One

Arrange a dozen matches as shown in Fig. 328 and try and raise them all by lifting the sticking out end of the lower match. If you are adroit enough, the trick will work, if not, practice a bit.

Figure 328



Is It Easy?

What do you make of what is shown in Fig. 329: is it easy to lift a match with two other matches?

Figure 329

It seems easy as pie, doesn't it? But try to do it yourself and you'll find that it requires patience and practice, the slightest jolt will turn the match over.

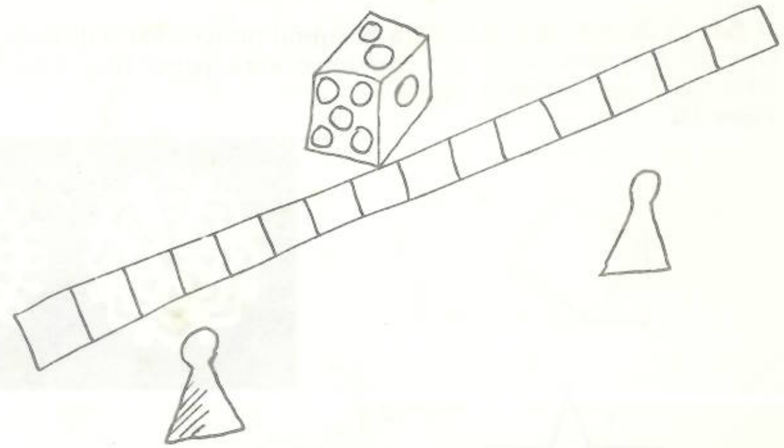
On a Narrow Path

Draw a narrow path of 15 squares on a sheet of paper (Fig. 330).

For the game you'll need a die and two counters or draughts (two coins or buttons will also do).

The rules of the game are simple. Each of the two partners places his counter at the either end of the path. Then they take turns to throw the die (the one with the largest number of points begins). Each partner shifts his counter forward by so many squares as there are points shown by the die, but he is not entitled to skip the square occupied by his opponent's counter. If the die shows more points than there are free squares left, he must retreat by the excess number of squares.

Figure 330



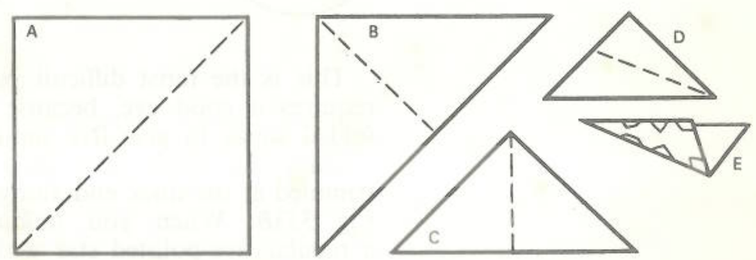
The counters thus alternatively appear in the middle of the path or at its extremes. The game ends when one of the partners is forced to leave the path. The winner is the one who stays.

Star-Like Patterns

Some people maybe don't know that just with a pair of scissors, without any drawing instruments it's possible to manufacture an infinite variety of beautiful paper patterns.

Take a sheet of white paper and fold it several times as shown in Fig. 331, *A*, *B*, *C*, *D*, and *E*. Having reached the stage *E*, cut the folded paper along some ornate lines, e.g. like those shown in the figure.

Figure 331

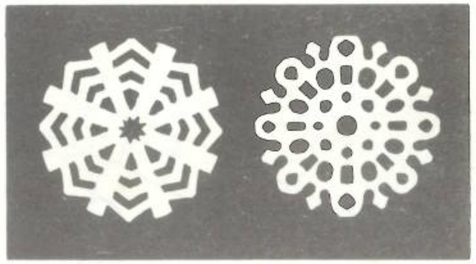


Now unfold and smooth out the paper to obtain

27

a beautiful design that will look yet better when glued on some dark paper (Fig. 332).

Figure 332

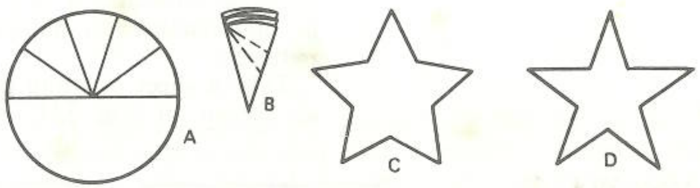


Five-Pointed Star

Can you cut out a paper five-pointed star? It is not simple and takes some practice, otherwise your star will have unequal points. There are two methods of cutting good regular stars.

In the first method, a circle is drawn on a sheet of paper using a pair of compasses or just a saucer. The circle is cut out and folded in two, the semicircle obtained is then folded four times as shown in Fig. 333*A*.

Figure 333



This is the most difficult part of the problem as it requires a good eye, because the semicircle must be folded so as to give five similar segments.

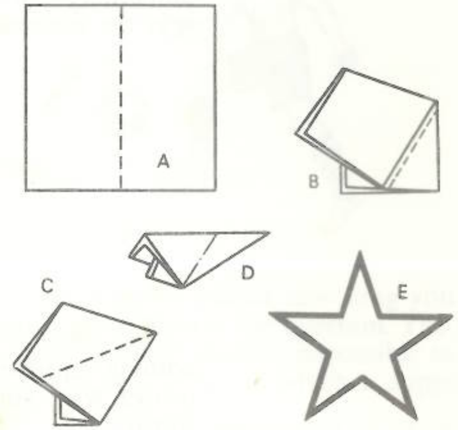
Once the circle has been folded correctly, it is trimmed at the thick end along one of the dash lines in Fig. 333*B*. When you unfold the paper, you get a regular five-pointed star with either shallow or deep notches (Fig. 333, *C* and *D*) depending on how you trimmed the semicircle.

The second method is perhaps simpler as we start with a square, not a circle. To begin with, a square sheet of paper (Fig. 334*A*) is folded in two. Then three

364–365

more folds are made as shown in Fig. 334, *B*, *C*, and *D*. The dot-and-dash line in Fig. 334*D* indicates the trim line. The resultant star is depicted in Fig. 334*E*.

Figure 334



What's Written Here?

Something is written in the circle (Fig. 335). Looking at it in the conventional way, you will, of course, perceive

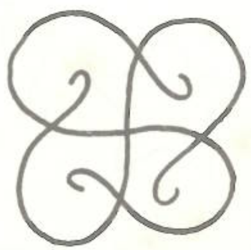
Figure 335

nothing sensible. However, if you view the circle in the proper way, you'll be able to read the words. Which words?

It's Simple. Or Is It?

Look carefully at the design in Fig. 336 and try to remember it so that you could reproduce it from memory. Have you remembered it?... Then begin drawing. At first, mark out the four end points of the

Figure 336



two lines. The first curve will probably come out adequately. Okay! Now draw the second curve. But the line is stubbornly unsuccess. This seemingly easy job does not now appear to be so easy.

On Which Foot

Look at Fig. 337 and say which leg the footballer is standing on, the right or left.

He seems to be standing on his right leg, but you can say that he is standing on his left leg with the same

Figure 337

measure of certainty. No matter how long you view the drawing you'll never answer the question. The artist has done his job so skillfully that it's impossible to establish which leg is doing the kick and which is supporting the man.

Perhaps you are asking, "But which is which, really?" I don't know. The artist doesn't know, either. It will remain an unsolvable mystery for ever.

How Many Fish?

You see a strange drawing here (Fig. 338). It might seem that the angler has caught nothing so far. But

Figure 338

look very attentively at the figure: three big fish are already here. Where are they?

Where Is the Tamer?

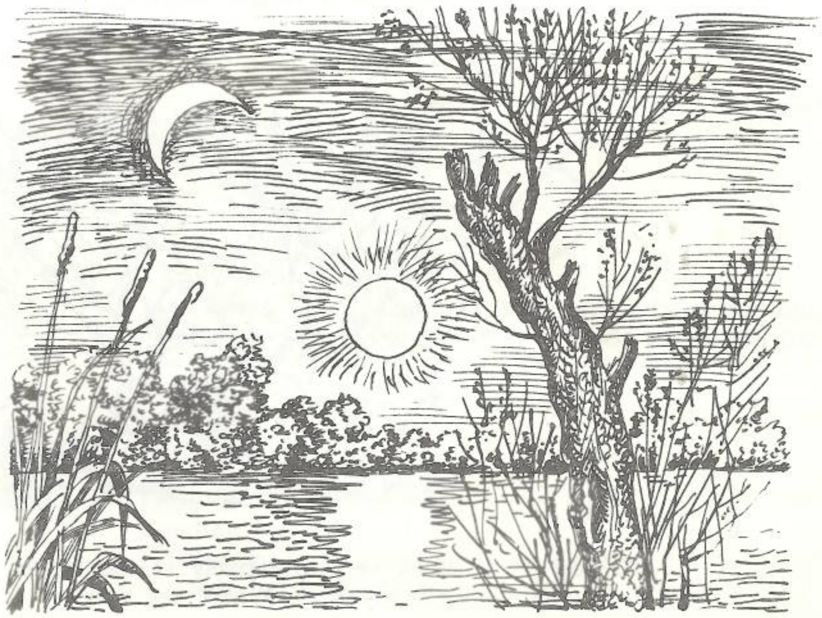
Where is the tamer of this tiger (Fig. 339)?
His portrait does appear in this figure. Find it.

Figure 339

Sunset

Look at the picture (Fig. 340) – a sunset – and say if it is correct.
The picture contains one incongruity that you should find.

Figure 340



Moonset

Figure 341 represents a tropical moonset. Is the picture correct?
Perhaps you can see something incongruous about it.

Figure 341

27 *Guessing Domino Points*

Here you use a secret language known only to you and one of your friends with whom you've preliminarily worked it out. You've agreed, say, that the following words have the meaning:

"I", and "my" – 1
"you", and "your" – 2
"he", and "his" – 3
"we", and "our" – 4
"they", and "their" – 5
"it", and "its" – 6

These conventions may be illustrated by some examples. Let the piece in question be 4–6. In that case, your companion calls out:

"We've thought of a piece, guess it."

In the secret language this will be: "we" – 4, "it" – 6, hence 4–6.

If the piece is 1–5, then your companion utters:

"I think this time they are difficult to guess."

Those uninitiated will never guess that these words contain the secret message: "I" – 1, "they" – 5.

A further example: 4–2. What "message" should your companion send? Something like this:

"Now we've thought of such a piece that you'll never guess."

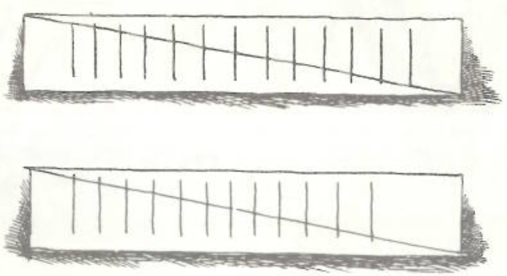
But how about the blanks? It might be denoted by some other word, say, friend. If the domino is 0–4, your fellow-conspirator calls out:

"Guess, friend, what we've thought now."

Disappearing Line

The jist of the trick is better illustrated in a simplified form. Figure 342 shows a piece of cardboard with 13 lines. The sheet is cut along the diagonal. If you shift one part relative to the other as shown in the figure, then instead of 13 lines you'll get only 12,

Figure 342



one will have disappeared. In this case we can easily see where it has gone since each of the 12 new lines has got somewhat longer than before, namely a 1/12th longer.

Clearly, when shifted one of the lines has been divided into 12 parts each of which went to lengthen the other lines. Reverse shifting brings the vanished line back into being by shortening the other lines.

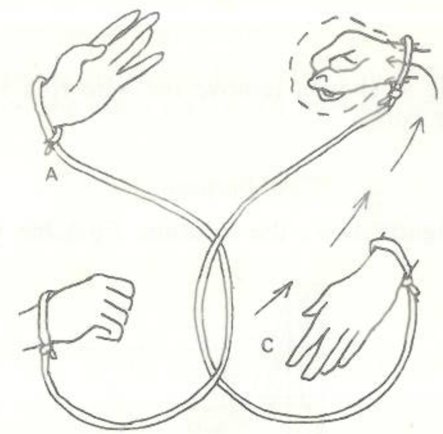
The lines in Fig. 321 are arranged in a circle and possess the same property: shifting the circle through an appropriate angle kills one of the lines (it is "smeared" over the other 12).

Escaping

Yes, it is.

String *A* is taken at point *C* and threaded through loop *B* in the direction indicated

Figure 343

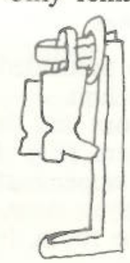


by the arrow. When a sufficient length of the string has already been tucked in, hand *B* is put into the loop formed and, when string *A* is pulled, the friends separate.

A Pair of Boots

The accompanying figure explains the answer. The frame is folded and the ring is put on the folded ends. Then the unfolded figure of "double boots" is threaded in-between the folded ends, refolded and pushed to the bend in the frame. Finally, the ring is slided onto the end. It now only remains to unfold the frame.

Figure 344



Corks on a Ring

Now that you know how to solve the previous problem, this one will be smooth sailing.

Figure 345

Fold the paper ring as shown, remove the wire ring by sliding it away to the free end, and remove the corks.

Two Buttons

The accompanying figure shows the solution. Fold the paper so that the upper and

Figure 346

lower ends of the narrow strip between the cuts will coincide. Then thread the strip through the oval hole and remove the buttons through the loop.

"Magic Purse"

The point is that you open the purse from the opposite side.

Guessing Matches

I was simply made a fool of. The student who pretended to control the guessing was my brother's conspirator and gave signals to him.

But how? That was the trick of it. It turned out that the matches were arranged not at random: the brother had so arranged them (Fig. 347) that the pattern would resemble the outlines of a human face. So the upper match marked the hair, the next below, the forehead, further down were the eyes, nose, mouth, chin, neck, and on either side the ears. When Alex walked into the room, he first of all cast a glance at

Figure 347

the "controller" who touched an appropriate feature of his face with his hand, thus indicating which match had been thought of.

What's Written Here?

Bring the ring up to your eyes as shown in Fig. 348. You'll clearly read the words MIR PUBLISHERS, then turn the circle and you'll now see the word PERELMAN.

Figure 348

The letters are extremely elongated and narrow, therefore it's impossible to make them out in the conventional way. In the suggested method the letters become much shorter, their width being the same. This imparts a normal aspect to the letters, thus simplifying reading them.

How Many Fish?

I'll help you discern the catch. One fish is one the angler's back, head down. Another is between the float and the end of the fishing rod. The third is under his feet.

Where Is the Tamer?

The tiger's eye doubles as the tamer's eye, the tamer is looking in the opposite direction.

Sunset

The incongruity is that the convex part of the crescent faces in the opposite direction from the sun, not towards it. The moon is illuminated by the sun, hence it by no means can be facing the sun with it dark side...

The French astronomer Flammarion wrote: "Most of painters are ignorant of this, because a year never passes without a large number of inverted crescents appearing at the Paris Salon".

27

Moonset

Strange as it may seem, the crescent in Fig. 341 is depicted correctly. It's a *tropical* landscape where the position of the crescent differs from that in the higher latitudes, where the hump of the new moon faces to the right and that of the old moon faces to the left. But in tropical lands the crescent hangs *horizontally* in the sky.

This is explained as follows. In the higher latitudes the sun and moon (indeed all the luminaries in general) execute their diurnal motion in inclined circles. Therefore during the evening the sun casts slanting rays at the moon, illuminating it from the right or left, so that the crescent faces to the right or left. But on the equator the celestial bodies move in normal trajectories with the result that the sun illuminating the moon sets below the horizon directly beneath the moon and not to the left or right of it. The moon is thus illuminated from below and that is why the crescent has the form of a gondola, as shown in the figure.

The End